Pre-Algebra: Unit 4
Problem Solving with Geometry
Course Philosophy/Description

In mathematics, students will focus on the application of concepts, skills and understandings. Students will solve problems involving the key knowledge and skills identified by the NJSLS; express mathematical reasoning, construct a mathematical argument and apply concepts to solve real world problems. The balanced math instructional model will be used as the basis for all mathematics instruction.

Pre-Algebra consists of the following domains: Ratios and Proportional Relationships (RP), The Number System (NS), Expressions and Equations (EE), Geometry (G), and Statistics and Probability (SP). Instructional time should focus on four critical areas: (1) building understanding of and applying proportional relationships; (2) building understanding of real numbers and working with expressions and linear equations; (3) solving problems involving scale drawings, informal geometric constructions, area, surface area, and volume; and (4) drawing inferences about populations based on samples.

1) Students extend their understanding of ratios and develop understanding of proportionality by exploring a wide variety of percent problems, including those involving discounts, interest, taxes, tips, and percent increase or decrease. Students solve problems about scale drawings, graph proportional relationships, and understand the unit rate informally as a measure of the steepness of the related line, called the slope. They distinguish proportional relationships from other relationships.

2) Students develop a unified understanding of real numbers, recognizing that numbers are rational or irrational, and understand that rational numbers can be expressed as fractions, decimals, and percents. Students extend mathematical operations to all rational numbers, maintaining the properties of operations and the relationships between operations. By applying these properties, and by viewing negative numbers in terms of everyday contexts, students explain and interpret the rules for mathematical operations with negative numbers. They use the arithmetic of rational numbers as they formulate expressions and equations with variables to solve problems. Students identify irrational numbers as well as a rational approximation for an irrational number.

3) Students work with three-dimensional figures, relating them to two-dimensional figures by examining cross-sections. They solve real-world mathematical problems involving circumference, area, surface area, and volume. In preparation for work with congruence and similarity in Grade 8, students reason about relationships among two-dimensional figures. They use scale drawings and informal geometric constructions to gain familiarity with the relationships between angles formed by intersecting lines. This understanding is deepened by learning transformations. Students are able to identify the sum of the angles in triangles of various configurations. Students understand the statement of the Pythagorean Theorem and its converse and can explain why the Pythagorean Theorem holds. They apply the Pythagorean Theorem to find distances between points on the coordinate plane, to find lengths, and to analyze polygons.

4) Students build on their previous work with single data distributions to compare two data distributions and address questions about differences between populations. They begin informal work with random sampling to generate data sets and learn about the importance of representative samples for drawing inferences.
This ESL framework was designed to be used by bilingual, dual language, ESL and general education teachers. Bilingual and dual language programs use the home language and a second language for instruction. ESL teachers and general education or bilingual teachers may use this document to collaborate on unit and lesson planning to decide who will address certain components of the SLO and language objective. ESL teachers may use the appropriate leveled language objective to build lessons for ELLs which reflects what is covered in the general education program. In this way, whether it is a pull-out or push-in model, all teachers are working on the same Student Learning Objective connected to the New Jersey Student Learning Standards. The design of language objectives are based on the alignment of the World-Class Instructional Design Assessment (WIDA) Consortium’s English Language Development (ELD) standards with the New Jersey Student Learning Standards (NJSLS). WIDA’s ELD standards advance academic language development across content areas ultimately leading to academic achievement for English learners. As English learners are progressing through the six developmental linguistic stages, this framework will assist all teachers who work with English learners to appropriately identify the language needed to meet the requirements of the content standard. At the same time, the language objectives recognize the cognitive demand required to complete educational tasks. Even though listening and reading (receptive) skills differ from speaking and writing (expressive) skills across proficiency levels the cognitive function should not be diminished. For example, an Entering Level One student only has the linguistic ability to respond in single words in English with significant support from their home language. However, they could complete a Venn diagram with single words which demonstrates that they understand how the elements compare and contrast with each other or they could respond with the support of their home language (L1) with assistance from a teacher, para-professional, peer or a technology program.

http://www.state.nj.us/education/modelcurriculum/ela/ELLOverview.pdf
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<tr>
<th>#</th>
<th>Student Learning Objective</th>
<th>NJSLS</th>
<th>Instruction: 8 weeks</th>
<th>Assessment: 1 week</th>
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<tbody>
<tr>
<td>1</td>
<td>Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms. Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.</td>
<td>7.G.B.6, 8.G.C.9</td>
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<td>2</td>
<td>Explain and model the properties of rotations, reflections, and translations with physical representations and/or geometry software using pre-images and resultant images of lines, line segments, and angles.</td>
<td>8.G.A.1</td>
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<td>3</td>
<td>Describe and perform a sequence of rotations, reflections, and/or translations on a two dimensional figure in order to prove that two figures are congruent.</td>
<td>8.G.A.2</td>
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<tr>
<td>4</td>
<td>Use the coordinate plane to locate images or pre-images of two-dimensional figures and determine the coordinates of a resultant image after applying dilations, rotations, reflections, and translations.</td>
<td>8.G.A.3</td>
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<tr>
<td>5</td>
<td>Apply an effective sequence of transformations to determine that figures are similar when corresponding angles are congruent and corresponding sides are proportional. Write similarity statements based on such transformations.</td>
<td>8.G.A.4</td>
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<td>6</td>
<td>Explain a proof of the Pythagorean Theorem and its converse.</td>
<td>8.G.B.6</td>
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<td></td>
<td>Activity</td>
<td>Standards</td>
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<td>--------------------------------------------------------------------------</td>
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<td>7</td>
<td>Apply the Pythagorean Theorem to determine unknown side lengths of right triangles in two and three dimensions to solve real-world and mathematical problems and to determine the distance between two points in the coordinate plane.</td>
<td>8.G.B.7, 8.G.B.8</td>
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<td>8</td>
<td>Use similar triangles to explain why the slope (m) is the same between any two points on a non-vertical line in the coordinate plane.</td>
<td>8.EE.B.6</td>
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<td>9</td>
<td>Apply the properties of integer exponents to write equivalent numerical expressions.</td>
<td>8.EE.A.1</td>
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<tr>
<td>10</td>
<td>Evaluate square roots and cubic roots of small perfect squares and cubes respectively and use square and cube root symbols to represent solutions to equations of the form ( x^2 = p ) and ( x^3 = p ) where ( p ) is a positive rational number; identify ( \sqrt{2} ) as irrational.</td>
<td>8.EE.A.2</td>
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<td>11</td>
<td>Estimate and express the values of very large or very small numbers with numbers expressed in the form of a single digit times an integer power of 10. Compare numbers expressed in this form, expressing how many times larger or smaller one is than the other.</td>
<td>8.EE.A.3</td>
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<tr>
<td>12</td>
<td>Perform operations using numbers expressed in scientific notation, including problems where both decimals and scientific notation are used. In real-world problem-solving situations, choose units of appropriate size for measurement of very small and very large quantities and interpret scientific notation generated when technology has been used for calculations.</td>
<td>8.EE.A.4</td>
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</table>
Research about Teaching and Learning Mathematics

Structure teaching of mathematical concepts and skills around problems to be solved (Checkly, 1997; Wood & Sellars, 1996; Wood & Sellars, 1997)
Encourage students to work cooperatively with others (Johnson & Johnson, 1975; Davidson, 1990)
Use group problem-solving to stimulate students to apply their mathematical thinking skills (Artzt & Armour-Thomas, 1992)
Students interact in ways that support and challenge one another’s strategic thinking (Artzt, Armour-Thomas, & Curcio, 2008)
Activities structured in ways allowing students to explore, explain, extend, and evaluate their progress (National Research Council, 1999)
There are three critical components to effective mathematics instruction (Shellard & Moyer, 2002):

- Teaching for conceptual understanding
- Developing children’s procedural literacy
- Promoting strategic competence through meaningful problem-solving investigations

Teachers should be:

- Demonstrating acceptance and recognition of students’ divergent ideas
- Challenging students to think deeply about the problems they are solving, extending thinking beyond the solutions and algorithms required to solve the problem
- Influencing learning by asking challenging and interesting questions to accelerate students’ innate inquisitiveness and foster them to examine concepts further
- Projecting a positive attitude about mathematics and about students’ ability to “do” mathematics

Students should be:

- Actively engaging in “doing” mathematics
- Solving challenging problems
- Investigating meaningful real-world problems
- Making interdisciplinary connections
- Developing an understanding of mathematical knowledge required to “do” mathematics and connect the language of mathematical ideas with numerical representations
- Sharing mathematical ideas, discussing mathematics with one another, refining and critiquing each other’s ideas and understandings
- Communicating in pairs, small group, or whole group presentations
- Using multiple representations to communicate mathematical ideas
- Using connections between pictures, oral language, written symbols, manipulative models, and real-world situations
- Using technological resources and other 21st century skills to support and enhance mathematical understanding
Mathematics is not a stagnant field of textbook problems; rather, it is a dynamic way of constructing meaning about the world around us, generating knowledge and understanding about the real world every day. Students should be metaphorically rolling up their sleeves and “doing mathematics” themselves, not watching others do mathematics for them or in front of them. (Protheroe, 2007)

Balanced Mathematics Instructional Model

Balanced math consists of three different learning opportunities: guided math, shared math, and independent math. Ensuring a balance of all three approaches will build conceptual understanding, problem solving, computational fluency, and procedural fluency. Building conceptual understanding is the focal point of developing mathematical proficiency. Students should frequently work on rigorous tasks, talk about the math, explain their thinking, justify their answer or process, build models with graphs or charts or manipulatives, and use technology.

When balanced math is used in the classroom it provides students opportunities to:

- solve problems
- make connections between math concepts and real-life situations
- communicate mathematical ideas (orally, visually and in writing)
- choose appropriate materials to solve problems
- reflect and monitor their own understanding of the math concepts
- practice strategies to build procedural and conceptual confidence

Teacher builds conceptual understanding by modeling through demonstration, explicit instruction, and think alouds, as well as guiding students as they practice math strategies and apply problem solving strategies. (whole group or small group instruction)

Teacher and students practice mathematics processes together through interactive activities, problem solving, and discussion. (whole group or small group instruction)

Students practice math strategies independently to build procedural and computational fluency. Teacher assesses learning and reteaches as necessary. (whole group instruction, small group instruction, or centers)
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<td>Interviews</td>
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<td>Pressing for Accuracy and Reasoning</td>
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<td>Maintain the Cognitive Demand</td>
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# Educational Technology Standards

8.1.8.A.1, 8.1.8.A.3, 8.1.8.E.1, 8.2.8.C.8

- **Technology Operations and Concepts**
  - **Demonstrate knowledge of a real world problem using digital tools.**
    - **Example:** Students can use math game websites to find the unknown side lengths of right triangles. [https://www.mathgames.com/skill/8.60-pythagorean-theorem](https://www.mathgames.com/skill/8.60-pythagorean-theorem)
    - **Example:** Students use an online tool to reinforce calculating area, radius and circumference of a circle. [https://www.mathgames.com/skill/7.1-circles-calculate-area-radius-circumference](https://www.mathgames.com/skill/7.1-circles-calculate-area-radius-circumference)

- **Use and/or develop a simulation that provides an environment to solve a real world problem or theory.**
  - **Example:** Students can create simulations for finding surface area and volume of cubes, triangular and rectangular prisms. [http://www.shodor.org/interactivate/activities/SurfaceAreaAndVolume/](http://www.shodor.org/interactivate/activities/SurfaceAreaAndVolume/)
    - **Example:** Students can use simulation to determine facts of angle-angle relationships. [http://www.shodor.org/interactivate/activities/Angles/](http://www.shodor.org/interactivate/activities/Angles/) [http://www.visnos.com/demos/basic-angles](http://www.visnos.com/demos/basic-angles)
    - **Example:** Students can use simulation to determine the volume of a cylinder. [https://www.learner.org/interactives/geometry/area_volume2.html](https://www.learner.org/interactives/geometry/area_volume2.html)
    - **Example:** Students can use math game websites to simulate and solve Pythagorean Theorem real world and mathematical problems. [http://www.interactive-maths.com/ladders-up-walls-ggb.html](http://www.interactive-maths.com/ladders-up-walls-ggb.html)
Educational Technology

- Research and Information Fluency
  - Use digital tools and online resources to explore a problem or issue.
    Example: Students can search through Learnzillion, Khan Academy, and other interactive sites for appropriate instructional videos and/or information pertaining to strategies and modeling for applying the Pythagorean Theorem, solving mathematical problems of volume, describing a 2-dimensional figure that results when a 3-dimensional figure is sliced from multiple angles, and understanding congruency and similarity.

- Design
  - Develop a proposal for a chosen solution that include models (physical, graphical or mathematical) to communicate the solution to peers.
    Example: Students can develop an image use to model the properties of rotation, reflection and translation and explain the resultant images.
    http://www.shodor.org/interactivate/activities/3DTransmographer/

    Example: Teachers can use geometry software to draw and construct triangles with given conditions.
    https://www.geogebra.org/m/JHgTXKrt
Career Ready Practices

Career Ready Practices describe the career-ready skills that all educators in all content areas should seek to develop in their students. They are practices that have been linked to increase college, career, and life success. Career Ready Practices should be taught and reinforced in all career exploration and preparation programs with increasingly higher levels of complexity and expectation as a student advances through a program of study.

- **CRP2. Apply appropriate academic and technical skills.**
  Career-ready individuals readily access and use the knowledge and skills acquired through experience and education to be more productive. They make connections between abstract concepts with real-world applications, and they make correct insights about when it is appropriate to apply the use of an academic skill in a workplace situation.

  **Example:** Students will apply prior knowledge when solving real world problems. Students will make sound judgements about the use of geometry software to explore and deepen understanding the concepts of angles, translation, rotation and reflection. Students will also make sound judgements about the use of specific tools, such as rulers and protractors to draw geometric shapes with given conditions.

- **CRP4. Communicate clearly and effectively and with reason.**
  Career-ready individuals communicate thoughts, ideas, and action plans with clarity, whether using written, verbal, and/or visual methods. They communicate in the workplace with clarity and purpose to make maximum use of their own and others’ time. They are excellent writers; they master conventions, word choice, and organization, and use effective tone and presentation skills to articulate ideas. They are skilled at interacting with others; they are active listeners and speak clearly and with purpose. Career-ready individuals think about the audience for their communication and prepare accordingly to ensure the desired outcome.

  **Example:** Students will on a daily basis communicate their reasoning behind their solution paths by making connections to the context and the quantities, using proper vocabulary, along with decontextualizing and/or contextualizing the problem. Students will create representations using protractors to justify facts about angles in a triangle and angles created when parallel lines are cut by a transversal. Students will create simulations for finding surface area and volume of cubes, triangular and rectangular prisms. They will also explain the meaning behind the quantities and units involved. Students will also ask probing questions to clarify and improve arguments.

- **CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.**
  Career-ready individuals readily recognize problems in the workplace, understand the nature of the problem, and devise effective plans to solve the problem. They are aware of problems when they occur and take action quickly to address the problem; they thoughtfully investigate the root cause of the problem prior to introducing solutions. They carefully consider the options to solve the problem. Once a solution is agreed upon, they follow through to ensure the problem is solved, whether through their own actions or the actions of others.
Career Ready Practices

**Example:** Throughout their daily lessons, students will understand the meaning of a problem and look for entry points into solving their problems by analyzing the relationships of the quantities, constraints and goals of the task. Plans for solution paths will be made and have meaning. Students will self-monitor, evaluate and critique their process and progress as they are working and make changes as necessary.

- **CRP12. Work productively in teams while using cultural global competence.**
  Career-ready individuals positively contribute to every team, whether formal or informal. They apply an awareness of cultural difference to avoid barriers to productive and positive interaction. They find ways to increase the engagement and contribution of all team members. They plan and facilitate effective team meetings.

  **Example:** Students will work in collaborative and whole group settings to develop various solutions to math tasks that are presented to them. They will work together to understand the terms of the problem, ask clarifying and challenging questions among each other, and develop agreed upon solutions using a variety of strategies and models. Students will listen to, read and discuss arguments with each other with respect and courtesy at all times and will be willing to assist those that may need assistance. Students will demonstrate and explain to a peer or small group the sequence of transformations with physical representations, explaining the proof of and how to use the Pythagorean Theorem, equations using angles, area and volume.
WIDA Proficiency Levels

At the given level of English language proficiency, English language learners will process, understand, produce or use:

<table>
<thead>
<tr>
<th>Level</th>
<th>Description</th>
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</table>
| 6- Reaching | - Specialized or technical language reflective of the content areas at grade level  
- A variety of sentence lengths of varying linguistic complexity in extended oral or written discourse as required by the specified grade level  
- Oral or written communication in English comparable to proficient English peers |
| 5- Bridging | - Specialized or technical language of the content areas  
- A variety of sentence lengths of varying linguistic complexity in extended oral or written discourse, including stories, essays or reports  
- Oral or written language approaching comparability to that of proficient English peers when presented with grade level material. |
| 4- Expanding | - Specific and some technical language of the content areas  
- A variety of sentence lengths of varying linguistic complexity in oral discourse or multiple, related sentences or paragraphs  
- Oral or written language with minimal phonological, syntactic or semantic errors that may impede the communication, but retain much of its meaning, when presented with oral or written connected discourse, with sensory, graphic or interactive support |
| 3- Developing | - General and some specific language of the content areas  
- Expanded sentences in oral interaction or written paragraphs  
- Oral or written language with phonological, syntactic or semantic errors that may impede the communication, but retain much of its meaning, when presented with oral or written, narrative or expository descriptions with sensory, graphic or interactive support |
| 2- Beginning | - General language related to the content area  
- Phrases or short sentences  
- Oral or written language with phonological, syntactic, or semantic errors that often impede the communication when presented with one to multiple-step commands, directions, or a series of statements with sensory, graphic or interactive support |
| 1- Entering | - Pictorial or graphic representation of the language of the content areas  
- Words, phrases or chunks of language when presented with one-step commands directions, WH-, choice or yes/no questions, or statements with sensory, graphic or interactive support |
# Language Development Supports For English Language Learners

To Increase Comprehension and Communication Skills

## Environment

- Welcoming and stress-free
- Respectful of linguistic and cultural diversity
- Honors students’ background knowledge
- Sets clear and high expectations
- Includes routines and norms
- Is thinking-focused vs. answer-seeking
- Offers multiple modalities to engage in content learning and to demonstrate understanding
- Includes explicit instruction of specific language targets
- Provides participation techniques to include all learners

- Integrates learning centers and games in a meaningful way
- Provides opportunities to practice and refine receptive and productive skills in English as a new language
- Integrates meaning and purposeful tasks/activities that:
  - Are accessible by all students through multiple entry points
  - Are relevant to students’ lives and cultural experiences
  - Build on prior mathematical learning
  - Demonstrate high cognitive demand
  - Offer multiple strategies for solutions
  - Allow for a language learning experience in addition to content

## Sensory Supports*

- Real-life objects (realia) or concrete objects
- Physical models
- Manipulatives
- Pictures & photographs
- Visual representations or models such as diagrams or drawings
- Videos & films
- Newspapers or magazines
- Gestures
- Physical movements
- Music & songs

## Graphic Supports*

- Graphs
- Charts
- Timelines
- Number lines
- Graphic organizers
- Graphing paper

## Interactive Supports*

- In a whole group
- In a small group
- With a partner such as Turn-and-Talk
- In pairs as a group (first, two pairs work independently, then they form a group of four)
- In triads
- Cooperative learning structures such as Think-Pair-Share
- Interactive websites or software
- With a mentor or coach

## Verbal and Textual Supports

- Labeling
- Students’ native language
- Modeling
- Repetitions
- Paraphrasing
- Summarizing
- Guiding questions
- Clarifying questions
- Probing questions
- Leveled questions such as What? When? Where? How? Why?
- Questioning prompts & cues
- Word Banks
- Sentence starters
- Sentence frames
- Discussion frames
- Talk moves, including Wait Time

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# Building Equity in Your Teaching Practice

How do the essential questions highlight the connection between the big ideas of the unit and equity in your teaching practice?

## Content Integration

Teachers use examples and content from a variety of cultures & groups.

- This unit / lesson is connected to other topics explored with students.
- There are multiple viewpoints reflected in the content of this unit / lesson.
- The materials and resources are reflective of the diverse identities and experiences of students.
- The content affirms students, as well as exposes them to experiences other than their own.

## Knowledge Construction

Teachers help students understand how knowledge is created and influenced by cultural assumptions, perspectives & biases.

- This unit / lesson provides context to the history of privilege and oppression.
- This unit / lesson addresses power relationships.
- This unit / lesson help students to develop research and critical thinking skills.
- This curriculum creates windows and mirrors* for students.

## Prejudice Reduction

Teachers implement lessons and activities to assert positive images of ethnic groups & improve intergroup relations.

- This unit / lesson help students question and unpack biases & stereotypes.
- This unit / lesson help students examine, research and question information and sources.
- The curriculum encourage discussion and understanding about the groups of people being represented.
- This unit / lesson challenges dominant perspectives.

## Equitable Pedagogy

Teachers modify techniques and methods to facilitate the academic achievement of students from diverse backgrounds.

- The instruction has been modified to meet the needs of each student.
- Students feel respected and their cultural identities are valued.
- Additional supports have been provided for students to become successful and independent learners.
- Opportunities are provided for student to reflect on their learning and provide feedback.

## Empowering School Culture

Using the other four dimensions to create a safe and healthy educational environment for all.

- There are opportunities for students to connect with the community.
- My classroom is welcoming and supportive for all students?
- I am aware of and sensitive to the needs of my students and their families.
- There are effective parent communication systems established. Parents can talk to me about issues as they arise in my classroom.

### Culturally Relevant Pedagogy Examples

- **Present New Concepts Using Student Vocabulary:** Use student diction to capture attention and build understanding before using academic terms.
  
  *Example:* Creating a scavenger hunt where students work together in groups to find new terms and their definition.  
  [http://www.classtools.net/QR/](http://www.classtools.net/QR/)

- **Bring in Guest Speakers:** Invite guest speakers who can add context to your lesson and speak from a specific culture’s general perspective.

  *Example:* Invite guest speakers who can engage and motivate students who share a sense of culture. For example, bring a construction worker, drafter or architect to infuse passion into content.

- **Use Media that Positively Depict a Range of Cultures:** Include different cultures and languages in your curriculum by presenting relevant materials, such as movies about them.

  *Example:* Children process content more effectively when their cultures and languages have places in the curriculum.  
  [https://www.youtube.com/results?search_query=donald+duck+in+mathmagic+land](https://www.youtube.com/results?search_query=donald+duck+in+mathmagic+land)

- **Run Problem-Based Learning Scenarios:** Present relatable real-world problems for your students to solve, explicitly referencing cultures and communities when applicable.

  *Example:* This allows you to bridge two cultural connections. First, include cultural links in the questions, whether they are explicit or students make it themselves. Second, allow students to apply different approaches to solve the question, using unique cultural perspectives.  
  [http://www.bie.org/object/video/geometric_sculpture_project](http://www.bie.org/object/video/geometric_sculpture_project)
## Differentiated Instruction

### Accommodate Based on Students Individual Needs: Strategies

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<th>Processing</th>
<th>Comprehension</th>
<th>Recall</th>
<th>Assistive Technology</th>
<th>Tests/Quizzes/Grading</th>
<th>Behavior/Attention</th>
<th>Organization</th>
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<tbody>
<tr>
<td>Extra time for assigned tasks</td>
<td>Extra Response time</td>
<td>Precise processes for balanced math instructional model</td>
<td>Teacher-made checklist</td>
<td>Computer/whiteboard</td>
<td>Extended time</td>
<td>Consistent daily structured routine</td>
<td>Individual daily planner</td>
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<td>Adjust length of assignment</td>
<td>Have students verbalize steps</td>
<td>Short manageable tasks</td>
<td>Use visual graphic organizers</td>
<td>Tape recorder</td>
<td>Study guides</td>
<td>Simple and clear classroom rules</td>
<td>Display a written agenda</td>
</tr>
<tr>
<td>Timeline with due dates for reports and projects</td>
<td>Repeat, clarify or reword directions</td>
<td>Brief and concrete directions</td>
<td>Reference resources to promote independence</td>
<td>Video Tape</td>
<td>Shortened tests</td>
<td>Frequent feedback</td>
<td>Note-taking assistance</td>
</tr>
<tr>
<td>Communication system between home and school</td>
<td>Mini-breaks between tasks</td>
<td>Provide immediate feedback</td>
<td>Visual and verbal reminders</td>
<td></td>
<td>Read directions aloud</td>
<td></td>
<td>Color code materials</td>
</tr>
<tr>
<td>Provide lecture notes/outline</td>
<td>Provide a warning for transitions</td>
<td>Small group instruction</td>
<td>Graphic organizers</td>
<td></td>
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<tr>
<td></td>
<td>Partnering</td>
<td>Emphasize multi-sensory learning</td>
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</tr>
</tbody>
</table>

### Assistive Technology
- Computer/whiteboard
- Tape recorder
- Video Tape

### Tests/Quizzes/Grading
- Extended time
- Study guides
- Shortened tests
- Read directions aloud

### Behavior/Attention
- Consistent daily structured routine
- Simple and clear classroom rules
- Frequent feedback

### Organization
- Individual daily planner
- Display a written agenda
- Note-taking assistance
- Color code materials
<table>
<thead>
<tr>
<th>Differentiated Instruction</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Accommodate Based on Students Individual Needs: Strategies</strong></td>
</tr>
</tbody>
</table>

- Anchor charts to model strategies and use of formulas
- Reference sheets that list formulas, step-by-step procedures and modeling strategies
- Conceptual word wall that contains definition, translation, pictures and/or examples
- Graphic organizers, examples include: Venn Diagrams, Four Square, KWL, etc.
- Translation dictionary
- Highlight and label the solution steps for multi-step problems in different colors
- Utilize technological programs which provide verbal and visual instruction in native and/or second language
- Three dimensional figures to find the volume of cones, cylinders and spheres
- Two dimensional figures to review area
- Area models to represent, understand and apply the Pythagorean Theorem
- Reflection Mirrors, Tracing Paper, Coordinate Planes to locate images or pre-images as a result of dilations, rotations, reflections and translations
- Protractors to justify facts about parallel lines cut by a transversal
- Explain in writing how to construct triangles using mechanical (i.e. ruler, protractor) and technological tools or free hand drawings from given conditions (scale factor, length, degrees) using manipulatives, demonstrations and math journal
- Use molding clay to create three dimensional shapes. Cut the shapes in order to determine the shapes of the cross sections
- Graph paper with labeled drawn figures to support understanding scaled drawings
Interdisciplinary Connections

Model interdisciplinary thinking to expose students to other disciplines.

Science Connection:
Population Equations: (MS-LS2-1)
- Students will apply properties of operations to add, subtract, multiply and divide rational numbers, rewrite expressions, and solve multi-step real-life world problems. Students will find the rate of reproduction and attribution for populations of wildlife.

Polly Gone: (MS-LS2-1)
- Students will create the dimensions for enclosures based on the amount of area needed for various animals at the zoo they live.

Social Studies Connection:
Take the Ancient Greek Challenge: (6.2.8.B.3 and 6.2.8.D.3f))
- Students will draw geometric shapes while discovering the history of geometry.

ELA Connection:
Fences: (W.7.2.A-F and W.7.4)
- Students will be required to write a letter to their client identifying and explaining their recommendation. They will need to cite specific evidence to support their reasoning.

Aaron’s Design (RI.7.1)
- Students will describe the Transformation(s) used to draw the given design.

Congruent Rectangles (W.7.4)
- Students will write a sequence of events that occurred in order to translate one rectangle to another rectangle on the graph.
Enrichment

What is the purpose of Enrichment?

- The purpose of enrichment is to provide extended learning opportunities and challenges to students who have already mastered, or can quickly master, the basic curriculum. Enrichment gives the student more time to study concepts with greater depth, breadth, and complexity.
- Enrichment also provides opportunities for students to pursue learning in their own areas of interest and strengths.
- Enrichment keeps advanced students engaged and supports their accelerated academic needs.
- Enrichment provides the most appropriate answer to the question, “What do you do when the student already knows it?”

Enrichment is...

- Planned and purposeful
- Different, or differentiated, work – not just more work
- Responsive to students’ needs and situations
- A promotion of high-level thinking skills and making connections within content
- The ability to apply different or multiple strategies to the content.
- The ability to synthesize concepts and make real world and cross-curricular connections
- Elevated contextual complexity
- Sometimes independent activities, sometimes direct instruction
- Inquiry based or open ended assignments and projects
- Using supplementary materials in addition to the normal range of resources
- Choices for students
- Tiered/Multi-level activities with flexible groups (may change daily or weekly)

Enrichment is not...

- Just for gifted students (some gifted students may need intervention in some areas just as some other students may need frequent enrichment)
- Worksheets that are more of the same (busywork)
- Random assignments, games, or puzzles not connected to the content areas or areas of student interest
- Extra homework
- A package that is the same for everyone
- Thinking skills taught in isolation
- Unstructured free time
## Assessments

### Required District/State Assessments
- Unit Assessments
- SGO Assessments
- PARCC

### Suggested Formative/Summative Classroom Assessments
- Describe Learning Vertically
- Identify Key Building Blocks
- Make Connections (between and among key building blocks)
- Short/Extended Constructed Response Items
- Multiple-Choice Items (where multiple answer choices may be correct)
- Drag and Drop Items
- Use of Equation Editor
- Quizzes
- Journal Entries/Reflections/Quick-Writes
- Accountable talk
- Projects
- Portfolio
- Observation
- Graphic Organizers/ Concept Mapping
- Presentations
- Role Playing
- Teacher-Student and Student-Student Conferencing
- Homework
# New Jersey Student Learning Standards

<table>
<thead>
<tr>
<th>Standard</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>7.G.B.6</strong></td>
<td>Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.</td>
</tr>
<tr>
<td><strong>8.G.C.9</strong></td>
<td>Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.</td>
</tr>
<tr>
<td><strong>8.G.A.1</strong></td>
<td>Verify experimentally the properties of rotations, reflections, and translations:</td>
</tr>
<tr>
<td><strong>8.G.A.1a</strong></td>
<td>Lines are transformed to lines, and line segments to line segments of the same length.</td>
</tr>
<tr>
<td><strong>8.G.A.1b</strong></td>
<td>Angles are transformed to angles of the same measure.</td>
</tr>
<tr>
<td><strong>8.G.A.1c</strong></td>
<td>Parallel lines are transformed to parallel lines.</td>
</tr>
<tr>
<td><strong>8.G.A.2</strong></td>
<td>Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.</td>
</tr>
<tr>
<td><strong>8.G.A.3</strong></td>
<td>Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.</td>
</tr>
<tr>
<td><strong>8.G.A.4</strong></td>
<td>Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.</td>
</tr>
<tr>
<td><strong>8.G.B.6</strong></td>
<td>Explain a proof of the Pythagorean Theorem and its converse.</td>
</tr>
<tr>
<td><strong>8.G.B.7</strong></td>
<td>Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.</td>
</tr>
<tr>
<td><strong>8.G.B.8</strong></td>
<td>Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.</td>
</tr>
<tr>
<td><strong>8.EE.B.6</strong></td>
<td>Use similar triangles to explain why the slope $m$ is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at $b$.</td>
</tr>
<tr>
<td><strong>8.EE.A.1</strong></td>
<td>Know and apply the properties of integer exponents to generate equivalent numerical expressions. <em>For example, $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.</em></td>
</tr>
</tbody>
</table>
New Jersey Student Learning Standards

8.EE.A.2: Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where $p$ is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.

8.EE.A.3: Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. For example, estimate the population of the United States as $3 \times 10^8$ and the population of the world as $7 \times 10^9$, and determine that the world population is more than 20 times larger.

8.EE.A.4: Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notations are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.
Mathematical Practices

1. Make sense of problems and persevere in solving them.

2. Reason abstractly and quantitatively.

3. Construct viable arguments and critique the reasoning of others.

4. Model with mathematics.

5. Use appropriate tools strategically.

6. Attend to precision.

7. Look for and make use of structure.

8. Look for and express regularity in repeated reasoning.
### New Jersey Student Learning Standard(s):

**7.G.B.6:** Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

**8.G.C.9:** Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

### Student Learning Objective 1:
Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms. Apply the formula for the volume of a cone, a cylinder, or a sphere to find a single unknown dimension when solving real-world and mathematical problems.

### Modified Student Learning Objectives/Standards:

**M.EE.7.G.B.6** Determine the area of a rectangle using the formula for length × width, and confirm the result using tiling or partitioning into unit squares.

**M.EE.8.G.C.9:** Use the formulas for perimeter, area, and volume to solve real-world and mathematical problems (limited to perimeter and area of rectangles and triangles and volume of rectangular prisms).
<table>
<thead>
<tr>
<th>MPs</th>
<th>Evidence Statement Key/ Clarifications</th>
<th>Skills, Strategies &amp; Concepts</th>
<th>Essential Understandings/ Questions (Accountable Talk)</th>
<th>Tasks/Activities</th>
</tr>
</thead>
<tbody>
<tr>
<td>MP 1</td>
<td>7.G.6</td>
<td>Solve real-world and mathematical problems involving area of two dimensional objects composed of triangles, quadrilaterals, and polygons.</td>
<td>Where do we see examples of two and three-dimensional objects in the real-world?</td>
<td>A Slimmer Jewel Case</td>
</tr>
<tr>
<td>MP 2</td>
<td>8.G.C.9</td>
<td>Solve real-world and mathematical problems involving volume of three dimensional objects composed of cubes and right prisms.</td>
<td>How do you determine volume and surface area of different three-dimensional figures?</td>
<td>Anna’s Room</td>
</tr>
<tr>
<td>MP 3</td>
<td></td>
<td>Solve real-world and mathematical problems involving surface area of three-dimensional objects composed of cubes and right prisms.</td>
<td>How can these formulas be used to solve mathematical and real-world problems?</td>
<td>Real-World Area Problems</td>
</tr>
<tr>
<td>MP 4</td>
<td></td>
<td>Solve a multi-step equation for a missing variable.</td>
<td>How would the volume and surface area be affected when dimensions of a figure are doubled and/or tripled?</td>
<td>Real-World Volume Problems</td>
</tr>
<tr>
<td>MP 5</td>
<td></td>
<td>Use the volume formula to find a single unknown dimension of cones, cylinders and spheres when solving real world problems.</td>
<td>How can comparing polygons help to solve problems involving two and three dimensional figures?</td>
<td>Pool Services</td>
</tr>
<tr>
<td>MP 6</td>
<td></td>
<td>Write and solve using the formula of a cone, cylinder or sphere.</td>
<td>How does the change in radius affect the volume of a cylinder or sphere?</td>
<td>Sand Under the Swing Set</td>
</tr>
<tr>
<td>MP 7</td>
<td></td>
<td>Apply previous knowledge of cones, cylinders, and spheres and their characteristics in order to explain how the formulas are derived.</td>
<td></td>
<td>Yum Yum Cereal</td>
</tr>
<tr>
<td>MP 8</td>
<td></td>
<td>Example: James wanted to plant pansies in his new planter. He wondered how much potting soil he should buy to fill</td>
<td></td>
<td>Comparing Snow Cones</td>
</tr>
<tr>
<td></td>
<td></td>
<td>where do we see examples of two and three-dimensional objects in the real-world?</td>
<td></td>
<td>Comparing Spheres and Cylinders</td>
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<td></td>
<td></td>
<td></td>
<td>Flower Vases</td>
</tr>
</tbody>
</table>
Use the measurements in the diagram below to determine the planter’s volume.

<table>
<thead>
<tr>
<th><strong>SPED Strategies:</strong></th>
<th>How does the change in height affect the volume of a cylinder or sphere?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-teach vocabulary using visual and verbal models.</td>
<td>How does the volume of a cylinder and sphere with the same radius change if it is doubled?</td>
</tr>
<tr>
<td>Review and provide pictorial guide of vocabulary words for students to make a connection (i.e. base, compose, decompose, height, polygon, quadrilateral, triangle).</td>
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<tr>
<td>Explain the formulas by breaking them down to illustrate the meaning of the parts.</td>
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<tr>
<td>Provide students with a graphic organizer with the formulas and the explanation of the parts.</td>
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<tr>
<td>Demonstrate and practice how to calculate the area of a triangle(s), and quadrilateral(s).</td>
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</tr>
<tr>
<td>Demonstrate and practice how to compose or decompose a polygon.</td>
<td></td>
</tr>
<tr>
<td>Review, demonstrate and practice determining the appropriate operation needed to calculate the area of a polygon composed of more than one two-dimensional shape.</td>
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</tr>
</tbody>
</table>
Use nets to calculate surface area. Real-world and mathematical multi-step problems that require finding area, perimeter, volume, and surface area of figures should reflect situations relevant to seventh graders. The computations should make use of formulas and involve whole numbers, fractions, decimals, ratios and various units of measure with same system conversions.

Create visual, verbal or tactile cues or reminders.

Review and demonstrate how to identify solids in the environment similar to cones, cylinders, and spheres.

**Resources UDL - Visual and Auditory Learner(s):**

*7.G.6 - Volume & Surface Area of Pyramid*
https://youtu.be/5qYrQ1Yy9LJ

*7.G.6 - Volume & Surface Area of Cylinders*
https://youtu.be/TabJ7vTS_JU

*Roosevelt Middle School Math Team - Common Core Math Tutorials - 8.EE.A.2*

*Roosevelt Middle School Math Team -- Common Core Tutorial Videos -- 8.G.C.9*
https://youtu.be/BiaBa0IHIDs

**ELL Strategies:**
Let students choose the language they prefer for arithmetic computation and discourse.
|   | Initiate discussions and provide opportunities for collaboration.  
|   | Pre-teach vocabulary using visual and verbal models.  
|   | Provide students with a graphic organizer with the formulas and the explanation of the parts.  
|   | Provide 3-dimensional figures that are cones, spheres or cylinders for students to use as a visual.  
|   | Provide reference sheets with the formulas.  
|   | Teacher partially completes the mathematical problem and labels essential terms.  
|   | Utilize interactive tools that can be used to illustrate solution methods, and build language as well as math skills.  
|   | Use whiteboards for students to write the equations dictated by the teacher.  
|   | Utilize visuals and photographs to show ELLs how to solve surface area problems of two and three dimensional objects and shapes.  

**Website: Teachers First**  
Adapt a Strategy. Adjusting Lessons for ESL/ELL students  
http://www.teachersfirst.com/content/esl/adaptstrat.cfm
New Jersey Student Learning Standard(s):

8.G.A.1: Verify experimentally the properties of rotations, reflections, and translations.
  
  8.G.A.1a: Lines are transformed to lines, and line segments to line segments of the same length.
  8.G.A.1b: Angles are transformed to angles of the same measure.
  8.G.A.1c: Parallel lines are transformed to parallel lines.

Student Learning Objective 2: Explain and model the properties of rotations, reflections, and translations with physical representations and/or geometry software using pre-images and resultant images of lines, line segments, and angles.

Modified Student Learning Objectives/Standards:


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<tbody>
<tr>
<td>MP 3</td>
<td>8.G.A.1a</td>
<td>Verify experimentally the properties of rotations, reflections, and translations.</td>
<td>How can I precisely describe the movement of a shape in a plane?</td>
<td>IFL Task(s) – Set of Related Lessons “Understanding Congruence and Similarity”</td>
</tr>
<tr>
<td>MP 5</td>
<td>8.G.A.1b</td>
<td>Students will use knowledge of rotations, reflections, translations, and graphing in order to create images on the coordinate plane.</td>
<td>How can I use mathematical procedures to enlarge, or shrink, a shape?</td>
<td>PBA: Outer Space</td>
</tr>
<tr>
<td>MP 8</td>
<td>8.G.A.1c</td>
<td>Use geometry software in order to create translations, rotations, and reflections.</td>
<td>How can I determine if two shapes are similar, congruent, or neither?</td>
<td>New Patterns</td>
</tr>
<tr>
<td></td>
<td>• Tasks do not have a context.</td>
<td>Students need multiple opportunities to explore the transformation of figures so that they can appreciate that points stay the same distance apart and lines stay at the same</td>
<td>How can we use rotations, reflections, and translations in order to create images?</td>
<td></td>
</tr>
<tr>
<td>Why is it important to create pre-images and images?</td>
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<td></td>
<td></td>
<td></td>
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<tr>
<td>-----------------------------------------------------</td>
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<tr>
<td>What characteristics are the same in the pre-image and the image?</td>
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<tr>
<td>Rigid motions—translations, rotations, and reflections—move a figure as a whole, without adding, deleting, shrinking, or stretching portions of the figure. As a result:</td>
<td></td>
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</tr>
<tr>
<td>- lines, line segments, and angles are moved to lines, line segments, and angles;</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>- length, distance, and angle measures remain the same</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>- parallel lines are taken to parallel lines.</td>
<td></td>
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<td></td>
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</tr>
</tbody>
</table>

| Explain that a property of rigid motion transformations is that the measure of a two-dimensional object under the transformation remains unchanged. |
| Measure angles using a protractor. |
| Show that parallel lines are translated, rotated or reflected parallel lines. |
| Identify corresponding lines after a translation, rotation, or reflection by using prime notation. |

**SPED Strategies:**
Pre-teach vocabulary using visual and verbal models.

Illustrate what happens to points, angles, lines, and line segments when they are transformed.

Practice identifying transformed line or line segment as a rotation, reflection, or translation.

Model how rotated, reflected, or translated lines or line segments are the same length as the original.

Practice and demonstrate how to determine that the rotated, reflected, and translated lines...
and line segments are the same as the original figure.

**Resources UDL - Visual and Auditory Learner(s):**
CCSS 8.G.1 - video 1 – Translations
https://youtu.be/9gwodxblOj4

**ELL Strategies:**
After applying transformations, identify the coordinates of the resultant image using modeling.

Have students visualize, create and model their own rotations, reflections, and translations.

Have students work with partners, to create sketch graphs, and label parts in English and Spanish.

Pre teach key vocabulary words.
New Jersey Student Learning Standard(s):

8.G.A.2: Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.

Student Learning Objective 3: Describe and perform a sequence of rotations, reflections, and/or translations on a two dimensional figure in order to prove that two figures are congruent.

Modified Student Learning Objectives/Standards:

M.EE.8.G.A.2: Identify shapes that are congruent.

<table>
<thead>
<tr>
<th>MPs</th>
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</tr>
</thead>
</table>
| MP 2 MP 7  | 8.G.2  
- Tasks do not have a context.  
- Figures may be drawn in the coordinate plane, but do not include the use of coordinates.  
- Tasks require students to make connections between congruence and transformations. | Have students discuss what makes two figures congruent and the qualities of congruent figures. Students should discuss and justify their reasoning as to why two figures are congruent or not congruent. Analyze the preimage to determine if a series of rotations, reflections, and translations could have been applied to create the image. A two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections and translations. Students need multiple opportunities to explore the transformation of figures so that they can appreciate that points stay the same distance | How can we prove that two figures are congruent? Two figures are congruent if they have congruent corresponding angles and congruent corresponding side lengths Can two figures still be congruent if they are facing in different directions? How can congruency between two figures be described by a series of rotations, reflections and/or translations? How can I determine if two shapes are similar, congruent, or neither? | IFL Task(s) – Set of Related Lessons “Understanding Congruence and Similarity” Additional Tasks: Congruent Rectangles Triangle Congruence with Coordinates Gamers for Life |
apart and lines stay at the same angle after they have been rotated, reflected, and/or translated. Is Figure A congruent to Figure A’? Explain how you know.

Describe the sequence of transformations that results in the transformation of Figure A to Figure A’.

Why is it important to create pre-images and images?

If two figures are congruent, a sequence of rotations, reflections, and translations that shows the congruence between them can be described.

Different rigid motions, or sequences, of rigid motion, can sometimes produce the same transformation of a figure.
**SPED Strategies:**
Provide students with graph paper and manipulatives to visualize and practice the thinking involved in transformations.

Use assessing and advancing questions to tailor instructional strategies to the needs of students.

Create a google doc/anchor chart/graphic organizer to highlight what students have learned about the nature of transformed figures.

**Resources UDL - Visual and Auditory Learner(s):**

8.G.2-2.0 Congruence
https://youtu.be/-QQ7Jy0gbUo

**ELL Strategies:**
Provide students with graph paper and manipulatives to visualize and practice the thinking involved in transformations.

Create graphic organizers/charts/posters for students to use a reference.
New Jersey Student Learning Standard(s):

8.G.A.3: Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

Student Learning Objective 4: Use the coordinate plane to locate images or pre-images of two-dimensional figures and determine the coordinates of a resultant image after applying dilations, rotations, reflections, and translations.

Modified Student Learning Objectives/Standards: N/A

<table>
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</thead>
<tbody>
<tr>
<td>MP 2 MP 3</td>
<td>8.G.3</td>
<td>Describe using coordinates, the resulting two-dimensional figure after applying dilations with scale factor greater than 1, less than, and equal to 1.</td>
<td>How can the coordinate plane help me understand properties of reflections, translations, and rotations?</td>
<td>Coordinating Reflections</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Apply understanding of translations, rotations, reflections and the coordinate plane to determine the coordinates of a resulting image after applying a series of rotations, reflections, and translations.</td>
<td>Are two figures congruent after applying a series of rotations, reflections, and translations?</td>
<td>Coordinating Translations</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Use scale factors in order to create dilations and help in determining the coordinates of the image.</td>
<td>What does it mean to dilate an image?</td>
<td>Coordinating Rotations</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Use technology to demonstrate how images can be dilated, rotated, reflected, and translated.</td>
<td>How is the pre-image related to the dilated image?</td>
<td>Dilations in the Coordinate Plane</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Know that a scale factor that is less than 1 will result in a smaller image, and a scale</td>
<td>In what ways can I represent the relationships that exist between similar figures using the scale factors, length ratios, and area ratios?</td>
<td>Changing Shapes</td>
</tr>
<tr>
<td></td>
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<td>Examples of Dilation</td>
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<td>Same Shape</td>
</tr>
</tbody>
</table>
factor greater than 1 will result in a larger image.

Use a ruler to create dilated images with a given scale factor and determine the relationship of the characteristics from the original figure to the dilated image.

**Examples:**

Translation:

Rotation
Reflection:

**SPED Strategies:**
Link new learning about dilations to prior learning and real-life experiences such as enlarging or reducing a photograph.

Model how to analyze the relationship between the preimage and image after dilation and document this information for future use as a graphic organizer, google doc or anchor chart.

Provide students with graph paper and manipulatives to visualize and practice the thinking involved in dilations, rotations, reflections and translations of figures on the coordinate plane.
<table>
<thead>
<tr>
<th><strong>Resources UDL - Visual and Auditory Learner(s):</strong></th>
<th></th>
</tr>
</thead>
</table>
| 8.G.3 *Rotation Coordinates*  
https://youtu.be/Q9CXsWRItwo |  |

**ELL Strategies:**

Have students visualize, create and model their own rotations, reflections, and translations.

Create large charts/posters with words & pictures on wall, include L1 (students’ native language) translations side by side.

Provide students with graph paper and manipulatives to visualize and practice the thinking involved in dilations, rotations, reflections and translations of figures on the coordinate plane.
New Jersey Student Learning Standard(s):

8.G.A.4: Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.

Student Learning Objective 5: Apply an effective sequence of transformations to determine that figures are similar when corresponding angles are congruent and corresponding sides are proportional. Write similarity statements based on such transformations.

Modified Student Learning Objectives/Standards:

M.EE.8.G.A.4: Identify similar shapes with and without rotation.

<table>
<thead>
<tr>
<th>MPs</th>
<th>Evidence Statement Key/ Clarifications</th>
<th>Skills, &amp; Concepts</th>
<th>Essential Understandings/ Questions (Accountable Talk)</th>
<th>Tasks/Activities</th>
</tr>
</thead>
<tbody>
<tr>
<td>MP 2</td>
<td>8.G.4</td>
<td>Emphasize the effects that different transformations have on figures and know that dilations create similar figures and that they can be translated, reflected or rotated. Use scale factors to dilate figures. Explain the preservation of similarity when a figure is dilated, rotated, reflected, and/or translated. Describe the sequence of transformations that occurred from the original 2-dimensional figure to the image to show similarity. A 2-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations and dilations.</td>
<td>How can similarity between two figures be described by a series of rotations, reflections, dilations and/or translations? How can I tell if two figures are similar? What has to be true about the corresponding sides and corresponding angles for two figures to be similar? How can you transform figures to make congruent figures? How can you transform figures to make similar figures? Because dilations shrink or expand a figure without altering...</td>
<td>IFL Task(s) – Set of Related Lessons named, “Understanding Congruence and Similarity” Additional Tasks: Aaron’s Designs Are They Similar? Fundamental Theorem of Similarity</td>
</tr>
<tr>
<td>MP 7</td>
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</tbody>
</table>

40 | Page
Examples:

Is Figure A similar to Figure A’? Explain how you know.

Describe the sequence of transformations that results in the transformation of Figure A to Figure A’.

the shape of the figure, when a figure is dilated by a scale factor of \( r \), the angle measures in the image remain the same as the corresponding angles in the original figure, and lengths (e.g., side lengths) in the image are changed by a factor of \( r \) of the corresponding lengths in the original figure. The dilated image is similar to the original figure.

Because dilations with a scale factor of \( r \) create an image that is similar to the original figure, and lengths (e.g., side lengths) in the image are changed by a factor of \( r \), all such images are therefore congruent to one another.

If two figures are similar, a sequence of dilations, rotations, reflections, and translations that shows the similarity between them can be described.
**SPED Strategies:**
Review and practice identifying the properties (i.e. dilations, reflections, rotations, and translations on a coordinate grid).

Review and describe the properties of similarity between two figures through a sequence of rotations, reflections, translations, and dilations.

Practice identifying the transformations of the image from the pre-image.

Review and practice how to determine the properties of similarity between two-dimensional shapes.

Provide students with an image and a preimage on a coordinate plane and manipulatives and have them work with a peer to determine different paths to transform the figure that would fit the scenario given. A graphic organizer that helps students document their thinking would facilitate this process.

**Resources UDL - Visual and Auditory Learner(s):**
8.G.4 Series of Transformations
https://youtu.be/AUJBSaW1fs0

**ELL Strategies:**
Provide multiple examples of similar figures so that students grasp the concept.
Practice identifying the transformations of the image from the pre-image.

Review and practice how to determine the properties of similarity between two-dimensional shapes.

Provide anchor charts/posters/illustrations of examples for rotations, reflections, translations and dilations.

**New Jersey Student Learning Standard(s):**


**Student Learning Objective 6:** Explain a proof of the Pythagorean Theorem and its converse.

**Modified Student Learning Objectives/Standards:** N/A

<table>
<thead>
<tr>
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<th>Essential Understandings/ Questions (Accountable Talk)</th>
<th>Tasks/Activities</th>
</tr>
</thead>
<tbody>
<tr>
<td>MP 2</td>
<td>NA</td>
<td>Students will determine the relationship between the hypotenuse and legs of a right triangle. Use deductive reasoning to prove the Pythagorean Theorem and its converse. Analyze various proofs of the Pythagorean Theorem to gain a deeper understanding of why the Pythagorean Theorem can be used.</td>
<td>The Pythagorean Theorem can be used both algebraically and geometrically to solve problems involving right triangles. What is Pythagorean Theorem and when does it apply?</td>
<td>The Pythagorean Relationship Finding Pythagorean Triples in Disguise The Converse of the Pythagorean Theorem</td>
</tr>
</tbody>
</table>
Students should verify, using a model, that the sum of the squares of the legs is equal to the square of the hypotenuse in a right triangle.

Students should also understand that if the sum of the squares of the 2 smaller legs of a triangle is equal to the square of the third leg, then the triangle is a right triangle.

**SPED Strategies:**
Pre-teach vocabulary using visual and verbal models.

Review and practice identifying legs and hypotenuse of right triangles.

Review, explain and demonstrate the proof of the Pythagorean Theorem using a visual example.

How can the Pythagorean Theorem and its converse be modeled?

**Informal Proof of the Pythagorean Theorem**

**Pythagoras Plus**
**Resources UDL - Visual and Auditory Learner(s):**
*Roosevelt Middle School Math Team -- Common Core Tutorial Videos - 8.G.B.6*
https://youtu.be/f6qIvqe-CIM

**ELL Strategies:**
Have students work with partners, small groups.
Pre-teach vocabulary using visual and verbal models.
Review and practice identifying legs and hypotenuse of right triangles.

---

**New Jersey Student Learning Standard(s):**

8.G.B.7: Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

8.G.B.8: Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

**Student Learning Objective 7:** Apply the Pythagorean Theorem to determine unknown side lengths of right triangles in two and three dimensions to solve real-world and mathematical problems and to determine the distance between two points in the coordinate plane.

**Modified Student Learning Objectives/Standards:** N/A

<table>
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<tr>
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<th>Tasks/Activities</th>
</tr>
</thead>
<tbody>
<tr>
<td>MP 2</td>
<td>8.G.7-1</td>
<td>Students should apply the Pythagorean Theorem to determine unknown side lengths in right triangles.</td>
<td>Right triangles have a special relationship among the side lengths which can be</td>
<td>Applications of the Pythagorean</td>
</tr>
<tr>
<td>MP 7</td>
<td>8.G.7-2</td>
<td></td>
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</tbody>
</table>
• Tasks have “thin context” or no context.
• An equal number of tasks require the answer to be given as a whole number or as an irrational number written to approximately three decimal places.

**8.G.8**

Review squaring numbers and taking square roots so students will effectively be able to apply the Pythagorean Theorem and find missing side lengths.

Through authentic experiences and exploration, students should use the Pythagorean Theorem to solve problems. Problems can include working in both two and three dimensions. Students should be familiar with the common Pythagorean Theorem e.g. $a^2 + b^2 = c^2$.

Students will apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

Students will solve problems involving the Pythagorean Theorem.

Students will create a right triangle from the two points given (as shown in the diagram below) and then use the Pythagorean Theorem to find the distance between the two given points.

![Diagram showing a right triangle with coordinates (-2, 4) and (-3, -6)](image)

**SPED Strategies:**

Review the Pythagorean Theorem, how to use it to find missing side lengths and the distance between two points as covered in Unit 3.

represented by a model and a formula.

Pythagorean Triples can be used to construct right triangles.

How can the Pythagorean Theorem be used to solve problems?

Both the Pythagorean Theorem and distance formula can be used to find missing side lengths in a coordinate plane and real-world situation.

What is the correlation between the Pythagorean Theorem and the distance formula?

How can we use the coordinate plane to determine the distance between two points using the Pythagorean Theorem?

How is the Pythagorean Theorem used to determine unknown side length in right triangles?
Model how to use the Pythagorean Theorem to solve real world problems involving right triangles in two and three dimensions.

**Resources UDL - Visual and Auditory Learner(s):**
*Compare linear and nonlinear functions* - 8.SP.3
https://youtu.be/ydefwk2xZI0

*Linear vs nonlinear relations*
https://youtu.be/F5RZak0dVi4

**ELL Strategies:**
Utilize authentic experiences and explanations to help students understand Pythagorean Theorem.

Review with students the thinking process used to calculate missing side lengths of a right triangle.

Review key vocabulary with the students; present pictorial representation when available.

How is the Pythagorean Theorem used to find distance between two points in a coordinate system?
New Jersey Student Learning Standard(s):

8.EE.B.6: Use similar triangles to explain why the slope \( m \) is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation \( y = mx \) for a line through the origin and the equation \( y = mx + b \) for a line intercepting the vertical axis at \( b \).

Student Learning Objective 8: Derive the equation of a line \((y = mx \text{ for a line through the origin and the equation } y = mx + b \text{ for a line intercepting the vertical axis at } b\) and use similar triangles to explain why the slope \((m)\) is the same between any two points on a non-vertical line in the coordinate plane.

Modified Student Learning Objectives/Standards:

M.EE.8.EE. B.5-6: Graph a simple ratio by connecting the origin to a point representing the ratio in the form of \( y/x \) for example, when given a ratio in standard form (2:1), convert to 2/1, and plot (1,2).

<table>
<thead>
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</tr>
</thead>
<tbody>
<tr>
<td>MP 2</td>
<td>8.EE.6</td>
<td>Show, using similar triangles, and explain why the slope, ( m ), is the same between any two distinct points on a non-vertical line.</td>
<td>How can patterns, relations, and functions be used as tools to best describe and help explain real-life relationships?</td>
<td>What’s My Line?</td>
</tr>
<tr>
<td>MP 4</td>
<td></td>
<td>Derive, from two points, the equation ( y = mx ) for a line through the origin.</td>
<td></td>
<td>Slopes Between Points on a Line</td>
</tr>
<tr>
<td>MP 5</td>
<td></td>
<td>Derive, from two points, the equation ( y = mx + b ) for a line intercepting the vertical axis at ( b )</td>
<td></td>
<td>Slippery Slopes</td>
</tr>
<tr>
<td>MP 6</td>
<td></td>
<td><strong>SPED Strategies:</strong> Review graphs of a proportional relationship and describe it attributes.</td>
<td></td>
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<tr>
<td>MP 7</td>
<td></td>
<td>Review and practice graphs and compare to the slope of the related line in a proportional relationship.</td>
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<tr>
<td>MP 8</td>
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</table>

*Tasks do not have a context.

- Given a non-vertical line in the coordinate plane, tasks might for example require students to choose two pairs of points and record the rise, run, and slope relative to each pair and verify that they are the same.
- For the explain aspect of 8.EE.6, see 8.C.5.1.
| Tasks may assess simple graphing of lines from a linear equation in slope-intercept form. | Review and provide different descriptions of slope.  
Review and practice draw the graph of the proportional relationship between the two quantities.  
Review and practice the graph of an equation written as $y = mx + b$ linear.  
Review, practice and provide strategy cards and scaffolds on the importance of using the slope and y-intercept.  
Identify the slope of an equation by counting rise/run on a graph.  
Identify where the graph of the equation crosses the y-axis.  
Review and practice solutions for a linear equation fall on a line on the coordinate plane.  
Review and practice points on the line are solutions for the linear equation.  
Review and practice linear equation a constant change in the x-value corresponds to a constant change in the y-value.  
Receive and practices finding a solution to a linear equation by selecting any x-value and finding the corresponding y-value. | How can similar triangles be used to model slope? |
<table>
<thead>
<tr>
<th>Resources UDL - Visual and Auditory Learner(s): Roosevelt Middle School Math Team -- Common Core Tutorial Videos -- 8.EE.B.5</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><a href="https://youtu.be/5mJIY1cK6yM">https://youtu.be/5mJIY1cK6yM</a></td>
<td></td>
</tr>
<tr>
<td>8.EE.6-1.0 Slope and Similar Triangles <a href="https://youtu.be/S0ENLYcM5AI">https://youtu.be/S0ENLYcM5AI</a></td>
<td></td>
</tr>
<tr>
<td><strong>ELL Strategies:</strong> Have students translate symbols into words, and write out the sentence.</td>
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<tr>
<td>Provide visual cues, graphic representations, gestures, realia, and pictures.</td>
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<tr>
<td>Provide and utilize manipulatives, such as graph paper, charts, and posters.</td>
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<tr>
<td>Identify key phrases or new vocabulary to pre-teach.</td>
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<tr>
<td>Build knowledge from real world examples.</td>
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<tr>
<td>Have students work with partners to share problem-solving strategies.</td>
<td></td>
</tr>
</tbody>
</table>
Create a "sentence frame" and post it on the board.

Identify key phrases or new vocabulary to pre-teach.

**Website:**
KHAN ACADEMY
*Expressions and Equations*
[https://www.khanacademy.org/commoncore/grade-8-EE](https://www.khanacademy.org/commoncore/grade-8-EE)

### New Jersey Student Learning Standard(s):

**8.EE.A.1:** Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example, $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.*

**Student Learning Objective 9:** Apply the properties of integer exponents to write equivalent numerical expressions.

### Modified Student Learning Objectives/Standards:

**M.EE.8.EE.A.1:** Identify the meaning of an exponent (limited to exponents of 2 and 3).

<table>
<thead>
<tr>
<th>MPs</th>
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<th>Essential Understandings/Questions (Accountable Talk)</th>
<th>Tasks/Activities</th>
</tr>
</thead>
<tbody>
<tr>
<td>MP 1</td>
<td><strong>8.EE.1</strong></td>
<td>Exponents can be defined as simplified representations of repeated multiplication.</td>
<td>How can you use exponents to write numbers?</td>
<td>Alien Attack</td>
</tr>
<tr>
<td>MP 2</td>
<td>Tasks do not have a context.</td>
<td>Apply properties of exponents to numerical expressions.</td>
<td>When are exponents used and why are they important?</td>
<td>Extending the Definitions of Exponents</td>
</tr>
<tr>
<td>MP 4</td>
<td>Tasks focus on the properties and equivalence, not on simplification.</td>
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<tr>
<td>MP 6</td>
<td>Half of the expressions involve one property; half of the expressions involve two or three properties.</td>
<td>Write a numerical expression with a negative exponent as an equivalent numerical expression with a positive exponent.</td>
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</tr>
<tr>
<td>MP 7</td>
<td>Tasks should involve a single common base or a potential common base, such as, a task that includes 3, 9 and 27.</td>
<td>Multiply numerical expressions with integer exponents with like bases by adding the exponents.</td>
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<tr>
<td>MP 8</td>
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<td>Evaluate numerical expressions with integer exponents.</td>
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<td></td>
<td>Generate equivalent numerical expressions using positive and negative integer exponents.</td>
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<td></td>
<td><strong>SPED Strategies:</strong></td>
<td><strong>Exponential Exponents</strong></td>
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<tr>
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<td></td>
<td>Write a numerical expression with a negative exponent as an equivalent numerical expression with a positive exponent.</td>
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<td>Why are exponents added when the bases are the same and they are being multiplied?</td>
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<td>Why are exponents subtracted when the bases are the same and they being divided?</td>
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<td>Why does a negative exponent become a fraction?</td>
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<td>How do I simplify and evaluate numeric expressions involving integer exponents?</td>
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<tr>
<td></td>
<td></td>
<td>How do you use patterns to understand mathematics and model situations?</td>
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<td></td>
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<td>Resources</td>
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<td>A Few Folds</td>
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<td>Integer Exponents</td>
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<td></td>
<td></td>
<td>Applying Properties of Exponents</td>
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<td>Provide a calculator.</td>
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</tbody>
</table>
Teach and model fundamental skills and procedures explicitly until they become automatic.

Present information through different modalities (i.e. visual, auditory, tactile, & kinesthetic).

**Resources UDL - Visual and Auditory Learner(s):**

Exponents - *The Power of Zero* | 8.EE.A.1 | 8th Grade Math
https://youtu.be/GGvgEM7SK7k

8.EE.1-1 - Rules of Exponents
https://youtu.be/LtWhMUkJ5U0

*Roosevelt Middle School Math Team -- Common Core Tutorial Videos -- 8.G.C.9*
https://youtu.be/BiaBa0IHIDs

**ELL Strategies:**

Utilize Manipulatives and develop hands-on activities.

Model structure and clarify unfamiliar syntax.

Utilize gestures, or L1(student native language) translations, to demonstrate vocabulary.

Provide students with translation dictionary.
<table>
<thead>
<tr>
<th>Have students work with partners, small groups.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Word/picture wall with L1 (students’ native language) translations side by side.</td>
</tr>
<tr>
<td>Provide supplemental resources with L1 (students’ native language) text and/or support.</td>
</tr>
<tr>
<td>Pictures/illustrations and have student write meaning in their Math Journals.</td>
</tr>
</tbody>
</table>

**Website:**

*Teachers first adapt a strategy. Adjusting lessons for ESL/ELL students*

[http://www.teachersfirst.com/content/esl/adaptstrat.cfm](http://www.teachersfirst.com/content/esl/adaptstrat.cfm)
New Jersey Student Learning Standards (s):

**8.EE.A.2:** Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where $p$ is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.

**Student Learning Objective 10:** Evaluate square roots and cubic roots of small perfect squares and cubes respectively and use square and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$ where $p$ is a positive rational number; identify $\sqrt{2}$ as irrational.

**Modified Student Learning Objectives/Standards:**

**M.EE.8.EE.A.2:** Identify a geometric sequence of whole numbers with a whole number common ratio.

<table>
<thead>
<tr>
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<th>Tasks/Activities</th>
</tr>
</thead>
<tbody>
<tr>
<td>MP 2 MP 4 MP 5 MP 6 MP 7 MP 8</td>
<td>8.EE.2</td>
<td>Square roots and cube roots; perfect squares and perfect cubes. There is an inverse relationship between powers and squares roots. Name numbers that are perfect squares and non-perfect squares. Evaluate a square root of a perfect square. A variable raised to the $n$th power can be isolated by taking the $n$th power of the solution.</td>
<td>How are properties of integer exponents used to generate equivalent numerical expressions? How can you determine if your answer will be rational or irrational?</td>
<td>Existence and Uniqueness of Square and Cube Roots Simplifying Square Roots Teacher/Student Square Roots Teacher/Student</td>
</tr>
</tbody>
</table>
**SPED Strategies:**

Pre-teach vocabulary (cube of a number, cube root, integer, irrational and rational number, etc.) using visual and verbal models.

Create a reference sheet with students to review the concept of cubes and squares of numbers. Including the perfect squares to 15 and the perfect cubes to 10 would be helpful for students as they progress through this learning.

Model the thinking process and arithmetic needed to solve equations for unknown squared or cubed values.

Demonstrate and explain why equations of the form \( x^2 = p \) have two (real) solutions when \( p \) is positive (e.g., the square root of 256 is ±16), one solution when \( p \) is zero (zero to any power, except zero, = 0), and none when \( p \) is negative (a negative number is not a perfect square).

Demonstrate and explain why equations of the form \( x^3 = p \) have ONLY one (real) solution regardless of the sign of \( p \).
**Resources UDL - Visual and Auditory Learner(s):**
*Roosevelt Middle School Math Team - Common Core Math Tutorials - 8.EE.A.2*  
https://www.youtube.com/watch?v=jA9jNYvfdO8

*Roosevelt Middle School Math Team -- Common Core Tutorial Videos -- 8.G.C.9*  
https://youtu.be/BiaBa0IHIDs

**ELL Strategies:**
Model structure and clarify unfamiliar syntax.

Utilizing gestures, or L1 (students’ native language) translations, to demonstrate vocabulary comprehension and word-meaning.

Provide supplemental resources with L1 (students’ native language) text and/or support.

Pre-teach vocabulary (cube of a number, cube root, integer, irrational and rational number, etc.) using visual and verbal models.
New Jersey Student Learning Standard(s):

8.EE.A.3. Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. For example, estimate the population of the United States as $3 \times 10^8$ and the population of the world as $7 \times 10^9$, and determine that the world population is more than 20 times larger.

Student Learning Objective 11: Estimate and express the values of very large or very small numbers with numbers expressed in the form of a single digit times an integer power of 10. Compare numbers expressed in this form, expressing how many times larger or smaller one is than the other.

Modified Student Learning Objectives/Standard:


<table>
<thead>
<tr>
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<th>Tasks/Activities</th>
</tr>
</thead>
<tbody>
<tr>
<td>MP 2</td>
<td>8.EE.3</td>
<td>Very large and very small quantities can be approximated with numbers expressed in the form of a single digit times an integer power of 10.</td>
<td>How can I use the powers of 10 to express very small and very large quantities?</td>
<td>Orders of Magnitude</td>
</tr>
<tr>
<td>MP 4</td>
<td></td>
<td>Estimate very large and very small quantities with numbers expressed in the form of a single digit times an integer power of 10.</td>
<td>Students will understand that if the exponent increases by 1, the value increases 10 times.</td>
<td>Exploring Powers of 10</td>
</tr>
<tr>
<td>MP 5</td>
<td></td>
<td>Compare numbers written in the form of a single digit times an integer power of 10 and express how many times as much one is than the other.</td>
<td>Using this system, how can you determine how much larger or how much smaller one quantity is compared to another?</td>
<td>Magnitude &amp; Estimating Quantities</td>
</tr>
<tr>
<td>MP 6</td>
<td></td>
<td></td>
<td>Why is this type of system necessary?</td>
<td></td>
</tr>
<tr>
<td>MP 7</td>
<td></td>
<td></td>
<td>How can you define zero and negative exponents?</td>
<td></td>
</tr>
<tr>
<td>MP 8</td>
<td>SPED Strategies: Review and practice the use of patterns in order to develop the understanding of powers of 10.</td>
<td>Review writing a number expressed as a single digit times an integer power of 10.</td>
<td>How Many Times in a Millennium</td>
<td></td>
</tr>
</tbody>
</table>
Review reading numbers that are powers of ten; create and practice writing numbers that are powers of ten.

Provide a calculator.

**Resources UDL - Visual and Auditory Learner(s):**
*Roosevelt Middle School Math Team -- Common Core Tutorial Videos -- 8.EE.A.3*
[https://youtu.be/nCBK7NTRUQ4](https://youtu.be/nCBK7NTRUQ4)

*Scientific Notation - Converting to Standard Form 8.EE.A.3 8th Grade Math*
[https://youtu.be/92sMn3yGlm8](https://youtu.be/92sMn3yGlm8)

**ELL Strategies:**
Incorporate lesson introductory writing activities using their math journals.

Model structure and clarify unfamiliar syntax.

Have students visualize and create their own conversion charts, and diagrams with measurement labels.

Utilize gestures, or L1(student native language) translations, to demonstrate vocabulary comprehension and word-meaning.

Have students work with partners, small groups.

Create word/picture wall with L1(student native language) translations side by side.

How can you read numbers that are written in scientific notation?
Provide supplemental resources with L1(student native language) text and/or support.

Utilize pictures/illustrations and have student write meaning in their Math Journals.

Develop interactive games and activities to promote retention.

New Jersey Student Learning Standard(s):

8.EE.A.4: Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.

Student Learning Objective 12: Perform operations using numbers expressed in scientific notation, including problems where both decimals and scientific notation are used. In real-world problem-solving situations, choose units of appropriate size for measurement of very small and very large quantities and interpret scientific notation generated when technology has been used for calculations.

Modified Student Learning Objectives/Standards:


<table>
<thead>
<tr>
<th>MP 2</th>
<th>Evidence Statement Key/ Clarifications</th>
<th>Skills, Strategies &amp; Concepts</th>
<th>Essential Understandings/Questions (Accountable Talk)</th>
<th>Tasks/Activities</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>8.EE.4</td>
<td>Multiply and divide numbers expressed in scientific notation, including problems in which one number is in decimal form and one is in scientific notation.</td>
<td>Why is it important to perform operations in scientific notation?</td>
<td>Ants vs. Humans</td>
</tr>
<tr>
<td></td>
<td>• Tasks have “thin context” 2 or no context.</td>
<td>Add and subtract numbers expressed in scientific notation, including problems in</td>
<td>Why is it important that everyone correctly puts their solutions in scientific notation?</td>
<td>E. Coli</td>
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<tr>
<td></td>
<td>• Rules or conventions for significant figures are not assessed.</td>
<td></td>
<td></td>
<td>Giant Burgers</td>
</tr>
</tbody>
</table>
### MP 5
- Some of the tasks involve both decimal and scientific notation

### MP 6
- which one number is in decimal form and one is in scientific notation.
- Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities.
- Interpret scientific notation that has been generated by technology (e.g. recognize 4.1E-2 and 4.1e-2 as 4.1 \times 10^{-2}).

**SPED Strategies:**
- Review and practice the use patterns in order to develop understanding of powers of 10.
- Review and drill writing a number expressed as a single digit times an integer power of 10.
- Review and provide strategies on how to read numbers in scientific notation.
- Review writing numbers in scientific notation.
- Review converting numbers from standard form to scientific notation and vice versa.
- Create and provide conversion numbers from standard form to scientific notation chart and/or flip chart.
- Provide a calculator.

### MP 7
- How do you change between scientific notation and standard notation?
- Is it always easier to perform an operation in scientific notation? Is it always easier to perform an operation in standard form?
- Justify your answer.
- Real world situation involving exponential relationships can be solved using multiple representations.

### MP 8
- 100 People
- Pennies to Heaven
- Estimating Length Using Scientific Notation
- Scientific Notation
- More Scientific Notation
- Expression and Equation-Short Task
Resources (UDL - Visual and Auditory Learner(s):
Roosevelt Middle School Math Team -- Common Core Tutorial Videos -- 8.EE.A.4
https://youtu.be/Pr5CEP56hZQ

Comparing Scientific Notated Numbers - How Many Times Bigger 8.EE.A.4 8th Grade Math
https://youtu.be/MvA7IVSWn4Y

ELL Strategies:
Incorporate lesson introductory writing activities using their math journals.

Model structure and clarify unfamiliar syntax.

Have students visualize and create their own conversion charts, and diagrams with measurement labels.

Utilizing gestures, or L1 (student native language) translations, to demonstrate vocabulary comprehension and word-meaning.

Have students work with partners, small groups.

Provide supplemental resources with L1 (students’ native language) text and/or support.
| | Provide pictures/illustrations and have students write meaning in their Math Journals. |
| | Develop interactive games and activities to promote retention. |
Integrated Evidence Statements

7.C.4: Base explanations/reasoning on a coordinate plane diagram (whether provided in the prompt or constructed by the student in her response).
   - Tasks use only coordinates in Quadrant 1 and use only a positive constant of proportionality.

7.C.5: Given an equation, present the solution steps as a logical argument that concludes with the set of solutions (if any).

7.C.8: Construct, autonomously, chains of reasoning that will justify or refute propositions or conjectures.
   - Tasks may have scaffolding, if necessary, in order to yield a degree of difficulty appropriate to Grade 7.

7.D.2: Solve multi-step contextual problems with degree of difficulty appropriate to grade 7, requiring application of knowledge and skills articulated in 6.RP.A, 6.EE.C, 6.G.
   - Tasks may have scaffolding, if necessary, in order to yield a degree of difficulty appropriate to grade 7.

7.D.3: Micro-models: Autonomously apply a technique from pure mathematics to a real-world situation in which the technique yields valuable results even though it is obviously not applicable in a strict mathematical sense (e.g., profitably applying proportional relationships to a phenomenon that is obviously nonlinear or statistical in nature).
   - Tasks may have scaffolding, if necessary, in order to yield a degree of difficulty appropriate to grade 7.

   - Tasks may have scaffolding, if necessary, in order to yield a degree of difficulty appropriate to grade 7.

8.C.5.3: Apply geometric reasoning in a coordinate setting, and/or use coordinates to draw geometric conclusions. Content Scope: Knowledge and skills articulated in 8.G.B
   - Some of tasks require students to use the converse of the Pythagorean Theorem.

8.C.6: Construct, autonomously, chains of reasoning that will justify or refute propositions or conjectures. Content Scope: Knowledge and skills articulated in 7.RP.A, 7NS.A, 7.EE.A.
### Integrated Evidence Statements

8.EE.C.Int.1: Solve word problems leading to linear equations in one variable whose solutions require expanding expressions using the distributive property and collecting like terms.

8. D.1: Solve multi-step contextual word problems with degree of difficulty appropriate to Grade 8, requiring application of knowledge and skills articulated in Type I, Sub-Claim A Evidence Statements.
   - Some of the tasks may use scaffolding.


Content Scope: Knowledge and skills articulated in Type I, Sub-Claim A Evidence Statements

Some of the tasks may use scaffolding.
<table>
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<th>Unit 4 Vocabulary</th>
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<td>Altitude (of a Triangle)</td>
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<td>Center of Rotation</td>
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<td>Circumference</td>
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<td>Clockwise</td>
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<td>Complementary Angle</td>
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<td>Cone</td>
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<td>Congruent</td>
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<td>Constant</td>
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<td>Constant of Proportionality</td>
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<td>Converse of Pythagorean Theorem</td>
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<td>Corresponding Angles</td>
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<td>Cross Section</td>
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<td>Cylinder</td>
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<td>Dimensions</td>
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<td>Discount</td>
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<td>Distance Formula</td>
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<td>Equivalent Fractions</td>
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<td>Exterior Angles</td>
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<td>Face</td>
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<td>Geometric Solid</td>
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<td>Height</td>
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<td>Hypotenuse</td>
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<td>Image</td>
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<tr>
<td>Inscribed</td>
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<tr>
<td>Interior Angle</td>
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</tbody>
</table>
## References & Suggested Instructional Websites

- [http://illuminations.nctm.org.index](http://illuminations.nctm.org.index)
- [www.internet4classrooms.com](http://www.internet4classrooms.com)
- [www.illustrativemathematics.org/](http://www.illustrativemathematics.org/)
- [http://www.katm.org/flipbooks/7%20FlipBook%20Final%20CCSS%202014.pdf](http://www.katm.org/flipbooks/7%20FlipBook%20Final%20CCSS%202014.pdf)
- [https://www.georgiastandards.org/Common-Core/Pages/Math-6-8.aspx](https://www.georgiastandards.org/Common-Core/Pages/Math-6-8.aspx)
- [https://learnzillion.com/](https://learnzillion.com/)
- [http://www.insidemathematics.org/](http://www.insidemathematics.org/)
- [https://www.engageny.org/](https://www.engageny.org/)
Field Trip Ideas

**Buehler Challenger & Science Center** – [http://www.bcsc.org/5-9th-grade/](http://www.bcsc.org/5-9th-grade/)

- Participants work as a team as they take on the role of astronauts and mission controllers to *Rendezvous with Comet Halley*, *Return to the Moon*, or *Voyage to Mars*. Students use team-building and hands-on learning with a focus on STEM to complete their mission goal.


**Fiery Hazards @ Liberty Science Center**

- Classify and model different types of volcanoes. Study a series of eruptions and construct an explanation of how lava flows have changed Earth’s surface at varying times and spatial scales.


- For more than 20 years, educators from around the country have been bringing students to the Museum to help them understand how finance impacts their daily lives. The Museum offers discounted admission for pre-booked groups of eight or more, as well as a variety of classes for students in middle school through college.


- Mathematics illuminates the patterns and structures all around us. Our dynamic exhibits, gallery, and programs will stimulate inquiry, spark curiosity, and reveal the wonders of mathematics. MoMath has innovative exhibits that will engage folks from 105 to 5 years old (and sometimes younger), but with a special emphasis on activities for 4th through 8th graders.