Pre-Algebra: Unit 3
Drawing Inferences about Populations & Probability Models
Course Philosophy/Description

In mathematics, students will focus on the application of concepts, skills and understandings. Students will solve problems involving the key knowledge and skills identified by the NJSLS: express mathematical reasoning, construct a mathematical argument and apply concepts to solve real world problems. The balanced math instructional model will be used as the basis for all mathematics instruction.

Pre-Algebra consists of the following domains: Ratios and Proportional Relationships (RP), The Number System (NS), Expressions and Equations (EE), Geometry (G), and Statistics and Probability (SP). Instructional time should focus on four critical areas: (1) building understanding of and applying proportional relationships; (2) building understanding of real numbers and working with expressions and linear equations; (3) solving problems involving scale drawings, informal geometric constructions, area, surface area, and volume; and (4) drawing inferences about populations based on samples.

1) Students extend their understanding of ratios and develop understanding of proportionality by exploring a wide variety of percent problems, including those involving discounts, interest, taxes, tips, and percent increase or decrease. Students solve problems about scale drawings, graph proportional relationships, and understand the unit rate informally as a measure of the steepness of the related line, called the slope. They distinguish proportional relationships from other relationships.

2) Students develop a unified understanding of real numbers, recognizing that numbers are rational or irrational, and understand that rational numbers can be expressed as fractions, decimals, and percents. Students extend mathematical operations to all rational numbers, maintaining the properties of operations and the relationships between operations. By applying these properties, and by viewing negative numbers in terms of everyday contexts, students explain and interpret the rules for mathematical operations with negative numbers. They use the arithmetic of rational numbers as they formulate expressions and equations with variables to solve problems. Students identify irrational numbers as well as a rational approximation for an irrational number.

3) Students work with three-dimensional figures, relating them to two-dimensional figures by examining cross-sections. They solve real-world mathematical problems involving circumference, area, surface area, and volume. In preparation for work with congruence and similarity in Grade 8, students reason about relationships among two-dimensional figures. They use scale drawings and informal geometric constructions to gain familiarity with the relationships between angles formed by intersecting lines. This understanding is deepened by learning transformations. Students are able to identify the sum of the angles in triangles of various configurations. Students understand the statement of the Pythagorean Theorem and its converse and can explain why the Pythagorean Theorem holds. They apply the Pythagorean Theorem to find distances between points on the coordinate plane, to find lengths, and to analyze polygons.

4) Students build on their previous work with single data distributions to compare two data distributions and address questions about differences between populations. They begin informal work with random sampling to generate data sets and learn about the importance of representative samples for drawing inferences.
ESL Framework

This ESL framework was designed to be used by bilingual, dual language, ESL and general education teachers. Bilingual and dual language programs use the home language and a second language for instruction. ESL teachers and general education or bilingual teachers may use this document to collaborate on unit and lesson planning to decide who will address certain components of the SLO and language objective. ESL teachers may use the appropriate leveled language objective to build lessons for ELLs which reflects what is covered in the general education program. In this way, whether it is a pull-out or push-in model, all teachers are working on the same Student Learning Objective connected to the New Jersey Student Learning Standard. The design of language objectives are based on the alignment of the World-Class Instructional Design Assessment (WIDA) Consortium’s English Language Development (ELD) standards with the New Jersey Student Learning Standards (NJSLS). WIDA’s ELD standards advance academic language development across content areas ultimately leading to academic achievement for English learners. As English learners are progressing through the six developmental linguistic stages, this framework will assist all teachers who work with English learners to appropriately identify the language needed to meet the requirements of the content standard. At the same time, the language objectives recognize the cognitive demand required to complete educational tasks. Even though listening and reading (receptive) skills differ from speaking and writing (expressive) skills across proficiency levels the cognitive function should not be diminished. For example, an Entering Level One student only has the linguistic ability to respond in single words in English with significant support from their home language. However, they could complete a Venn diagram with single words which demonstrates that they understand how the elements compare and contrast with each other or they could respond with the support of their home language (L1) with assistance from a teacher, para-professional, peer or a technology program.

http://www.state.nj.us/education/modelcurriculum/ela/ELLOverview.pdf
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<th>#</th>
<th>Student Learning Objective</th>
<th>NJSLS</th>
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<tbody>
<tr>
<td>1</td>
<td>Distinguish between representative and non-representative samples of a population (e.g. if the class had 50% girls and the sample had 10% girls, then that sample was not representative of the population).</td>
<td>7.SP.A.1</td>
</tr>
<tr>
<td>2</td>
<td>Use random sampling to produce a representative sample.</td>
<td>7.SP.A.2</td>
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<tr>
<td>3</td>
<td>Develop inferences about a population using data from a random sample and assess the variation in estimates after generating multiple samples of the same size.</td>
<td>7.SP.A.2</td>
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<tr>
<td>4</td>
<td>Visually compare the means of two distributions that have similar variability; express the difference between the centers as a multiple of a measure of variability.</td>
<td>7.SP.B.3</td>
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<tr>
<td>5</td>
<td>Draw informal comparative inferences about two populations using their measures of center and measures of variability.</td>
<td>7.SP.B.4</td>
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<td>6</td>
<td>Interpret and express the likelihood of a chance event as a number between 0 and 1, relating that the probability of an unlikely event happening is near 0, a likely event is near 1, and 1/2 is neither likely nor unlikely.</td>
<td>7.SP.C.5</td>
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<tr>
<td>7</td>
<td>Approximate the probability of a chance event by collecting data and observing long-run relative frequency; predict the approximate relative frequency given the probability.</td>
<td>7.SP.C.6</td>
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<tr>
<td>8</td>
<td>Develop a uniform probability model by assigning equal probability to all outcomes; develop probability models by observing frequencies and use the models to determine probabilities of events; compare probabilities from a model to observed frequencies and explain sources of discrepancy when agreement is not good.</td>
<td>7.SP.C.7</td>
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<tr>
<td>9</td>
<td>Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams, identifying the outcomes in the sample space which compose the event. Use the sample space to find the probability of a compound event.</td>
<td>7.SP.C.8a, 8b</td>
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<tr>
<td>10</td>
<td>Design and use a simulation to generate frequencies for compound events.</td>
<td>7.SP.C.8c</td>
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<tr>
<td>11</td>
<td>Know the formulas for the area and circumference of a circle and use them to solve problems. Give an informal derivation of the relationship between the circumference and area of a circle.</td>
<td>7.G.B.4</td>
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<tr>
<td>12</td>
<td>Describe all of the 2-dimensional figures that result when a 3-dimensional figure is sliced from multiple angles.</td>
<td>7.G.A.3</td>
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</table>
Research about Teaching and Learning Mathematics

Structure teaching of mathematical concepts and skills around problems to be solved (Checkly, 1997; Wood & Sellars, 1996; Wood & Sellars, 1997)

Encourage students to work cooperatively with others (Johnson & Johnson, 1975; Davidson, 1990)

Use group problem-solving to stimulate students to apply their mathematical thinking skills (Artzt & Armour-Thomas, 1992)

Students interact in ways that support and challenge one another’s strategic thinking (Artzt, Armour-Thomas, & Curcio, 2008)

Activities structured in ways allowing students to explore, explain, extend, and evaluate their progress (National Research Council, 1999)

There are three critical components to effective mathematics instruction (Shellard & Moyer, 2002):

- Teaching for conceptual understanding
- Developing children’s procedural literacy
- Promoting strategic competence through meaningful problem-solving investigations

Teachers should be:

- Demonstrating acceptance and recognition of students’ divergent ideas
- Challenging students to think deeply about the problems they are solving, extending thinking beyond the solutions and algorithms required to solve the problem
- Influencing learning by asking challenging and interesting questions to accelerate students’ innate inquisitiveness and foster them to examine concepts further
- Projecting a positive attitude about mathematics and about students’ ability to “do” mathematics

Students should be:

- Actively engaging in “doing” mathematics
- Solving challenging problems
- Investigating meaningful real-world problems
- Making interdisciplinary connections
- Developing an understanding of mathematical knowledge required to “do” mathematics and connect the language of mathematical ideas with numerical representations
- Sharing mathematical ideas, discussing mathematics with one another, refining and critiquing each other’s ideas and understandings
- Communicating in pairs, small group, or whole group presentations
- Using multiple representations to communicate mathematical ideas
- Using connections between pictures, oral language, written symbols, manipulative models, and real-world situations
- Using technological resources and other 21st century skills to support and enhance mathematical understanding
Mathematics is not a stagnate field of textbook problems; rather, it is a dynamic way of constructing meaning about the world around us, generating knowledge and understanding about the real world every day. Students should be metaphorically rolling up their sleeves and “doing mathematics” themselves, not watching others do mathematics for them or in front of them. (Protheroe, 2007)

**Balanced Mathematics Instructional Model**

Balanced math consists of three different learning opportunities: guided math, shared math, and independent math. Ensuring a balance of all three approaches will build conceptual understanding, problem solving, computational fluency, and procedural fluency. Building conceptual understanding is the focal point of developing mathematical proficiency. Students should frequently work on rigorous tasks, talk about the math, explain their thinking, justify their answer or process, build models with graphs or charts or manipulatives, and use technology.

When balanced math is used in the classroom it provides students opportunities to:

- solve problems
- make connections between math concepts and real-life situations
- communicate mathematical ideas (orally, visually and in writing)
- choose appropriate materials to solve problems
- reflect and monitor their own understanding of the math concepts
- practice strategies to build procedural and conceptual confidence

**Teacher builds conceptual understanding by modeling through demonstration, explicit instruction, and think alouds, as well as guiding students as they practice math strategies and apply problem solving strategies. (whole group or small group instruction)**

**Teacher and students practice mathematics processes together through interactive activities, problem solving, and discussion. (whole group or small group instruction)**

**Students practice math strategies independently to build procedural and computational fluency. Teacher assesses learning and reteaches as necessary. (whole group instruction, small group instruction, or centers)**
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<td>Making Thinking Visible</td>
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<td>Develop and Demonstrate Mathematical Practices</td>
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<td>Inquiry-Oriented and Exploratory Approach</td>
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<td>Multiple Solution Paths and Strategies</td>
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<td>Explain the Rationale of your Math Work</td>
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<td>Interviews</td>
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<td>Role Playing</td>
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<td>Diagrams, Charts, Tables, and Graphs</td>
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<td>Anticipate Likely and Possible Student Responses</td>
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<td>Collect Different Student Approaches</td>
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<td>Challenging</td>
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<td>Pressing for Accuracy and Reasoning</td>
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<td>Maintain the Cognitive Demand</td>
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Educational Technology

Standards

8.1.8.A.1, 8.1.8.A.3, 8.1.8.E.1, 8.2.8.C.8, 8.2.8.E.3

- **Technology Operations and Concepts**
  - Demonstrate knowledge of a real world problem using digital tools.
    - Example: Students use the Random Number Generator to attain a random sample and develop inferences to a larger population. [http://stattrek.com/statistics/random-number-generator.aspx](http://stattrek.com/statistics/random-number-generator.aspx)
  - Use and/or develop a simulation that provides an environment to solve a real world problem or theory.

- **Research and Information Fluency**
  - Effectively use a variety of search tools and filters in professional public databases to find information to solve a real world problem.
    - Example: Students can search through Learnzillion, Khan Academy and other interactive sites for appropriate instructional videos and/or information pertaining to strategies and modeling for creating sample spaces for compound events.

- **Design**
  - Develop a proposal for a chosen solution that include models (physical, graphical or mathematical) to communicate the solution to peers.

- **Computational Thinking: Programming**:
  - Develop an algorithm to solve an assigned problem using a specified set of commands and use peer review to critique the solution.
    - Example: Student can use conceptual investigations to create algorithms for finding measures of center and measures of variability.
Career Ready Practices

Career Ready Practices describe the career-ready skills that all educators in all content areas should seek to develop in their students. They are practices that have been linked to increase college, career, and life success. Career Ready Practices should be taught and reinforced in all career exploration and preparation programs with increasingly higher levels of complexity and expectation as a student advances through a program of study.

- **CRP2. Apply appropriate academic and technical skills.**
  Career-ready individuals readily access and use the knowledge and skills acquired through experience and education to be more productive. They make connections between abstract concepts with real-world applications, and they make correct insights about when it is appropriate to apply the use of an academic skill in a workplace situation.

  **Example:** Students will apply prior knowledge when solving real-world problems. Students will make sound judgements about the use of specific tools, such as Random Number Generator to attain a random sample and develop inferences to a larger population.

- **CRP4. Communicate clearly and effectively and with reason.**
  Career-ready individuals communicate thoughts, ideas, and action plans with clarity, whether using written, verbal, and/or visual methods. They communicate in the workplace with clarity and purpose to make maximum use of their own and others’ time. They are excellent writers; they master conventions, word choice, and organization, and use effective tone and presentation skills to articulate ideas. They are skilled at interacting with others; they are active listeners and speak clearly and with purpose. Career-ready individuals think about the audience for their communication and prepare accordingly to ensure the desired outcome.

  **Example:** Students will on a daily basis communicate their reasoning behind their solution paths by making connections to the context and the quantities, using proper vocabulary, along with decontextualizing and/or contextualizing the problem. Students will create organized lists, tables, and tree diagrams to represent sample spaces for compound events. They will also explain the meaning behind the quantities and units involved. Students will also ask probing questions to clarify and improve arguments.

- **CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.**
  Career-ready individuals readily recognize problems in the workplace, understand the nature of the problem and devise effective plans to solve the problem. They are aware of problems when they occur and take action quickly to address the problem; they thoughtfully investigate the root cause of the problem prior to introducing solutions. They carefully consider the options to solve the problem. Once a solution is agreed upon, they follow through to ensure the problem is solved, whether through their own actions or the actions of others.
Example: Throughout their daily lessons, students will understand the meaning of a problem and look for entry points into solving their problems by analyzing the relationships of the quantities, constraints and goals of the task. Plans for solution paths will be made and have meaning. Students will self-monitor, evaluate and critique their process and progress as they are working and make changes as necessary.

- **CRP12. Work productively in teams while using cultural global competence.**
  Career-ready individuals positively contribute to every team, whether formal or informal. They apply an awareness of cultural difference to avoid barriers to productive and positive interaction. They find ways to increase the engagement and contribution of all team members. They plan and facilitate effective team meetings.

Example: Students will work in collaborative and whole group settings to develop various solutions to math tasks that are presented to them. They will work together to understand the terms of the problem, ask clarifying and challenging questions among each other, and develop agreed upon solutions using a variety of strategies and models. Students will listen to, read and discuss arguments with each other with respect and courtesy at all times and will be willing to assist those that may need assistance. In this unit students will demonstrate and explain to a peer or small group how to distinguish between a representative and non-representative sample of a population.
## WIDA Proficiency Levels

At the given level of English language proficiency, English language learners will process, understand, produce or use:

<table>
<thead>
<tr>
<th>Level</th>
<th>Language Proficiency</th>
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</table>
| 6- Reaching | - Specialized or technical language reflective of the content areas at grade level  
- A variety of sentence lengths of varying linguistic complexity in extended oral or written discourse as required by the specified grade level  
- Oral or written communication in English comparable to proficient English peers |
| 5- Bridging | - Specialized or technical language of the content areas  
- A variety of sentence lengths of varying linguistic complexity in extended oral or written discourse, including stories, essays or reports  
- Oral or written language approaching comparability to that of proficient English peers when presented with grade level material. |
| 4- Expanding | - Specific and some technical language of the content areas  
- A variety of sentence lengths of varying linguistic complexity in oral discourse or multiple, related sentences or paragraphs  
- Oral or written language with minimal phonological, syntactic or semantic errors that may impede the communication, but retain much of its meaning, when presented with oral or written connected discourse, with sensory, graphic or interactive support |
| 3- Developing | - General and some specific language of the content areas  
- Expanded sentences in oral interaction or written paragraphs  
- Oral or written language with phonological, syntactic or semantic errors that may impede the communication, but retain much of its meaning, when presented with oral or written, narrative or expository descriptions with sensory, graphic or interactive support |
| 2- Beginning | - General language related to the content area  
- Phrases or short sentences  
- Oral or written language with phonological, syntactic, or semantic errors that often impede the communication when presented with one to multiple-step commands, directions, or a series of statements with sensory, graphic or interactive support |
| 1- Entering | - Pictorial or graphic representation of the language of the content areas  
- Words, phrases or chunks of language when presented with one-step commands directions, WH-, choice or yes/no questions, or statements with sensory, graphic or interactive support |
Language Development Supports For English Language Learners To Increase Comprehension and Communication Skills

Environment
- Welcoming and stress-free
- Respectful of linguistic and cultural diversity
- Honors students' background knowledge
- Sets clear and high expectations
- Includes routines and norms
- Is thinking-focused vs. answer-seeking
- Offers multiple modalities to engage in content learning and to demonstrate understanding
- Includes explicit instruction of specific language targets
- Provides participation techniques to include all learners
- Integrates learning centers and games in a meaningful way
- Provides opportunities to practice and refine receptive and productive skills in English as a new language
- Integrates meaning and purposeful tasks/activities that:
  - Are accessible by all students through multiple entry points
  - Are relevant to students' lives and cultural experiences
  - Build on prior mathematical learning
  - Demonstrate high cognitive demand
  - Offer multiple strategies for solutions
  - Allow for a language learning experience in addition to content

Sensory Supports*
- Real-life objects (realia) or concrete objects
- Physical models
- Manipulatives
- Pictures & photographs
- Visual representations or models such as diagrams or drawings
- Videos & films
- Newspapers or magazines
- Gestures
- Physical movements
- Music & songs

Graphic Supports*
- Graphs
- Charts
- Timelines
- Number lines
- Graphic organizers
- Graphing paper

Interactive Supports*
- In a whole group
- In a small group
- With a partner such as Turn-and-Talk
- In pairs as a group (first, two pairs work independently, then they form a group of four)
- In triads
- Cooperative learning structures such as Think-Pair-Share
- Interactive websites or software
- With a mentor or coach

Verbal and Textual Supports
- Labeling
- Students' native language
- Modeling
- Repetitions
- Paraphrasing
- Summarizing
- Guiding questions
- Clarifying questions
- Probing questions
- Leveled questions such as What? When? Where? How? Why?
- Questioning prompts & cues
- Word Banks
- Sentence starters
- Sentence frames
- Discussion frames
- Talk moves, including Wait Time

# Building Equity in Your Teaching Practice

How do the essential questions highlight the connection between the big ideas of the unit and equity in your teaching practice?

## Content Integration
Teachers use examples and content from a variety of cultures & groups.

- This unit / lesson is connected to other topics explored with students.
- There are multiple viewpoints reflected in the content of this unit / lesson.
- The materials and resources are reflective of the diverse identities and experiences of students.
- The content affirms students, as well as exposes them to experiences other than their own.

## Knowledge Construction
Teachers help students understand how knowledge is created and influenced by cultural assumptions, perspectives & biases.

- This unit / lesson provides context to the history of privilege and oppression.
- This unit / lesson addresses power relationships.
- This unit / lesson help students to develop research and critical thinking skills.
- This curriculum creates windows and mirrors* for students.

## Prejudice Reduction
Teachers implement lessons and activities to assert positive images of ethnic groups & improve intergroup relations.

- This unit / lesson help students question and unpack biases & stereotypes.
- This unit / lesson help students examine, research and question information and sources.
- The curriculum encourage discussion and understanding about the groups of people being represented.
- This unit / lesson challenges dominant perspectives.

## Equitable Pedagogy
Teachers modify techniques and methods to facilitate the academic achievement of students from diverse backgrounds.

- The instruction has been modified to meet the needs of each student.
- Students feel respected and their cultural identities are valued.
- Additional supports have been provided for students to become successful and independent learners.
- Opportunities are provided for student to reflect on their learning and provide feedback.

## Empowering School Culture
Using the other four dimensions to create a safe and healthy educational environment for all.

- There are opportunities for students to connect with the community.
- My classroom is welcoming and supportive for all students?
- I am aware of and sensitive to the needs of my students and their families.
- There are effective parent communication systems established. Parents can talk to me about issues as they arise in my classroom.

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Culturally Relevant Pedagogy Examples

- **Present New Concepts Using Student Vocabulary:** Use student diction to capture attention and build understanding before using academic terms.
  
  **Example:** Work with students to create a variety of sorting and matching games of vocabulary words in this unit. Students can work in teams or individually to play these games for approximately 10-15 minutes each week. This will provide students a hands-on opportunity to familiarize themselves with new vocabulary.

- **Integrate Relevant Word Problems:** Contextualize equations using word problems that reference students’ interest.
  
  **Example:** Create and use word problems that will promote students’ curiosity and self-motivation to explore concepts. The following provides a real world application problem related to the World Cup.
  
  [Link](https://www.yummymath.com/wp-content/uploads/WorldCup-Part-II%E2%80%93Pots2018.pdf)

- **Encourage Students to Propose Ideas for Projects:** Let students take projects from concept to completion by pitching your idea, allowing them to showcase their strengths.
  
  **Example:** Students will develop project ideas that meet grade level standards. Assist student in choosing from a list of options to refine their ideas in order to meet standards.
  
  [Link](https://sites.google.com/a/leanderisd.org/cpms-7th-grade-math/pace-math/statistics-project)

- **Call on Each Student:** Encourage each student to share his or her thoughts through call-and-response, keeping the class’s attention in the process.
  
  **Example:** Foster confidence. Make the assessment process less intimidating by offering different ways to demonstrate skills and understanding. For example, students conduct surveys to find out their classmates’ favorite TV shows and determine whether the sample is representative or biased.
SOCIAL AND EMOTIONAL LEARNING (SEL) COMPETENCIES

SELF-AWARENESS
The ability to accurately recognize one’s own emotions, thoughts, and values and how they influence behavior. The ability to accurately assess one’s strengths and limitations, with a well-grounded sense of confidence, optimism, and a “growth mindset.”
- Identifying Emotions
- Accurate self-perception
- Recognizing strengths
- Self-confidence
- Self-efficacy

SOCIAL AWARENESS
The ability to take the perspective of and empathize with others, including those from diverse backgrounds and cultures. The ability to understand social and ethical norms for behavior and to recognize family, school, and community resources and supports.
- Perspective-taking
- Empathy
- Appreciating diversity
- Respect for others

RESPONSIBLE DECISION-MAKING
The ability to make constructive choices about personal behavior and social interactions based on ethical standards, safety concerns, and social norms. The realistic evaluation of consequences of various actions, and a consideration of the well-being of oneself and others.
- Identifying problems
- Analyzing situations
- Solving problems
- Evaluating
- Reflecting
- Ethical responsibility

SELF-MANAGEMENT
The ability to successfully regulate one’s emotions, thoughts, and behaviors in different situations — effectively managing stress, controlling impulses, and motivating oneself. The ability to set and work toward personal and academic goals.
- Impulse control
- Stress management
- Self-discipline
- Self-motivation
- Goal setting
- Organizational skills

RELATIONSHIP SKILLS
The ability to establish and maintain healthy and rewarding relationships with diverse individuals and groups. The ability to communicate clearly, listen well, cooperate with others, resist inappropriate social pressure, negotiate conflict constructively, and seek and offer help when needed.
- Communication
- Social engagement
- Relationship building
- Teamwork

Diagram:
- Homes and communities
- Schools
- Classrooms
- Self-awareness
- Self-management
- Social awareness
- Relationship skills
- Responsible decision-making

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<table>
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<th>SEL Competency</th>
<th>Examples</th>
<th>Content Specific Activity &amp; Approach to SEL</th>
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</table>
| ✔ Self-Awareness  
Self-Management  
Social-Awareness  
Relationship Skills  
Responsible Decision-Making | **Example practices that address Self-Awareness:**  
• Clearly state classroom rules  
• Provide students with specific feedback regarding academics and behavior  
• Offer different ways to demonstrate understanding  
• Create opportunities for students to self-advocate  
• Check for student understanding / feelings about performance  
• Check for emotional wellbeing  
• Facilitate understanding of student strengths and challenges | Provide feedback on activities that is timely, reinforce what students did well, and outline specific strategies for students to practice in order for them to improve upon their performance for the next assigned task.  
Students see connections between current tasks and their personal interests.  
Students analyze and distinguish between representative and non-representative samples of a population of their choosing. |
| ✔ Self-Management  
Self-Awareness  
Social-Awareness Skills  
Relationship Skills  
Responsible Decision-Making | **Example practices that address Self-Management:**  
• Encourage students to take pride/ownership in work and behavior  
• Encourage students to reflect and adapt to classroom situations  
• Assist students with being ready in the classroom  
• Assist students with managing their own emotional states | Students reflect on barriers they may encounter when completing an assignment by asking clarifying questions, trying out others’ strategies, and describing the approaches when drawing inferences about populations and probability models.  
Have students create a graph that shows progress in ALEKS or Imagine Math Facts.  
Have students brainstorm ways to motivate themselves; perhaps finding the probability of winning a game of chance. |
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<th>Self-Awareness</th>
<th>Social-Awareness</th>
<th>Relationship Skills</th>
<th>Responsible Decision-Making</th>
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<td>Self-Management</td>
<td><strong>Social-Awareness</strong></td>
<td>Relationship Skills</td>
<td>Responsible Decision-Making</td>
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<tr>
<th>Example practices that address Social-Awareness:</th>
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<tr>
<td>• Encourage students to reflect on the perspective of others</td>
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<td>• Assign appropriate groups</td>
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<tr>
<td>• Help students to think about social strengths</td>
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<td>• Provide specific feedback on social skills</td>
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<tr>
<td>• Model positive social awareness through metacognition activities</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Establish Math Centers as a strategy to encourage students to self-assess progress toward their learning goals. This strategy promotes academic growth.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lead a discussion about the difference between theoretical and experimental probability, and how students can use these ideas to make predictions.</td>
</tr>
<tr>
<td>Build respect for diversity in the classroom by having students share their different perspectives on situations or solution strategies.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Self-Awareness</th>
<th>Social-Awareness</th>
<th>Relationship Skills</th>
<th>Responsible Decision-Making</th>
</tr>
</thead>
<tbody>
<tr>
<td>✓ <strong>Relationship Skills</strong></td>
<td></td>
<td></td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Example practices that address Relationship Skills:</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Engage families and community members</td>
</tr>
<tr>
<td>• Model effective questioning and responding to students</td>
</tr>
<tr>
<td>• Plan for project-based learning</td>
</tr>
<tr>
<td>• Assist students with discovering individual strengths</td>
</tr>
<tr>
<td>• Model and promote respecting differences</td>
</tr>
<tr>
<td>• Model and promote active listening</td>
</tr>
<tr>
<td>• Help students develop communication skills</td>
</tr>
<tr>
<td>• Demonstrate value for a diversity of opinions</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Teach lessons that develop mathematical discourse and engage students in purposeful sharing of mathematical ideas, reasoning, and approaches using varied representations.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model and reinforce effective communication, relationship, and conflict-resolution skills that encourage students to feel comfortable taking mathematical risks.</td>
</tr>
<tr>
<td>Students work with a partner to review, demonstrate, and practice how to create tree diagrams, frequency tables, organized lists, and simulations to determine the probability of compound events.</td>
</tr>
<tr>
<td>Self-Awareness</td>
</tr>
<tr>
<td>----------------------</td>
</tr>
<tr>
<td>Self-Management</td>
</tr>
<tr>
<td>Social-Awareness</td>
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</tbody>
</table>
## Differentiated Instruction

**Accommodate Based on Students Individual Needs: Strategies**

<table>
<thead>
<tr>
<th>Time/General</th>
<th>Processing</th>
<th>Comprehension</th>
<th>Recall</th>
</tr>
</thead>
<tbody>
<tr>
<td>Extra time for assigned tasks</td>
<td>Extra Response time</td>
<td>Precise processes for balanced math instructional model</td>
<td>Teacher-made checklist</td>
</tr>
<tr>
<td>Adjust length of assignment</td>
<td>Have students verbalize steps</td>
<td>Short manageable tasks</td>
<td>Use visual graphic organizers</td>
</tr>
<tr>
<td>Timeline with due dates for reports and projects</td>
<td>Repeat, clarify or reword directions</td>
<td>Brief and concrete directions</td>
<td>Reference resources to promote independence</td>
</tr>
<tr>
<td>Communication system between home and school</td>
<td>Mini-breaks between tasks</td>
<td>Provide immediate feedback</td>
<td>Visual and verbal reminders</td>
</tr>
<tr>
<td>Provide lecture notes/outline</td>
<td>Provide a warning for transitions</td>
<td>Small group instruction</td>
<td>Graphic organizers</td>
</tr>
<tr>
<td></td>
<td>Partnering</td>
<td>Emphasize multi-sensory learning</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Assistive Technology</th>
<th>Tests/Quizzes/Grading</th>
<th>Behavior/Attention</th>
<th>Organization</th>
</tr>
</thead>
<tbody>
<tr>
<td>Computer/whiteboard</td>
<td>Extended time</td>
<td>Consistent daily structured routine</td>
<td>Individual daily planner</td>
</tr>
<tr>
<td>Tape recorder</td>
<td>Study guides</td>
<td>Simple and clear classroom rules</td>
<td>Display a written agenda</td>
</tr>
<tr>
<td>Video Tape</td>
<td>Shortened tests</td>
<td>Frequent feedback</td>
<td>Note-taking assistance</td>
</tr>
<tr>
<td></td>
<td>Read directions aloud</td>
<td></td>
<td>Color code materials</td>
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</table>

- **Recall**
  - Teacher-made checklist
  - Use visual graphic organizers
  - Reference resources to promote independence
  - Visual and verbal reminders
  - Graphic organizers
Interdisciplinary Connections

Model interdisciplinary thinking to expose students to other disciplines.

Science Connection:  
*Counting Trees* (MS-LS2-4)  
- Students will determine how scientists make estimates about a population sizing using a representative sample by applying ratio and proportion, and organizing data in a frequency table.

*Predicting Population* (MS-ESS3-4)  
- In this task, students will use populations, samples, and proportions in order to make predictions about total population sizes.

Social Studies Connection:  
*Election Pole* (6.1.4.A.7)  
- This task introduces the fundamental statistical ideas of using summaries (statistics) from random samples to draw inferences (reasoned conclusions) about population characteristics (parameters).

ELA Connection:  
*Shakespeare vs. Harry Potter* (RL.8.1)  
- Students will analyze text from two periods of time to determine which author used longer words. Students will randomly sample given text from two pieces of literature. Students will find the mean, mean absolute deviation, and create box plots.

Various Tasks: (RL.7.1 and RI.7.1)  
- Students will be able to read, analyze, and cite informational text to solve problems and explain their reasoning of how the task was solved. Students will also focus on vocabulary, mechanics and grammar in effective writing.
## Enrichment

### What is the purpose of Enrichment?

- The purpose of enrichment is to provide extended learning opportunities and challenges to students who have already mastered, or can quickly master, the basic curriculum. Enrichment gives the student more time to study concepts with greater depth, breadth, and complexity.
- Enrichment also provides opportunities for students to pursue learning in their own areas of interest and strengths.
- Enrichment keeps advanced students engaged and supports their accelerated academic needs.
- Enrichment provides the most appropriate answer to the question, “What do you do when the student already knows it?”

### Enrichment is...

- Planned and purposeful
- *Different*, or differentiated, work – not just *more* work
- Responsive to students’ needs and situations
- A promotion of high-level thinking skills and making connections within content
- The ability to apply different or multiple strategies to the content
- The ability to synthesize concepts and make real world and cross-curricular connections
- Elevated contextual complexity
- Sometimes independent activities, sometimes direct instruction
- Inquiry based or open ended assignments and projects
- Using supplementary materials in addition to the normal range of resources
- Choices for students
- Tiered/Multi-level activities with flexible groups (may change daily or weekly)

### Enrichment is not...

- Just for gifted students (some gifted students may need intervention in some areas just as some other students may need frequent enrichment)
- Worksheets that are more of the same (busywork)
- Random assignments, games, or puzzles not connected to the content areas or areas of student interest
- Extra homework
- A package that is the same for everyone
- Thinking skills taught in isolation
- Unstructured free time
Assessments

**Required District/State Assessments**
- Unit Assessments
- SGO Assessments
- NJSLA

**Suggested Formative/Summative Classroom Assessments**
- Describe Learning Vertically
- Identify Key Building Blocks
- Make Connections (between and among key building blocks)
- Short/Extended Constructed Response Items
- Multiple-Choice Items (where multiple answer choices may be correct)
- Drag and Drop Items
- Use of Equation Editor
- Quizzes
- Journal Entries/Reflections/Quick-Writes
- Accountable talk
- Projects
- Portfolio
- Observation
- Graphic Organizers/Concept Mapping
- Presentations
- Role Playing
- Teacher-Student and Student-Student Conferencing
- Homework
New Jersey Student Learning Standards

7.SP.A.1: Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.

7.SP.A.2: Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.

7.SP.B.3: Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.

7.SP.B.4: Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.

7.SP.C.5: Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.

7.SP.C.6: Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.
New Jersey Student Learning Standards

**7.SP.C.7:** Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.

- **7.SP.C.7a:** Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. *For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.*

- **7.SP.C.7b:** Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. *For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?*

**7.SP.C.8:** Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.

- **7.SP.C.8a:** Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.

- **7.SP.C.8b:** Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., "rolling double sixes"), identify the outcomes in the sample space which compose the event.

- **7.SP.C.8c:** Design and use a simulation to generate frequencies for compound events. *For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?*

**7.G.B.4:** Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.

**7.G.A.3:** Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.
## Mathematical Practices

1. Make sense of problems and persevere in solving them.

2. Reason abstractly and quantitatively.

3. Construct viable arguments and critique the reasoning of others.

4. Model with mathematics.

5. Use appropriate tools strategically.

6. Attend to precision.

7. Look for and make use of structure.

8. Look for and express regularity in repeated reasoning.
Course: Grade 7 Pre-Algebra
Unit: 3 (Three)
Topic: Drawing Inferences about Populations & Probability Models

NJSLS:
7.SP.A.1, 7.SP.A.2, 7.SP.B.3, 7.SP.B.4, 7.SP.C.5, 7.SP.C.6, 7.SP.C.7a,b, 7.SP.C.8a,b,c, 7.G.B.4, 7.G.A.3

Unit Focus:
- Use random sampling to draw inferences about a population.
- Draw informal comparative inferences about two populations.
- Investigate chance processes and develop, use, and evaluate probability models.
- Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

New Jersey Student Learning Standard(s):

7.SP.A.1: Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.

Student Learning Objective 1: Distinguish between representative and non-representative samples of a population (e.g. if the class had 50% girls and the sample had 10% girls, then that sample was not representative of the population).

Modified Student Learning Objectives/Standards:
M.EE.7.SP.A.1–2 Answer a question related to the collected data from an experiment, given a model of data, or from data collected by the student.

<table>
<thead>
<tr>
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<th>Skills, Strategies &amp; Concepts</th>
<th>Essential Understandings/ Questions (Accountable Talk)</th>
<th>Tasks/Activities</th>
</tr>
</thead>
<tbody>
<tr>
<td>MP 1 MP 2</td>
<td>7.SP.1</td>
<td>Statistics can be used to gain information about a population by examining a sample of the population.</td>
<td>Explain how statistics is used to gain information about a population.</td>
<td>Is it Valid?</td>
</tr>
</tbody>
</table>
Generalizations about a population from a sample are valid only if the sample is representative of that population.

Random sampling tends to produce representative samples.

Evaluate the validity of a statistical sample from a population.

**Students are able to:**
- Analyze and distinguish between representative and non-representative samples of a population.

**SPED Strategies:**
- Review and practice how to define a population.
- Review and provide picture cards (i.e. data, hypothesis, population, random sampling, sample, sample size, statistics).
- Review city and state populations numbers (i.e. Paterson, New Jersey, New York).
- Use technology (i.e. to analyze population, data etc.).
- Review and practice how to determine a sample size.
- Identify and demonstrate how to choose a method for selecting respondents.

How do you evaluate the validity of a statistical sample from a population?

What factors affect the outcomes of a survey or study?

What is sampling?

How do you select a valid sample to survey or study?

What are the criteria for determining if a survey is biased?

How can we gather, organize and display data to communicate and justify results in the real world?

How can samples be used to make inferences about a population?

What sampling techniques can we use to increase validity of population inferences?
<table>
<thead>
<tr>
<th></th>
<th>Review and practice how to use statistics, test hypotheses and estimate population parameters.</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Practice and determine why a random sample gives more accurate results.</td>
</tr>
<tr>
<td></td>
<td>Provide examples, review and practice how to design and conduct surveys using sampling methods.</td>
</tr>
<tr>
<td></td>
<td>Demonstrate and practice making generalizations about a population from a sample of that population.</td>
</tr>
<tr>
<td></td>
<td>Practice conducting a random sampling of a population.</td>
</tr>
<tr>
<td></td>
<td>Practice determining how to validate inferences based on random sampling.</td>
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<tr>
<td></td>
<td>Teach and model fundamental skills and procedures explicitly until they become automatic.</td>
</tr>
<tr>
<td></td>
<td>Vary means to assess mastery of materials taught.</td>
</tr>
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<td></td>
<td>Present information through different modalities (i.e. visual, auditory, tactile, and kinesthetic).</td>
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</tbody>
</table>

**ELL Strategies:**

Develop graphic representations of sample populations and have students create tables showing compared characteristics through visual drawings.

Discuss if the answer is reasonable using white boards and charts so that students can visualize.
Develop word walls with translations side by side.

Utilize a KWL graphic organizer that has the parts listed in both their L1 (students’ native language) and L2 (students’ target language) to clarify understanding.

Provide students a translated math word bank and math reference sheet.

Have students conduct activities in small groups, pairs/triads to share and discuss solutions for real-world problems.

Create math journals for students, who can write meanings and note vocabulary in both languages.

<table>
<thead>
<tr>
<th>New Jersey Student Learning Standard(s):</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>7.SP.A.2:</strong> Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.</td>
</tr>
</tbody>
</table>

| **Student Learning Objective 2:** Use random sampling to produce a representative sample. |

<table>
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<tr>
<th><strong>Modified Student Learning Objectives/Standards:</strong></th>
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<tbody>
<tr>
<td>M.EE.7.SP.A.1–2 Answer a question related to the collected data from an experiment, given a model of data, or from data collected by the student.</td>
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<th>MPs</th>
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<th>Skills, Strategies &amp; Concepts</th>
<th>Essential Understandings/Questions (Accountable Talk)</th>
<th>Tasks/Activities</th>
</tr>
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<tbody>
<tr>
<td>MP 1 MP 2</td>
<td>7.SP.2</td>
<td>Inferences can be drawn from random sampling.</td>
<td>What is a random sample?</td>
<td>Candy Populations Election Polls</td>
</tr>
</tbody>
</table>

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**Evidence Statement/Key Clarifications:**

- **Skills, Strategies & Concepts:**
  - Inferences can be drawn from random sampling.

**Essential Understandings/Questions (Accountable Talk):**

- What is a random sample?
| MP 3 | Analyze data from a sample to draw inference about the population.  
Generate multiple random samples of the same size.  
Analyze the variation in multiple random samples of the same size. |
|------|------------------------------------------------------------------|
| MP 4 | **SPED Strategies:** Review and practice what makes a sample random.  
Review, practice and provide samples of random sampling when collecting data for a survey.  
Conduct a model survey and analyze the data within your classroom or school.  
Review and provide a graphic organizer for data charts and surveys.  
Practice determining how a sample should be selected.  
Practice determining how well a random sample represents a population.  
Review and provide picture vocabulary words (i.e., biased sample, inferences, population, random, sample, sample size). |
| MP 6 | How can random samples be used to make predictions about populations?  
Does a sample have to be random to make accurate predictions about populations?  
How are proportions used to estimate information about populations?  
How can samples be used to make inferences about a population?  
What sampling techniques can we use to increase validity of population inferences? |

**Predicting Population**  
Shakespeare vs. Harry Potter  
Valentine’s Marbles
<table>
<thead>
<tr>
<th><strong>How does the variation in samples relate to the sampling method used for the survey and why?</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Review sampling methods that provide the most accurate sample.</td>
</tr>
<tr>
<td>Review different sampling methods of the same size that can help define the variation in the prediction.</td>
</tr>
<tr>
<td>Practice classifying and organizing data (i.e., sport, stock market, grades etc.).</td>
</tr>
<tr>
<td>Review and demonstrate how to analyze and interpret data that relates student’s interest.</td>
</tr>
<tr>
<td>Practice and review identifying the variation(s) in estimates or predictions of samples.</td>
</tr>
<tr>
<td>Create visual, verbal or tactile cues or reminders.</td>
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<tr>
<td>Link new learning to prior learning.</td>
</tr>
<tr>
<td>Connect to real-life experiences</td>
</tr>
<tr>
<td>Pre-teach prerequisite skills and concepts.</td>
</tr>
<tr>
<td>Embed links to websites for additional knowledge.</td>
</tr>
<tr>
<td>Design web quests to search for background information.</td>
</tr>
</tbody>
</table>
Teach and model fundamental skills and procedures explicitly until they become automatic.

Vary means to assess mastery of materials taught.

Present information through different modalities (i.e. visual, auditory, tactile, and kinesthetic).

Adjust color of text, graphs and visual content

Visuals, anchor charts and desk decals.

**ELL Strategies:**
Provide students with graph paper and have them work in groups on comparing random samples and populations.

Small group/ triads- group highly proficient math students with low-level ELLs.

Provide a variety of ways to respond: oral, choral, student boards, concrete models (e.g., fingers), pictorial models (e.g., ten-frame), pair share, small group share.

Provide students with reference sheets with L1 (students’ native language) text materials.

Utilize white boards to have students draw mathematical representations of real-world problems.
Math journals for students to practice writing skills with math terminology.

Use of translation dictionary or software.

Teach students how to ask questions (such as, “Do you agree?” and “Why do you think so?”) to extend think-pair-share conversations.

**Model and post conversation “starters” such as:**

- “I agree because….”
- “Can you explain how you solved it?”
- “I noticed that…”
- “Your solution is different from/ the same as mine”
- because…”
- “My mistake was to…”
New Jersey Student Learning Standard(s):

7.SP.A.2: Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.

Student Learning Objective 3: Develop inferences about a population using data from a random sample and assess the variation in estimates after generating multiple samples of the same size.

Modified Student Learning Objectives/Standards:

EE.7.SP.A.1–2 Answer a question related to the collected data from an experiment, given a model of data, or from data collected by the student.

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<td>MP 2</td>
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<td>MP 3</td>
<td></td>
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<td>Predicting Population</td>
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<tr>
<td>MP 4</td>
<td></td>
<td></td>
<td></td>
<td>Shakespeare vs. Harry Potter</td>
</tr>
<tr>
<td>MP 6</td>
<td></td>
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<td>Draw inferences about a population using data from a random sample and assess the variation in estimates after generating multiple samples of the same size.</td>
<td>How can random samples be used to make predictions about populations?</td>
</tr>
<tr>
<td></td>
<td>Gather data from multiple random samples of the same size in reference to a certain characteristic.</td>
<td>How do you explain real-world problems using statistics?</td>
</tr>
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<td><strong>SPED Strategies:</strong></td>
<td>How are proportions used to estimate information about populations?</td>
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<td>Review and practice what makes a sample random.</td>
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<td>Conduct a model survey and analyze the data within your classroom or school.</td>
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<td>Review and provide a graphic organizer for data charts and surveys.</td>
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<tr>
<td>Practice classifying and organizing data (i.e. sport, stock market, grades etc.).</td>
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<tr>
<td>Review and demonstrate how to analyze and interpret data that relates student’s interest.</td>
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<tr>
<td>What sampling techniques can we use to increase validity of population inferences?</td>
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</tbody>
</table>
| Practice and review identifying the variation(s) in estimates or predictions of samples.  
| Create visual, verbal or tactile cues or reminders.  
| Link new learning to prior learning.  
| Connect to real-life experiences.  
| Pre-teach prerequisite skills and concepts.  
| Embed links to websites for additional knowledge.  
| Design web quests to search for background information.  
| Teach and model fundamental skills and procedures explicitly until they become automatic.  
| Vary means to assess mastery of materials taught.  
| Present information through different modalities (i.e. visual, auditory, tactile, and kinesthetic).  
| Adjust color of text, graphs and visual content.  
| Visuals, anchor charts and desk decals.  

**ELL Strategies:**  
Provide students with graph paper and have them work in groups to compare random samples and populations.
| Small group/ triads- group highly proficient math students with low-level ELLs.  
| Provide a variety of ways to respond: oral, choral, student boards, concrete models (e.g., fingers), pictorial models (e.g., ten-frame), pair share, small group share.  
| Provide students with reference sheets with L1(students’ native language) text materials.  
| Utilize white boards to have students draw mathematical representations of real-world problems.  
| Use translation dictionary or software.  
| Teach students how to ask questions (such as, “Do you agree?” and “Why do you think so?”) to extend think-pair-share conversations.  
| **Model and post conversation “starters” such as:**  
| - “I agree because….”  
| - “Can you explain how you solved it?”  
| - “I noticed that…”  
| - “Your solution is different from/ the same as mine  
| - because…”  
| - “My mistake was to…” |
New Jersey Student Learning Standard(s):

7.SP.B.3: Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.

Student Learning Objective 4: Visually compare the means of two distributions that have similar variability; express the difference between the centers as a multiple of a measure of variability.

Modified Student Learning Objectives/Standards:

M.EE.7.SP.B.3 Compare two sets of data within a single data display such as a picture graph, line plot, or bar graph.

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<tr>
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<th>Tasks/Activities</th>
</tr>
</thead>
<tbody>
<tr>
<td>MP 1</td>
<td>7.SP.3</td>
<td>Determine the validity of two numerical data sets.</td>
<td>How do scientists make estimations about a population size using a representative sample?</td>
<td>College Athletes</td>
</tr>
<tr>
<td>MP 2</td>
<td>•Tasks may use mean absolute deviation, range, or interquartile range as a measure of variability</td>
<td>Visually compare two numerical data distributions and describe the degree of overlap.</td>
<td>How are the measures of variability used to analyze and compare data?</td>
<td>Counting Trees</td>
</tr>
<tr>
<td>MP 3</td>
<td></td>
<td>Compute the mean absolute deviation, range and interquartile range.</td>
<td>How can variability affect the overlap of two data sets?</td>
<td>Emergency 911</td>
</tr>
<tr>
<td>MP 4</td>
<td></td>
<td>Describe how many times larger/smaller the variability of one data set is to another.</td>
<td>How can we use the measures of variability to compare two sets of data?</td>
<td>Got Friends?</td>
</tr>
<tr>
<td>MP 5</td>
<td></td>
<td>Read and interpret data from statistical representations (box-and-whisker plot, line/dot plot.</td>
<td></td>
<td>Offensive Linemen</td>
</tr>
<tr>
<td>MP 6</td>
<td></td>
<td>Locate, approximately, the measure of center (mean or median) of a distribution.</td>
<td></td>
<td>Travel Times to Work</td>
</tr>
<tr>
<td>MP 7</td>
<td></td>
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<tr>
<td>Measure or approximate the difference between the measures centers and express it as a multiple of a measure of variability.</td>
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<tr>
<td>How can samples be used to make inferences about a population?</td>
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<tr>
<td><strong>SPED Strategies:</strong> Review and practice recalling measures of central tendency (i.e., mean, median, and mode) in a data distribution.</td>
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<tr>
<td>What sampling techniques can we use to increase validity of population inferences?</td>
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<tr>
<td>Review and practice recalling measures of variation (i.e. upper quartile, lower quartile, upper extreme-maximum, lower extreme minimum, range, interquartile range, and mean deviation).</td>
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<tr>
<td>Demonstrate and practice how to compare the differences in the measure of central tendency (mean, median, and mode) in two numerical data distributions.</td>
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<tr>
<td>Practice how to determine the variability of data (range mean deviation, variance, and standard deviation).</td>
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<tr>
<td>Provide a graphic organizer on mean, median, and mode.</td>
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<tr>
<td>Review vocabulary with picture cards (i.e. mean, median, mode, upper quartile, lower quartile, upper extreme-maximum, lower extreme minimum, range, interquartile range, and mean deviation, data distribution, measures of variability, mean deviation, similar variabilities,</td>
<td></td>
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</tbody>
</table>
standard deviation, upper extreme-maximum, variance).

Review and practice identifying the center, or median, of a numerical data set can vary from set to set and two separate sample populations.

Review vocabulary with picture cards (i.e., measures of center, median, random samples).

Create visual, verbal or tactile cues or reminders.

Link new learning to prior learning.

Connect content to real-life experiences.

Pre-teach prerequisite skills and concepts.

Embed links to websites for additional knowledge.

Design web quests to search for background information.

Teach and model fundamental skills and procedures explicitly until they become automatic.

Vary means to assess mastery of materials taught.

Present information through different modalities (i.e. visual, auditory, tactile, and kinesthetic).
**ELL Strategies:**
Initiate discussions and provide opportunities for collaboration.

Highlight solution steps for comparing probability events and model solution samples for students to visualize.

Teacher partially completes the mathematical problem and labels essential terms.

Utilize interactive tools that can be used to illustrate solution methods, and build language as well as math skills.

Have students use white boards to write the equations dictated by the teacher.

Utilize pictures and photographs to show ELLs examples of class vocabulary and concepts.

Describe and explain orally to students in small groups how to graph solutions, and allow them to interpret in their L1 (student native language) and/or L2 (student target language).
New Jersey Student Learning Standard(s):

7.SP.B.4: Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. *For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.*

**Student Learning Objective 5:** Draw informal comparative inferences about two populations using their measures of center and measures of variability.

**Modified Student Learning Objectives/Standards:** N/A

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</tr>
</thead>
<tbody>
<tr>
<td>MP 1</td>
<td>7.SP.4</td>
<td>Compare/contrast measures of central tendency to draw conclusions about two random samples.</td>
<td>How are the measures of variability used to analyze and compare data?</td>
<td>Counting Athletes</td>
</tr>
<tr>
<td>MP 2</td>
<td>7.SP.4</td>
<td>Compare/contrast variability of two data sets to draw conclusions about two random samples.</td>
<td>How can variability affect the overlap of two data sets?</td>
<td>Counting Trees</td>
</tr>
<tr>
<td>MP 3</td>
<td>7.SP.4</td>
<td>Read and interpret data from statistical representation (box-and-whisker plot, line/dot plot).</td>
<td>How do I use the measures of central tendency to compare two sets of data?</td>
<td>Emergency 911</td>
</tr>
<tr>
<td>MP 4</td>
<td>7.SP.4</td>
<td><strong>SPED Strategies:</strong> Review and practice recalling measures of central tendency (i.e., mean, median, and mode) in a data distribution.</td>
<td>How can we use data displays, measures of center and measures of variability from random samples to draw informal competitive inferences about two populations?</td>
<td>Got Friends?</td>
</tr>
<tr>
<td>MP 5</td>
<td>7.SP.4</td>
<td>Review and practice recalling measures of variation (i.e. upper quartile, lower quartile, upper extreme-maximum, lower extreme minimum, range, interquartile range, and mean deviation).</td>
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<td>Offensive Linemen</td>
</tr>
<tr>
<td>MP 6</td>
<td>7.SP.4</td>
<td></td>
<td></td>
<td>Travel Times to Work</td>
</tr>
<tr>
<td>MP 7</td>
<td>7.SP.4</td>
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</tbody>
</table>
Practice and demonstrate how to compare the differences in the measure of central tendency (mean, median, and mode) in two numerical data distributions.

Practice how to determine the variability of data (range mean deviation, variance, and standard deviation).

Provide a graphic organizer on mean, median, and mode.

Review vocabulary with picture cards (i.e. mean, median, mode, upper quartile, lower quartile, upper extreme-maximum, lower extreme minimum, range, interquartile range, and mean deviation, data distribution, measures of variability, mean deviation, similar variability, standard deviation, upper extreme-maximum, variance).

Review and practice identifying that the center, or median, of a numerical data set can vary from set to set and two separate sample populations.

Review vocabulary with picture cards (i.e., measures of center, median, random samples).

**ELL Strategies:**
Initiate discussions and provide opportunities for collaboration.
Highlight solution steps for comparing probability events and model solution samples for students to visualize.

Teacher partially completes the mathematical problem and labels essential terms. Utilize interactive tools that can be used to illustrate solution methods, and build language as well as math skills.

Have students use white boards to write the equations dictated by the teacher.

Utilize pictures and photographs to show ELLs examples of class vocabulary and concepts.

Describe and explain orally to students in small groups how to graph solutions, and allow them to interpret in their L1 (student native language) and/or L2 (student target language).
New Jersey Student Learning Standard(s):

7.SP.C.5: Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.

Student Learning Objective 6: Interpret and express the likelihood of a chance event as a number between 0 and 1, relating that the probability of an unlikely event happening is near 0, a likely event is near 1, and 1/2 is neither likely nor unlikely.

Modified Student Learning Objectives/Standards:

M.EE.7.SP.C.5–7: Describe the probability of events occurring as possible or impossible.

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</thead>
<tbody>
<tr>
<td>MP 4 MP 5</td>
<td>7.SP.5 • Tasks may involve probabilities that are certain (1) or impossible (0).</td>
<td>Probability of a chance event is a number between 0 and 1. Probability expresses the likelihood of the event occurring. Larger probability indicates greater likelihood. Describe a situation in which the event is likely. Identify the probability of a likely event as a number near 1. <strong>SPED Strategies:</strong> Review and practice factors that can affect the outcome of events.</td>
<td>How can events be described using probability? What is probability and how can I use it to describe the likelihood of an event occurring? How do you explain real-world problems using statistics? How do you interpret data from statistical representations?</td>
<td>Chance Experiments Fair Game Probability on the Number Line</td>
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<tr>
<td>MP 6 MP 7</td>
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</tbody>
</table>
Review and practice how probability can be expressed in terms (i.e., never/impossible, unlikely, likely, equally likely, or always/certain).

Review and practice how probability can be expressed as a number between 0 and 1 or 0% and 100%, inclusive.

Review and practice probability of an event uses the benchmarks of probability, 0, 1, \( \frac{1}{2} \). The closer the probability is to 0, the more unlikely the event; and the closer to 1, the more likely the event.

Review vocabulary with picture cards (i.e., benchmarks, event, outcomes, probability).

Provide hands on examples for students to determine probability (deck of cards, dice).

**ELL Strategies:**
Allow arranged groups to confer frequently with each other and share feedback.

Have students listen to and take notes on videos about probability.

Native language videos if available should be viewed.

Provide hands on examples for students to determine probability (deck of cards, dice).

Pre-teach to ELLs specific mathematical terms prior to the lesson, and provide translations for words.
New Jersey Student Learning Standard(s):

**7.SP.C.6**: Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. *For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.*

**Student Learning Objective 7**: Approximate the probability of a chance event by collecting data and observing long-run relative frequency; predict the approximate relative frequency given the probability

**Modified Student Learning Objectives/Standards:**

**M.EE.7.SP.C.5–7**: Describe the probability of events occurring as possible or impossible.

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</thead>
<tbody>
<tr>
<td>MP 1 MP 2 MP 3 MP 4 MP 5</td>
<td><strong>7.SP.6</strong> • Tasks require the student to make a prediction based on long-run relative frequency in data from a chance process.</td>
<td>Predict the number of times an event occurs by multiplying the theoretical probability by the number of trials. Compute the experimental probability of an event occurring through repeated trials. Compare theoretical probability of an event occurring and the experimental probability. <strong>SPED Strategies:</strong> Review and practice factors that can affect the outcome of events. Review and demonstrate understanding of probability of events that can be determined by chance and/or skill using numbers from 0 to 1 to indicate the likelihood of the event occurring.</td>
<td>How can you represent the likelihood of an event occurring? How can data be used to make predictions? The experimental probability or relative frequency of outcomes of an event can be used to estimate the exact probability of an event. Experimental probability approaches theoretical probability when the number of trials is large.</td>
<td>Blood Groups Heads or Tails Tossing Cylinders</td>
</tr>
<tr>
<td>Review and practice probability of an event using the benchmark of probability, $0, \frac{1}{2}, 1$. The closer the probability is to 0, the more unlikely the event; and the closer to 1, the more likely the event.</td>
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<tr>
<td>Review and practice how probability can be expressed as a number between 0 and 1 or 0% and 100%, inclusive.</td>
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<tr>
<td>Connect content to real-life experiences.</td>
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<tr>
<td><strong>ELL Strategies:</strong></td>
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<tr>
<td>Use of translation dictionary or software for vocabulary and word problems.</td>
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<tr>
<td>Implement strategy groups; group high-level with low-level students.</td>
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<tr>
<td>Allow arranged groups to confer frequently with each other and share feedback.</td>
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<tr>
<td>Have students listen to and take notes on videos about probability.</td>
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<tr>
<td><strong>How can I develop and compare probability models and use them to find probabilities of events?</strong></td>
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<tr>
<td><strong>What is the difference between experimental and theoretical probability and how can I use these ideas to make predictions?</strong></td>
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</table>
New Jersey Student Learning Standard(s):

7.SP.C.7: Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.

- **7.SP.C.7a.** Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.

- **7.SP.C.7b.** Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?

**Student Learning Objective 8:** Develop a uniform probability model by assigning equal probability to all outcomes; develop probability models by observing frequencies and use the models to determine probabilities of events; compare probabilities from a model to observed frequencies and explain sources of discrepancy when agreement is not good.

**Modified Student Learning Objectives/Standards:**

M.EE.7.SP.C.5–7 Describe the probability of events occurring as possible or impossible.

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</tr>
</thead>
<tbody>
<tr>
<td>MP 1</td>
<td>7.SP.7a</td>
<td>Calculate simple probabilities of events.</td>
<td>How are the outcomes of given events distinguished as possible?</td>
<td>Calculating Probabilities</td>
</tr>
<tr>
<td>MP 2</td>
<td>Simple events only.</td>
<td>Compare the results of a series of trials and draw conclusions.</td>
<td>How can you determine the likelihood that an event will occur?</td>
<td>Rolling Dice</td>
</tr>
<tr>
<td>MP 4</td>
<td>7.SP.7b</td>
<td>Uniform (equally likely) and non-uniform probability models.</td>
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<tr>
<td>MP 6</td>
<td>Develop a uniform probability model.</td>
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<tr>
<td>MP 6</td>
<td>Use a uniform probability model to determine the probabilities of events.</td>
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</tbody>
</table>
Develop (non-uniform) probability models by observing frequencies in data that has been generated from a chance process.

**SPED Strategies:**
Review and practice how to compare predictions of the outcomes of an experiment.

Review, demonstrate and practice how to compare outcomes from theoretical probability to experimental probability (e.g. Do the outcomes for a spinning penny appear to be equally likely based on the observed frequencies?).

Review and practice probability experiments and compare the results to theoretical probability.

Demonstrate how to replicate experiments to compare results (e.g. experiments can be replicated by the same group or by compiling class data).

Practice conducting experiments using various random generating devices (e.g. bag pulls spinners, number cubes, coin toss, colored chips/marbles, etc.).

Practice collecting data using physical objects, graphing calculators, and web-based simulations.

Create visual, verbal or tactile cues or reminders.

Connect content to real-life experiences.
<table>
<thead>
<tr>
<th>ELL Strategies:</th>
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</thead>
<tbody>
<tr>
<td>Clarify, compare, and make connections to math words in discussion, particularly during and after practice.</td>
</tr>
</tbody>
</table>

| Highlight critical vocabulary in discussion. For example, have students highlight key words in a real-world problem and translate. |

| Provide students with partially completed solutions prior to having students work in groups to complete, share and discuss. |

| Create a word/picture wall with translations of key words side by side. |

| Utilize resources with L1 (student native language) text and/or support for lesson objectives. |

| Utilize technological programs which provide verbal and visual instruction in native and/or second language. |
New Jersey Student Learning Standard(s):

7.SP.C.8: Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.

7.SP.C.8a: Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.

7.SP.C.8b: Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., "rolling double sixes"), identify the outcomes in the sample space which compose the event.

Student Learning Objective 9: Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams, identifying the outcomes in the sample space which compose the event. Use the sample space to find the probability of a compound event.

Modified Student Learning Objectives/Standards: N/A

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</thead>
<tbody>
<tr>
<td>MP 1</td>
<td>7.SP.8a</td>
<td>Use organized lists, tables, and tree diagrams to represent sample space.</td>
<td>How can you represent the probability of compound events by constructing models?</td>
<td>How I Roll</td>
</tr>
<tr>
<td>MP 2</td>
<td>7.SP.8b</td>
<td>Define compound probabilities as fractions of the sample space taken from.</td>
<td>Sometimes the outcome of one event does not affect the outcome of another event. (This is when the outcomes are called independent.)</td>
<td>Skittles</td>
</tr>
<tr>
<td>MP 4</td>
<td></td>
<td>Determine the total number of possible outcomes. (sample space or Counting Principle)</td>
<td>Tree diagrams and arrays are useful for describing relatively small sample spaces and computing probabilities, as well as for visualizing why the number</td>
<td>The Probability of Twinning</td>
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<tr>
<td>MP 5</td>
<td></td>
<td>Given a description of an event using everyday language, identify the outcomes in a sample space that make up the described event.</td>
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<td>Using a Tree Diagram</td>
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<tr>
<td>MP 7</td>
<td></td>
<td>Construct a tree diagram, list, or table to illustrate all possible outcome or compound events.</td>
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<td>Waiting Times</td>
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<tr>
<td>MP 8</td>
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<tr>
<td>SPED Strategies:</td>
<td>ELL Strategies:</td>
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<tr>
<td>Review and practice how to find the probability of simple events.</td>
<td>Clarify, compare, and make connections to math words in discussion, particularly during and after practice.</td>
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<tr>
<td>Review that the definition of a compound event is the probability of two or more things happening at once.</td>
<td>Highlight critical vocabulary in discussion. For example, have students highlight key words in a real-world problem and translate.</td>
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<tr>
<td>Review, demonstrate and practice how to create tree diagrams, frequency tables, organized lists, and simulations to determine the probability of compound events.</td>
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<td>Demonstrate and practice the relationship between various representations of compound probabilities (i.e. tree diagrams, frequency tables, etc.).</td>
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<td>Have students practice using a four-function calculator to assist in determining probability.</td>
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<tr>
<td>Create task that will ask students to complete a table to display possible outcomes of a compound event.</td>
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</table>

Simulations can be used to collect data and estimate probabilities for real situations that are sufficiently complex when theoretical probabilities are not obvious.

How can I find probabilities of compound event using simple spaces represented by organized lists, tables, tree diagrams, and simulations?
Provide students with partially completed solutions prior to having students work in groups to complete, share and discuss.

Create word/picture wall with translations of key words side by side.

Utilize resources with L1(student native language) text and/or support for lesson objectives.

Utilize technological programs which provide verbal and visual instruction in native and/or second language.

New Jersey Student Learning Standard(s):

7.SP.C.8c: Design and use a simulation to generate frequencies for compound events. For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?

Student Learning Objective 10: Design and use a simulation to generate frequencies for compound events.

Modified Student Learning Objectives/Standards: N/A

<table>
<thead>
<tr>
<th>MPs</th>
<th>Evidence Statement Key/ Clarifications</th>
<th>Skills, Strategies &amp; Concepts</th>
<th>Essential Understandings/ Questions (Accountable Talk)</th>
<th>Tasks/Activities</th>
</tr>
</thead>
<tbody>
<tr>
<td>MP 1 MP 2 MP 4 MP 5 MP 7 MP 8</td>
<td>7.SP.8c</td>
<td>Just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space. Use organized lists, tables, and tree diagrams to represent sample spaces.</td>
<td>How can you represent the probability of compound events by constructing models? Sometimes the outcome of one event does not affect the outcome of another event.</td>
<td>How I Roll Skittles The Probability of Twinning</td>
</tr>
</tbody>
</table>
Design simulations.
Use designed simulations to generate frequencies for compound events.

**SPED Strategies:**
Practice and demonstrate how to choose to roll two number cubes to simulate the number of strikes two bowling partners will get the next games.

Review that the definition of a compound event is the probability of two or more things happening at once.

Review, demonstrate and practice how to create tree diagrams, frequency tables, organized lists, and simulations to determine the probability of compound events.

Demonstrate and practice the relationship between various representations of compound probabilities (i.e. tree diagrams, frequency tables, etc.).

Create a task that will ask students to complete a table to display possible outcomes of a compound event.

Practice giving two spinners, students will find the probability of the compound event.

**ELL Strategies:**
Provide ELL students with additional language support through the use of sentence frames and starters.

(This is when the outcomes are called independent.)

Simulations can be used to collect data and estimate probabilities for real situations that are sufficiently complex when theoretical probabilities are not obvious.

How can I find probabilities of compound event using simple spaces represented by organized lists, tables, tree diagrams, and simulations?

**Using a Tree Diagram**

**Waiting Times**
<table>
<thead>
<tr>
<th>Highlight and label the solution steps for multi-step problems in different colors.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Students can utilize math journals to write notes, copy solution steps, and translate terms and key vocabulary.</td>
</tr>
<tr>
<td>Have students practice math procedures utilizing technical programs.</td>
</tr>
<tr>
<td>Provide students with paper, math journals, and have them view videos on sample probability activities and lessons and take notes as they view.</td>
</tr>
<tr>
<td>Provide dual language/bilingual dictionary. Have students translate unfamiliar terms.</td>
</tr>
</tbody>
</table>
New Jersey Student Learning Standard(s):

7.G.B.4: Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.

Student Learning Objective 11: Know the formulas for the area and circumference of a circle and use them to solve problems. Give an informal derivation of the relationship between the circumference and area of a circle.

Modified Student Learning Objectives/Standards:

M.EE.7.GB.4: Determine the perimeter of a rectangle by adding the measures of the sides.

<table>
<thead>
<tr>
<th>MPs</th>
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<th>Tasks/Activities</th>
</tr>
</thead>
</table>
| MP 1 MP 2 MP 3 MP 4 MP 5 MP 6 MP 7 MP 8 | 7.G.4  
- Tasks may or may not have context.  
- Tasks may require answers to be written in terms of π.  
- Tasks require students to identify or produce a logical conclusion about the relationship between the circumference and the area of a circle. | Students will understand how to use the formula for area and circumference of a circle.  
Solve problems by finding the area and circumference of circles.  
Show that the area of a circle can be derived from the circumference.  
Students understand the relationship between radius and diameter. Students also understand the ratio of circumference to diameter can be expressed as pi. Building on these understandings, students generate the formulas for circumference and area. | How are the diameter and circumference of a circle related?  
What is π?  
How does pi relate to the circumference and diameter of a circle?  
How can the circumference of a circle be calculated?  
How do the areas of squares relate to the area of circles? | Area and Circumference  
Eight Circles  
Historic Bicycle  
Stained Glass |
Provide formulas (i.e. area, circumference, radius, and diameter).

Review and practice using formula for the area of a circle and circumference of a circle.

Review and demonstrate the common measurement for \( \pi \approx 3.14 \).

Review identifying the diameter and radius of a circle.

Allow students to use a calculator.

Review, demonstrate and practice areas means the amount of space inside a two-dimensional figure.

Review and demonstrate that a circle is created by connecting all points equidistant from a point (center) is essential.

Link new learning to prior learning.

**Resources (UDL - Visual and Auditory Learner(s):**
Math Antics - Circles, Circumference And Area  
https://www.youtube.com/watch?v=O-cawByg2aA&list=PLSvbMuTKPH33jqTavXLPmaZvoj3nNxPMP&index=5

**ELL Strategies:**
Scaffold circumference and area problems and distribute based on students’ level.

Point to visuals while speaking, using your hands to clearly indicate the image that corresponds to your words.
Develop graphic representations of circles and circumference problems and have students create their own representations through visual drawings. Discuss if the answer is reasonable using white boards and charts which students can visualize and share with each other.

Develop word walls with translations side by side.

Math word bank and math reference sheet/translated/copied for students.

Create math journals for students, who can write meanings and note vocabulary in both languages.
New Jersey Student Learning Standards (s):

7.G.A.3: Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.

Student Learning Objective 12: Describe all of the 2-dimensional figures that result when a 3-dimensional figure is sliced from multiple angles.

Modified Student Learning Objectives/Standards:

M.EE.7.G.A.3: Match a two-dimensional shape with a three-dimensional shape that shares an attribute.

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</thead>
<tbody>
<tr>
<td>MP 5</td>
<td>7.G.3</td>
<td>Analyze three dimensional shapes (right rectangular pyramids and prisms) by examining and describing all of the 2-dimensional figures that result from slicing it at various angles.</td>
<td>Describe the two-dimensional figures that result from slicing three-dimensional figures.</td>
<td>Cool Cross Sections</td>
</tr>
<tr>
<td>MP 6</td>
<td>• Tasks have “thin context” or no context.</td>
<td>Use molding clay to create three dimensional shapes. Cut the shapes in order to determine the shapes of the cross sections.</td>
<td>What two-dimensional figures can be made by slicing a cube by planes?</td>
<td>Cube Ninjas</td>
</tr>
<tr>
<td>MP 7</td>
<td></td>
<td></td>
<td>What two-dimensional figures can be made by slicing cones, prisms, cylinders, and pyramids by planes?</td>
<td>Slicing on an Angle</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>How can all possible cross-sections of a solid be determined?</td>
<td>Slicing a Right Rectangular Prism</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>How is visualization essential to the study of geometry?</td>
<td>Slicing a Right Rectangular Pyramid</td>
</tr>
</tbody>
</table>

**SPED Strategies:**
Review and provide pictorial guide of vocabulary words for students to make a connection (i.e. construction, included angle, non-included angle, unique triangle, cross, section, cylinder, edge, face, net, prism, pyramid, rectangular prism, rectangular pyramid solid, vertex).
Practice drawing and creating polygons. Review and identifying properties of two-dimensional figures.
Create visual, verbal or tactile cues or reminders.
Link new learning to prior learning.
Connect to real-life experiences.
Pre-teach prerequisite skills and concepts.

**ELL Strategies:**
Use of translation dictionary or software for vocabulary and word problems.

Implement strategy groups, group high-level with low-level students.
Allow arranged groups to confer frequently with each other and share feedback.
Pre-teach to ELL’s specific mathematical terms prior to the lesson, and provide translations for words.

Explain in writing how to construct triangles using mechanical (i.e. ruler, protractor) and technological tools or free hand drawings from given conditions (scale factor, length, degrees) using manipulatives, demonstrations and math journal.
<table>
<thead>
<tr>
<th>Website: LEARNZILLION</th>
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<tbody>
<tr>
<td><em>Draw Geometric Shapes Given the Length of Sides</em></td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>MATH’s FUN</th>
</tr>
</thead>
<tbody>
<tr>
<td><em>Geometry</em></td>
</tr>
<tr>
<td><a href="http://www.mathsisfun.com/geometry/index.html">http://www.mathsisfun.com/geometry/index.html</a></td>
</tr>
</tbody>
</table>
7.D.1: Solve multi-step contextual word problems with degree of difficulty appropriate to Grade 7, requiring application of knowledge and skills articulated in Type I, Sub-Claim A Evidence Statements.
   - Tasks may have scaffolding, if necessary, in order to yield a degree of difficulty appropriate to grade 7.

7.D.2: Solve multi-step contextual problems with degree of difficulty appropriate to grade 7, requiring application of knowledge and skills articulated in 6.RP.A, 6.EE.C, 6.G.
   - Tasks may have scaffolding, if necessary, in order to yield a degree of difficulty appropriate to grade 7.

7.D.3: Micro-models: Autonomously apply a technique from pure mathematics to a real-world situation in which the technique yields valuable results even though it is obviously not applicable in a strict mathematical sense (e.g., profitably applying proportional relationships to a phenomenon that is obviously nonlinear or statistical in nature).
   - Tasks may have scaffolding, if necessary, in order to yield a degree of difficulty appropriate to grade 7.

   - Tasks may have scaffolding, if necessary, in order to yield a degree of difficulty appropriate to grade 7.
## Unit 3 Vocabulary

<table>
<thead>
<tr>
<th>Terms</th>
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<tbody>
<tr>
<td>Box and Whisker Plot</td>
<td>Minimum Value</td>
</tr>
<tr>
<td>Experimental Probability</td>
<td>Mode</td>
</tr>
<tr>
<td>Favorable Outcomes</td>
<td>Mutually Exclusive</td>
</tr>
<tr>
<td>Frequency</td>
<td>Outlier</td>
</tr>
<tr>
<td>Grouped Frequency Table</td>
<td>Outcomes</td>
</tr>
<tr>
<td>Histogram</td>
<td>Population</td>
</tr>
<tr>
<td>Inferences</td>
<td>Random Sampling</td>
</tr>
<tr>
<td>Inter-Quartile Range</td>
<td>Range</td>
</tr>
<tr>
<td>Likely Event</td>
<td>Representative Sample</td>
</tr>
<tr>
<td>Maximum Value</td>
<td>Sample</td>
</tr>
<tr>
<td>Mean</td>
<td>Sample Space</td>
</tr>
<tr>
<td>Mean Absolute Deviation</td>
<td>Simple Random Sampling</td>
</tr>
<tr>
<td>Measures of Center</td>
<td>Stem and Leaf Plot</td>
</tr>
<tr>
<td>Measure of Central Tendency</td>
<td>Theoretical Probability</td>
</tr>
<tr>
<td>Measures of Spread</td>
<td>Unlikely Event</td>
</tr>
<tr>
<td>Median</td>
<td></td>
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<td></td>
<td></td>
</tr>
</tbody>
</table>
References & Suggested Instructional Websites

https://bigbrainz.com/login/ (Imagine Math Facts)

https://www.imaginelearning.com/programs/math-facts

https://www.aleks.com/

https://paterson1991.smhost.net/lms/sm.view (SuccessMaker)

www.internet4classrooms.com


www.illustrativemathematics.org/

http://www.katm.org/flipbooks/7%20FlipBook%20Final%20CCSS%202014.pdf

http://www.ncpublicschools.org/

https://www.georgiastandards.org/Common-Core/Pages/Math-6-8.aspx

https://learnzillion.com/

http://www.insidemathematics.org/

https://www.engageny.org/
Field Trip Ideas

**Buehler Challenger & Science Center** – [http://www.bcsc.org/5-9th-grade/](http://www.bcsc.org/5-9th-grade/)
- Participants work as a team as they take on the role of astronauts and mission controllers to *Rendezvous with Comet Halley*, *Return to the Moon*, or *Voyage to Mars*. Students use team-building and hands-on learning with a focus on STEM to complete their mission goal.

**Liberty Science Center** - [https://lsc.org/visit/plan-your-visit](https://lsc.org/visit/plan-your-visit)
- An interactive science museum and learning center with math connections. There is a math guidebook for teachers to make connections with math.

- For more than 20 years, educators from around the country have been bringing students to the Museum to help them understand how finance impacts their daily lives. The Museum offers discounted admission for pre-booked groups of eight or more, as well as a variety of classes for students in middle school through college.

- Mathematics illuminates the patterns and structures all around us. Our dynamic exhibits, gallery, and programs will stimulate inquiry, spark curiosity, and reveal the wonders of mathematics. MoMath has innovative exhibits that will engage folks from 105 to 5 years old (and sometimes younger), but with a special emphasis on activities for 4th through 8th graders.