MATHEMATICS

Geometry: Unit 2
Congruence, Similarity & Proof
Course Philosophy/Description

Geometry stresses the ability to reason logically and to think critically, using spatial sense. A major part of the course will be devoted to teaching the student how to present a formal proof. Geometric properties of both two and three dimensions are emphasized as they apply to points, lines, planes, and solids. In this course students learn to recognize and work with geometric concepts in various contexts. They build on ideas of inductive and deductive reasoning, logic, concepts, and techniques of Euclidean plane and solid geometry and develop an understanding of mathematical structure, method, and applications of Euclidean plane and solid geometry. Students use visualizations, spatial reasoning, and geometric modeling to solve problems. Topics of study include points, lines, and angles; triangles; quadrilaterals and other polygons; circles; coordinate geometry; three-dimensional solids; geometric constructions; symmetry; similarity; and the use of transformations.

Upon successful completion of this course, students will be able to: Use and prove basic theorems involving congruence and similarity of figures; determine how changes in dimensions affect perimeter and area of common geometric figures; apply and use the properties of proportion; perform basic constructions with straight edge and compass; prove the Pythagorean Theorem; use the Pythagorean Theorem to determine distance and find missing dimensions of right triangles; know and use formulas for perimeter, circumference, area, volume, lateral and surface area of common figures; find and use measures of sides, interior and exterior angles of polygons to solve problems; use relationships between angles in polygons, complementary, supplementary, vertical and exterior angle properties; use special angle and side relationships in special right triangles; understand, apply, and solve problems using basic trigonometric functions; prove and use relationships in circles to solve problems; prove and use theorems involving properties of parallel lines cut by a transversal, quadrilaterals and circles; write geometric proofs, including indirect proofs; construct and judge validity of logical arguments; prove theorems using coordinate geometry including the midpoint of a segment and distance formula; understand transformations in the coordinate plane; construct logical verifications to test conjectures and counterexamples; and write basic mathematical arguments in paragraph and statement-reason form.
This ESL framework was designed to be used by bilingual, dual language, ESL and general education teachers. Bilingual and dual language programs use the home language and a second language for instruction. ESL teachers and general education or bilingual teachers may use this document to collaborate on unit and lesson planning to decide who will address certain components of the SLO and language objective. ESL teachers may use the appropriate leveled language objective to build lessons for ELLs which reflects what is covered in the general education program. In this way, whether it is a pull-out or push-in model, all teachers are working on the same Student Learning Objective connected to the New Jersey Student Learning Standard. The design of language objectives are based on the alignment of the World-Class Instructional Design Assessment (WIDA) Consortium’s English Language Development (ELD) standards with the New Jersey Student Learning Standards (NJSLS). WIDA’s ELD standards advance academic language development across content areas ultimately leading to academic achievement for English learners. As English learners are progressing through the six developmental linguistic stages, this framework will assist all teachers who work with English learners to appropriately identify the language needed to meet the requirements of the content standard. At the same time, the language objectives recognize the cognitive demand required to complete educational tasks. Even though listening and reading (receptive) skills differ from speaking and writing (expressive) skills across proficiency levels the cognitive function should not be diminished. For example, an Entering Level One student only has the linguistic ability to respond in single words in English with significant support from their home language. However, they could complete a Venn diagram with single words which demonstrates that they understand how the elements compare and contrast with each other or they could respond with the support of their native language with assistance from a teacher, para-professional, peer or a technology program.

http://www.state.nj.us/education/modelcurriculum/ela/ELLOverview.pdf
<table>
<thead>
<tr>
<th>#</th>
<th>Student Learning Objective</th>
<th>NJSLS</th>
<th>Big Ideas Math Correlation</th>
<th>Instruction: 8 weeks</th>
<th>Assessment: 1 week</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Verify the properties of dilations given by a center and a scale factor.</td>
<td>G.SRT.A.1,</td>
<td>4.5</td>
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<td></td>
<td></td>
<td>G.SRT.A.1a,</td>
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<td></td>
<td></td>
<td>G.SRT.A.1b</td>
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<tr>
<td>2</td>
<td>Use the definition of similarity in terms of similarity transformations to decide if two</td>
<td>G.SRT.A.2</td>
<td>4.6, 8.1</td>
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<tr>
<td></td>
<td>given figures are similar and explain, using similarity transformations, the meaning of</td>
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<tr>
<td></td>
<td>triangle similarity.</td>
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<tr>
<td>3</td>
<td>Use the properties of similarity transformations to establish the Angle-Angle criterion</td>
<td>G.SRT.A.3</td>
<td>8.2</td>
<td></td>
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<tr>
<td></td>
<td>for two triangles to be similar.</td>
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<tr>
<td>4</td>
<td>Construct and explain formal proofs of theorems involving lines, angles, triangles, and</td>
<td>G.CO.C.9,</td>
<td>2.5, 2.6, 3.2, 3.3, 3.4,</td>
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<tr>
<td></td>
<td>parallelograms.</td>
<td>G.CO.C.10,</td>
<td>5.1, 5.4, 6.2, 6.3, 6.4,</td>
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<tr>
<td></td>
<td></td>
<td>G.CO.C.11</td>
<td>6.5, 6.6, 7.2, 7.3, 7.4</td>
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</tr>
<tr>
<td>5</td>
<td>Prove theorems about triangles.</td>
<td>G.SRT.B.4</td>
<td>8.3, 8.4, 9.1</td>
<td></td>
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</tr>
<tr>
<td>6</td>
<td>Use congruence and similarity criteria for triangles to solve problems and to prove</td>
<td>G.SRT.B.5</td>
<td>5.7, 7.2, 7.3, 7.4, 7.5,</td>
<td></td>
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<tr>
<td></td>
<td>relationships in geometric figures.</td>
<td></td>
<td>8.2, 8.3, 8.4, 9.3</td>
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</tbody>
</table>
Research about Teaching and Learning Mathematics

Structure teaching of mathematical concepts and skills around problems to be solved (Checkly, 1997; Wood & Sellars, 1996; Wood & Sellars, 1997)

Encourage students to work cooperatively with others (Johnson & Johnson, 1975; Davidson, 1990)

Use group problem-solving to stimulate students to apply their mathematical thinking skills (Artzt & Armour-Thomas, 1992)

Students interact in ways that support and challenge one another’s strategic thinking (Artzt, Armour-Thomas, & Curcio, 2008)

Activities structured in ways allowing students to explore, explain, extend, and evaluate their progress (National Research Council, 1999)

There are three critical components to effective mathematics instruction (Shellard & Moyer, 2002):

- Teaching for conceptual understanding
- Developing children’s procedural literacy
- Promoting strategic competence through meaningful problem-solving investigations

Teachers should be:

- Demonstrating acceptance and recognition of students’ divergent ideas.
- Challenging students to think deeply about the problems they are solving, extending thinking beyond the solutions and algorithms required to solve the problem
- Influencing learning by asking challenging and interesting questions to accelerate students’ innate inquisitiveness and foster them to examine concepts further.
- Projecting a positive attitude about mathematics and about students’ ability to “do” mathematics

Students should be:

- Actively engaging in “doing” mathematics
- Solving challenging problems
- Investigating meaningful real-world problems
- Making interdisciplinary connections
- Developing an understanding of mathematical knowledge required to “do” mathematics and connect the language of mathematical ideas with numerical representations
- Sharing mathematical ideas, discussing mathematics with one another, refining and critiquing each other’s ideas and understandings
- Communicating in pairs, small group, or whole group presentations
- Using multiple representations to communicate mathematical ideas
- Using connections between pictures, oral language, written symbols, manipulative models, and real-world situations
- Using technological resources and other 21st century skills to support and enhance mathematical understanding.
Mathematics is not a stagnate field of textbook problems; rather, it is a dynamic way of constructing meaning about the world around us, generating knowledge and understanding about the real world every day. Students should be metaphorically rolling up their sleeves and “doing mathematics” themselves, not watching others do mathematics for them or in front of them. (Protheroe, 2007)

Balanced Mathematics Instructional Model

Balanced math consists of three different learning opportunities; guided math, shared math, and independent math. Ensuring a balance of all three approaches will build conceptual understanding, problem solving, computational fluency, and procedural fluency. Building conceptual understanding is the focal point of developing mathematical proficiency. Students should frequently work on rigorous tasks, talk about the math, explain their thinking, justify their answer or process, build models with graphs or charts or manipulatives, and use technology.

When balanced math is used in the classroom it provides students opportunities to:

- solve problems
- make connections between math concepts and real-life situations
- communicate mathematical ideas (orally, visually and in writing)
- choose appropriate materials to solve problems
- reflect and monitor their own understanding of the math concepts
- practice strategies to build procedural and conceptual confidence

Teacher builds conceptual understanding by modeling through demonstration, explicit instruction, and think alouds, as well as guiding students as they practice math strategies and apply problem solving strategies. (whole group or small group instruction)

Teacher and students practice mathematics processes together through interactive activities, problem solving, and discussion. (whole group or small group instruction)

Students practice math strategies independently to build procedural and computational fluency. Teacher assesses learning and reteaches as necessary. (whole group instruction, small group instruction, or centers)
# Effective Pedagogical Routines/Instructional Strategies

<table>
<thead>
<tr>
<th>Collaborative Problem Solving</th>
<th>Analyze Student Work</th>
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<tbody>
<tr>
<td>Connect Previous Knowledge to New Learning</td>
<td>Identify Student’s Mathematical Understanding</td>
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<tr>
<td>Making Thinking Visible</td>
<td>Identify Student’s Mathematical Misunderstandings</td>
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<tr>
<td>Develop and Demonstrate Mathematical Practices</td>
<td>Interviews</td>
</tr>
<tr>
<td>Inquiry-Oriented and Exploratory Approach</td>
<td>Role Playing</td>
</tr>
<tr>
<td>Multiple Solution Paths and Strategies</td>
<td>Diagrams, Charts, Tables, and Graphs</td>
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<tr>
<td>Use of Multiple Representations</td>
<td>Anticipate Likely and Possible Student Responses</td>
</tr>
<tr>
<td>Explain the Rationale of your Math Work</td>
<td>Collect Different Student Approaches</td>
</tr>
<tr>
<td>Quick Writes</td>
<td>Multiple Response Strategies</td>
</tr>
<tr>
<td>Pair/Trio Sharing</td>
<td>Asking Assessing and Advancing Questions</td>
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<td>Turn and Talk/Charting/Gallery Walks</td>
<td>Revoicing</td>
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<td>Small Group and Whole Class Discussions</td>
<td>Marking/Recapping</td>
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<tr>
<td>Student Modeling</td>
<td>Challenging</td>
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<td></td>
<td>Pressing for Accuracy and Reasoning</td>
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<td></td>
<td>Maintain the Cognitive Demand</td>
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</tbody>
</table>
Educational Technology

Standards


➢ Technology Operations and Concepts
   ● Create a report from a relational database consisting of at least two tables and describe the process, and explain the report results.

Example: Microsoft Excel can be used to create a 2-column formal proof of theorems involving lines, angles, triangles, and parallelograms.

➢ Creativity and Innovation
   ● Apply previous content knowledge by creating and piloting a digital learning game or tutorial.

Example: Students can create tutorials for their classmates proving geometric relationships in figures using criteria for triangle congruence and/or solving problems using triangle congruence and similarity criteria using programs like Prezi, PowerPoint, or You Tube.

➢ Design
   ● Create scaled engineering drawings of products both manually and digitally with materials and measurements labeled.

Example: Students can digitally create dilations that verify the concept that corresponding sides from a pre-image to an image are parallel and length is affected by the scale factor. https://illuminations.nctm.org/activity.aspx?id=4182
Career Ready Practices

Career Ready Practices describe the career-ready skills that all educators in all content areas should seek to develop in their students. They are practices that have been linked to increase college, career, and life success. Career Ready Practices should be taught and reinforced in all career exploration and preparation programs with increasingly higher levels of complexity and expectation as a student advances through a program of study.

- **CRP2. Apply appropriate academic and technical skills.**
  Career-ready individuals readily access and use the knowledge and skills acquired through experience and education to be more productive. They make connections between abstract concepts with real-world applications, and they make correct insights about when it is appropriate to apply the use of an academic skill in a workplace situation.
  
  **Example:** Students will apply prior knowledge when solving real-world problems. Students will make sound judgements about the use of specific tools, such as rulers to draw straight lines and graph paper to construct pre-image and dilation images.

- **CRP4. Communicate clearly and effectively and with reason.**
  Career-ready individuals communicate thoughts, ideas, and action plans with clarity, whether using written, verbal, and/or visual methods. They communicate in the workplace with clarity and purpose to make maximum use of their own and others’ time. They are excellent writers; they master conventions, word choice, and organization, and use effective tone and presentation skills to articulate ideas. They are skilled at interacting with others; they are active listeners and speak clearly and with purpose. Career-ready individuals think about the audience for their communication and prepare accordingly to ensure the desired outcome.
  
  **Example:** Students will on a daily basis communicate their reasoning behind their solution paths by making connections to the context and the quantities, using proper vocabulary, along with decontextualizing and/or contextualizing the problem. Students will create similarity transformations to determine the similarity of figures and explain the meaning of similarity for triangles. They will also explain proofs of theorems about triangles, parallelograms, and lines and angles. Students will also ask probing questions to clarify and improve arguments.
Career Ready Practices

● CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.
Career-ready individuals readily recognize problems in the workplace, understand the nature of the problem and devise effective plans to solve the problem. They are aware of problems when they occur and take action quickly to address the problem; they thoughtfully investigate the root cause of the problem prior to introducing solutions. They carefully consider the options to solve the problem. Once a solution is agreed upon, they follow through to ensure the problem is solved, whether through their own actions or the actions of others.

Example: Throughout their daily lessons, students will understand the meaning of a problem and look for entry points into solving their problems by analyzing the relationships of the quantities, constraints and goals of the task. Plans for solution paths will be made and have meaning. Students will self-monitor, evaluate and critique their process and progress as they are working and make changes as necessary.

● CRP12. Work productively in teams while using cultural global competence.
Career-ready individuals positively contribute to every team, whether formal or informal. They apply an awareness of cultural difference to avoid barriers to productive and positive interaction. They find ways to increase the engagement and contribution of all team members. They plan and facilitate effective team meetings.

Example: Students will work in collaborative and whole group settings to develop various solutions to math tasks that are presented to them. They will work together to understand the terms of the problem, ask clarifying and challenging questions among each other, and develop agreed upon solutions using a variety of strategies and models. Students will listen to, read and discuss arguments with each other with respect and courtesy at all times and will be willing to assist those that may need assistance. In this unit students will demonstrate and explain to a peer or small group the criteria that can be used to explain and prove that corresponding parts of congruent triangles are congruent and discuss the relationships among sides and other segments in a triangle are always true.
## WIDA Proficiency Levels

At the given level of English language proficiency, English language learners will process, understand, produce or use:

<table>
<thead>
<tr>
<th>Level</th>
<th>Proficiency Description</th>
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</table>
| 6- Reaching | - Specialized or technical language reflective of the content areas at grade level  
- A variety of sentence lengths of varying linguistic complexity in extended oral or written discourse as required by the specified grade level  
- Oral or written communication in English comparable to proficient English peers |
| 5- Bridging | - Specialized or technical language of the content areas  
- A variety of sentence lengths of varying linguistic complexity in extended oral or written discourse, including stories, essays or reports  
- Oral or written language approaching comparability to that of proficient English peers when presented with grade level material. |
| 4- Expanding | - Specific and some technical language of the content areas  
- A variety of sentence lengths of varying linguistic complexity in oral discourse or multiple, related sentences or paragraphs  
- Oral or written language with minimal phonological, syntactic or semantic errors that may impede the communication, but retain much of its meaning, when presented with oral or written connected discourse, with sensory, graphic or interactive support |
| 3- Developing | - General and some specific language of the content areas  
- Expanded sentences in oral interaction or written paragraphs  
- Oral or written language with phonological, syntactic or semantic errors that may impede the communication, but retain much of its meaning, when presented with oral or written, narrative or expository descriptions with sensory, graphic or interactive support |
| 2- Beginning | - General language related to the content area  
- Phrases or short sentences  
- Oral or written language with phonological, syntactic, or semantic errors that often impede of the communication when presented with one to multiple-step commands, directions, or a series of statements with sensory, graphic or interactive support |
| 1- Entering | - Pictorial or graphic representation of the language of the content areas  
- Words, phrases or chunks of language when presented with one-step commands directions, WH-, choice or yes/no questions, or statements with sensory, graphic or interactive support |
Language Development Supports For English Language Learners To Increase Comprehension and Communication Skills

**Environment**
- Welcoming and stress-free
- Respectful of linguistic and cultural diversity
- Honors students’ background knowledge
- Sets clear and high expectations
- Includes routines and norms
- Is thinking-focused vs. answer-seeking
- Offers multiple modalities to engage in content learning and to demonstrate understanding
- Includes explicit instruction of specific language targets
- Provides participation techniques to include all learners
- Integrates learning centers and games in a meaningful way
- Provides opportunities to practice and refine receptive and productive skills in English as a new language
- Integrates meaning and purposeful tasks/activities that:
  - Are accessible by all students through multiple entry points
  - Are relevant to students’ lives and cultural experiences
  - Build on prior mathematical learning
  - Demonstrate high cognitive demand
  - Offer multiple strategies for solutions
  - Allow for a language learning experience in addition to content

**Sensory Supports**
- Real-life objects (realia) or concrete objects
- Physical models
- Manipulatives
- Pictures & photographs
- Visual representations or models such as diagrams or drawings
- Videos & films
- Newspapers or magazines
- Gestures
- Physical movements
- Music & songs

**Graphic Supports**
- Graphs
- Charts
- Timelines
- Number lines
- Graphic organizers
- Graphing paper

**Interactive Supports**
- In a whole group
- In a small group
- With a partner such as Turn-and-Talk
- In pairs as a group (first, two pairs work independently, then they form a group of four)
- In triads
- Cooperative learning structures such as Think-Pair-Share
- Interactive websites or software
- With a mentor or coach

**Verbal and Textual Supports**
- Labeling
- Students’ native language
- Modeling
- Repetitions
- Paraphrasing
- Summarizing
- Guiding questions
- Clarifying questions
- Probing questions
- Leveled questions such as What? Where? How? Why?
- Questioning prompts & cues
- Word Banks
- Sentence starters
- Sentence frames
- Discussion frames
- Talk moves, including Wait Time

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BUILDING EQUITY IN YOUR TEACHING PRACTICE

How do the essential questions highlight the connection between the big ideas of the unit and equity in your teaching practice?

**CONTENT INTEGRATION**
Teachers use examples and content from a variety of cultures & groups.

- This unit / lesson is connected to other topics explored with students.
- There are multiple viewpoints reflected in the content of this unit / lesson.
- The materials and resources are reflective of the diverse identities and experiences of students.
- The content affirms students, as well as exposes them to experiences other than their own.

**KNOWLEDGE CONSTRUCTION**
Teachers help students understand how knowledge is created and influenced by cultural assumptions, perspectives & biases.

- This unit / lesson provides context to the history of privilege and oppression.
- This unit / lesson addresses power relationships.
- This unit / lesson help students to develop research and critical thinking skills.
- This curriculum creates windows and mirrors* for students.

**PREJUDICE REDUCTION**
Teachers implement lessons and activities to assert positive images of ethnic groups & improve intergroup relations.

- This unit / lesson help students question and unpack biases & stereotypes.
- This unit / lesson help students examine, research and question information and sources.
- The curriculum encourage discussion and understanding about the groups of people being represented.
- This unit / lesson challenges dominant perspectives.

**EQUITABLE PEDAGOGY**
Teachers modify techniques and methods to facilitate the academic achievement of students from diverse backgrounds.

- The instruction has been modified to meet the needs of each student.
- Students feel respected and their cultural identities are valued.
- Additional supports have been provided for students to become successful and independent learners.
- Opportunities are provided for student to reflect on their learning and provide feedback.

**EMPOWERING SCHOOL CULTURE**
Using the other four dimensions to create a safe and healthy educational environment for all.

- There are opportunities for students to connect with the community.
- My classroom is welcoming and supportive for all students?
- I am aware of and sensitive to the needs of my students and their families.
- There are effective parent communication systems established. Parents can talk to me about issues as they arise in my classroom.

*Windows and mirrors model developed by Derman-Sparks and Lazard, 1998
### Culturally Relevant Pedagogy Examples

- **Integrate Relevant Word Problems:** Contextualize equations using word problems that reference student interests and cultures.  
  **Example:** When learning about dilations and scale factor, problems that relate to student interests such as music, sports and art enable the students to understand and relate to the concept in a more meaningful way.

- **Everyone has a Voice:** Create a classroom environment where students know that their contributions are expected and valued.  
  **Example:** Norms for sharing are established that communicate a growth mindset for mathematics. All students are capable of expressing mathematical thinking and contributing to the classroom community. Students learn new ways of looking at problem solving by working with and listening to each other.

- **Run Problem Based Learning Scenarios:** Encourage mathematical discourse among students by presenting problems that are relevant to them, the school and/or the community.  
  **Example:** Using a Place Based Education (PBE) model, students explore math concepts while determining ways to address problems that are pertinent to their neighborhood, school or culture.

- **Encourage Student Leadership:** Create an avenue for students to propose problem solving strategies and potential projects.  
  **Example:** Students can deepen their understanding of proofs by creating problems together and deciding if the problems fit the necessary criteria. This experience will allow students to discuss and explore their current level of understanding.

- **Present New Concepts Using Student Vocabulary:** Use student diction to capture attention and build understanding before using academic terms.  
  **Example:** Teach math vocabulary in various modalities for students to remember. Use multi-modal activities, analogies, realia, visual cues, graphic representations, gestures, pictures and cognates. Directly explain and model the idea of vocabulary words having multiple meanings. Students can create the Word Wall with their definitions and examples to foster ownership.
# Differentiated Instruction

## Accommodate Based on Students Individual Needs: Strategies

<table>
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<tr>
<th>Time/General</th>
<th>Processing</th>
<th>Comprehension</th>
<th>Recall</th>
</tr>
</thead>
<tbody>
<tr>
<td>Extra time for assigned tasks</td>
<td>Extra Response time</td>
<td>Precise processes for balanced math instructional model</td>
<td>Teacher-made checklist</td>
</tr>
<tr>
<td>Adjust length of assignment</td>
<td>Have students verbalize steps</td>
<td>Short manageable tasks</td>
<td>Use visual graphic organizers</td>
</tr>
<tr>
<td>Timeline with due dates for reports and projects</td>
<td>Repeat, clarify or reword directions</td>
<td>Brief and concrete directions</td>
<td>Reference resources to promote independence</td>
</tr>
<tr>
<td>Communication system between home and school</td>
<td>Mini-breaks between tasks</td>
<td>Provide immediate feedback</td>
<td>Visual and verbal reminders</td>
</tr>
<tr>
<td>Provide lecture notes/outline</td>
<td>Provide a warning for transitions</td>
<td>Small group instruction</td>
<td>Graphic organizers</td>
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<tr>
<td></td>
<td>Partnering</td>
<td>Emphasize multi-sensory learning</td>
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</tbody>
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<thead>
<tr>
<th>Assistive Technology</th>
<th>Tests/Quizzes/Grading</th>
<th>Behavior/Attention</th>
<th>Organization</th>
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</thead>
<tbody>
<tr>
<td>Computer/whiteboard</td>
<td>Extended time</td>
<td>Consistent daily structured routine</td>
<td>Individual daily planner</td>
</tr>
<tr>
<td>Tape recorder</td>
<td>Study guides</td>
<td>Simple and clear classroom rules</td>
<td>Display a written agenda</td>
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<tr>
<td>Video Tape</td>
<td>Shortened tests</td>
<td>Frequent feedback</td>
<td>Note-taking assistance</td>
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<tr>
<td></td>
<td>Read directions aloud</td>
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<td>Color code materials</td>
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</tbody>
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<table>
<thead>
<tr>
<th>Organization</th>
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<tr>
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</tbody>
</table>
Differentiated Instruction
Accommodate Based on Content Specific Needs

- Teacher modeling of thinking processes involved in constructing a two column or paragraph proof
- Reference sheets that list formulas
- Pre-teach vocabulary using visual models that are connected to real-life situations
- Use a ruler to draw lines from the center of dilation to some vertices of the image and pre-image
- Use graph paper to represent the coordinate plane and construct pre-image and dilation image
- Record theorems and postulates in reference notebooks
- Word wall with visual representations of geometric terms
- Calculator to assist with computations
- Utilize technology through interactive sites to explore Plane Geometry, Constructions, and Coordinate Geometry

www.mathopenref.com  https://www.geogebra.org/
Interdisciplinary Connections

Model interdisciplinary thinking to expose students to other disciplines.

Physical Education Connection:

Backyard Basketball (2.5.12.A.1)

- This task uses similar triangles to calculate the height of a basketball net.

Engineering and Technology Connection:

Two Wheels And a Belt (9.3.ST.ET.4)

- This task combines two skills from domain G-C: making use of the relationship between a tangent segment to a circle and the radius touching that tangent segment (G-C.2), and computing lengths of circular arcs given the radii and central angles (G-C.5). It also requires students to create additional structure within the given problem, producing and solving a right triangle to compute the required central angles (G-SRT.8).

Theodolite (9.3.ST.ET.4)

- This task has students explore indirect measurement.
# Enrichment

## What is the Purpose of Enrichment?

- The purpose of enrichment is to provide extended learning opportunities and challenges to students who have already mastered, or can quickly master, the basic curriculum. Enrichment gives the student more time to study concepts with greater depth, breadth, and complexity.
- Enrichment also provides opportunities for students to pursue learning in their own areas of interest and strengths.
- Enrichment keeps advanced students engaged and supports their accelerated academic needs.
- Enrichment provides the most appropriate answer to the question, “What do you do when the student already knows it?”

## Enrichment is…

- Planned and purposeful
- *Different*, or differentiated, work – not just *more* work
- Responsive to students’ needs and situations
- A promotion of high-level thinking skills and making connections within content
- The ability to apply different or multiple strategies to the content
- The ability to synthesize concepts and make real world and cross-curricular connections
- Elevated contextual complexity
- Sometimes independent activities, sometimes direct instruction
- Inquiry based or open-ended assignments and projects
- Using supplementary materials in addition to the normal range of resources
- Choices for students
- Tiered/Multi-level activities with flexible groups (may change daily or weekly)

## Enrichment is not…

- Just for gifted students (some gifted students may need intervention in some areas just as some other students may need frequent enrichment)
- Worksheets that are more of the same (busywork)
- Random assignments, games, or puzzles not connected to the content areas or areas of student interest
- Extra homework
- A package that is the same for everyone
- Thinking skills taught in isolation
- Unstructured free time
Assessments

**Required District/State Assessments**
- Unit Assessment
- PARCC
- SGO Assessments

**Suggested Formative/Summative Classroom Assessments**
- Describe Learning Vertically
- Identify Key Building Blocks
- Make Connections (between and among key building blocks)
- Short/Extended Constructed Response Items
- Multiple-Choice Items (where multiple answer choices may be correct)
- Drag and Drop Items
- Use of Equation Editor
- Quizzes
- Journal Entries/Reflections/Quick-Writes
- Accountable talk
- Projects
- Portfolio
- Observation
- Graphic Organizers/Concept Mapping
- Presentations
- Role Playing
- Teacher-Student and Student-Student Conferencing
- Homework
New Jersey State Learning Standards

<table>
<thead>
<tr>
<th>Standard</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>G.SRT.A.1:</td>
<td>Verify experimentally the properties of dilations given by a center and a scale factor:</td>
</tr>
<tr>
<td>G.SRT.A.1a:</td>
<td>A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.</td>
</tr>
<tr>
<td>G.SRT.A.1b:</td>
<td>The dilation of a line segment is longer or shorter in the ratio given by the scale factor.</td>
</tr>
<tr>
<td>G.SRT.A.2:</td>
<td>Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.</td>
</tr>
<tr>
<td>G.SRT.A.3:</td>
<td>Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.</td>
</tr>
<tr>
<td>G.CO.C.9:</td>
<td>Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment’s endpoints.</td>
</tr>
<tr>
<td>G.CO.C.10:</td>
<td>Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.</td>
</tr>
<tr>
<td>G.CO.C.11:</td>
<td>Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.</td>
</tr>
<tr>
<td>G.SRT.B.4:</td>
<td>Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity</td>
</tr>
<tr>
<td>G.SRT.B.5:</td>
<td>Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.</td>
</tr>
<tr>
<td>Mathematical Practices</td>
<td></td>
</tr>
<tr>
<td>------------------------</td>
<td></td>
</tr>
<tr>
<td>1. Make sense of problems and persevere in solving them.</td>
<td></td>
</tr>
<tr>
<td>2. Reason abstractly and quantitatively.</td>
<td></td>
</tr>
<tr>
<td>3. Construct viable arguments and critique the reasoning of others.</td>
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</tr>
<tr>
<td>4. Model with mathematics.</td>
<td></td>
</tr>
<tr>
<td>5. Use appropriate tools strategically.</td>
<td></td>
</tr>
<tr>
<td>6. Attend to precision.</td>
<td></td>
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<tr>
<td>7. Look for and make use of structure.</td>
<td></td>
</tr>
<tr>
<td>8. Look for and express regularity in repeated reasoning.</td>
<td></td>
</tr>
</tbody>
</table>
## Geometry

<table>
<thead>
<tr>
<th>Unit: 1 (One)</th>
<th>Topic: Congruence, Similarity &amp; Proof</th>
</tr>
</thead>
</table>

### NJSLS:


### Unit Focus:

- Understand similarity in terms of similarity transformations
- Prove geometric theorems.
- Prove theorems involving similarity

### New Jersey Student Learning Standard(s):

**G.SRT.A.1:** Verify experimentally the properties of dilations given by a center and a scale factor.

- **G.SRT.A.1a:** A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.

- **G.SRT.A.1b:** The dilation of a line segment is longer or shorter in the ratio given by the scale factor.

**Student Learning Objective 1:** Verify the properties of dilations given by a center and a scale factor.

### Modified Student Learning Objectives/Standards: N/A

<table>
<thead>
<tr>
<th>MPs</th>
<th>Evidence Statement Key/Clarifications</th>
<th>Skills, Strategies &amp; Concepts</th>
<th>Essential Understandings/Questions (Accountable Talk)</th>
<th>Tasks/Activities</th>
</tr>
</thead>
<tbody>
<tr>
<td>MP 1 MP 3 MP 5 MP 8</td>
<td>G-SRT.1a G-SRT.1b</td>
<td>Dilation is a transformation in which a figure is enlarged or reduced with respect to a fixed point C called the center of dilation and the scale factor, k, which is the ratio of the lengths</td>
<td>Dilations are transformations that preserve angle measure but not distance.</td>
<td>Type II, III:</td>
</tr>
<tr>
<td>HS.C.14.5</td>
<td>HS.D. 3-2a</td>
<td></td>
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<tr>
<td>Construct, autonomously, chains of reasoning that will justify or refute geometric propositions or conjectures. Content scope: G-SRT.A.</td>
<td>Micro-models: Autonomously apply a technique from pure mathematics to a real-world situation in which the technique yields valuable results even though it is obviously not applicable in a strict mathematical sense (e.g., profitably applying proportional relationships to a phenomenon that is obviously nonlinear or not).</td>
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</tbody>
</table>

**Coordinate Rule for Dilations:** If $P(x, y)$ is the preimage of a point, then its image after a dilation centered at the origin $(0, 0)$ with scale factor $k$ is the point $P'(kx, ky)$. The scale factor of dilation is the ratio of corresponding sides of the image and the preimage. This ratio is used to solve real-life applications.

When a transformation, such as dilation, changes the shape or size of a figure, the transformation is nonrigid. It is important to pay attention to whether a nonrigid transformation is applied.

<table>
<thead>
<tr>
<th>Dilating a Line</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Additional Tasks:</strong></td>
</tr>
<tr>
<td><strong>Lesson Seed</strong></td>
</tr>
</tbody>
</table>

What happens to a figure once it has been dilated?

Why do dilations involve scale factors?

How can you determine if a line in a figure will be changed by dilation?

What are the key properties of dilations?

How do dilations affect the various parts of a figure and their relationship to each other?

How do you draw the image of a figure under a dilation?
<table>
<thead>
<tr>
<th>Reasoned estimates: Use reasonable estimates of known quantities in a chain of reasoning that yields an estimate of an unknown quantity. Content Scope: Knowledge and skills articulated in the Geometry Type I, Sub-Claim A Evidence Statements.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>HS.D. 3-4a</strong></td>
</tr>
<tr>
<td>Centers of dilation do not need to be at the origin, and students are constructing dilations with a straightedge and a compass.</td>
</tr>
<tr>
<td>Dilation of a line that passes through the center of dilation results in the same line.</td>
</tr>
<tr>
<td>Dilation of a line that does not pass through the center of dilation results in a line that is parallel to the original line.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Horizontal Stretch</th>
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<tbody>
<tr>
<td><img src="image1" alt="" /></td>
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</table>

<table>
<thead>
<tr>
<th>Vertical Stretch</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image2" alt="" /></td>
</tr>
</tbody>
</table>
Dilation of a line segment results in a longer line segment when, for scale factor $k$, $|k|$ is greater than 1.

Students may be confused because there is no “center” specified. The scale factor is the ratio of corresponding sides comparing preimage side length to image side length.

Dilation of a line segment results in a shorter line segment when, for scale factor $k$, $|k|$ is less than 1.

Perform dilations in order to verify the impact of dilations on lines and line segments.

Perform dilations and examine properties affecting sides from a pre-image to an image (scale factor affects side length)

Create dilations that verify the concept that corresponding sides from a preimage to an image are parallel and length is affected by the scale factor.

**SPED Strategies:**

Review the concept of scale factor with students using real life examples involving dilations.
Encourage students to add this concept to their reference notebook by providing notes or guiding notetaking.

Provide students with hands on opportunities to explore and extend their understanding of dilations.

**ELL Strategies:**

Provide students with translation dictionary.

Demonstrate comprehension of written word problems in student’s native language and/or using simplified word problems with visuals.

Explain in writing the properties of dilations in student’s native language and/or use gestures, examples and selected technical words.

---

**New Jersey Student Learning Standard(s):**

G.SRT.A.2: Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.

**Student Learning Objective 2:** Use the definition of similarity in terms of similarity transformations to decide if two given figures are similar and explain, using similarity transformations, the meaning of triangle similarity.

**Modified Student Learning Objectives/Standards:** N/A

<table>
<thead>
<tr>
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<th>Essential Understandings/Questions (Accountable Talk)</th>
<th>Tasks/Activities</th>
</tr>
</thead>
<tbody>
<tr>
<td>MP 3 MP 5</td>
<td>G-SRT.2</td>
<td>In grade 8, similar figures were defined as figures that have the same shape but not necessarily the same size. Two figures are</td>
<td>What does it mean for two figures to be similar?</td>
<td>Type I:</td>
</tr>
<tr>
<td><strong>MP 8</strong></td>
<td></td>
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<tr>
<td>● The “explain” part of the standard G-SRT.2 is not assessed here.</td>
<td>Similar when corresponding side lengths are proportional and corresponding angles are congruent</td>
<td>How do triangle similarity criteria follow from similarity transformations?</td>
<td>Backyard Basketball</td>
<td></td>
</tr>
<tr>
<td><strong>HS.C.14.5</strong></td>
<td></td>
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<td></td>
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</tr>
<tr>
<td>● Construct, autonomously, chains of reasoning that will justify or refute geometric propositions or conjectures. Content scope: G-SRT.A</td>
<td>Similarity is defined in terms of similarity transformations. Students need to understand that a similarity transformation is a dilation or a composition of rigid motions and dilations. To decide (prove) whether two figures are similar, you need to map one figure onto the other by a single dilation or a composition of rigid motions and dilations.</td>
<td>Type II, III:</td>
<td></td>
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<tr>
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<tr>
<td></td>
<td>Given two figures, determine, using transformations, if they are similar.</td>
<td>Are They Similar Congruent and Similar Triangles</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Explain, using similarity transformations, the meaning of similarity for triangles.</td>
<td>Similar Quadrilaterals</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>When figures are similar, corresponding angles are congruent and corresponding segments are proportional.</td>
<td>Similar Triangles</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<p>| <strong>HS.D. 3-2a</strong> |<br />
| --- | --- |
| ● Micro-models: Autonomously apply a technique from pure mathematics to a real-world situation in which the technique yields valuable results even though it is obviously not applicable in a strict mathematical sense (e.g., profitably applying proportional relationships to a similar when corresponding side lengths are proportional and corresponding angles are congruent |  |  |</p>
<table>
<thead>
<tr>
<th>phenomenon that is obviously nonlinear or statistical in nature). Content Scope: Knowledge and skills articulated in the Geometry Type I, Sub-Claim A Evidence Statements.</th>
</tr>
</thead>
<tbody>
<tr>
<td>There are criteria that allow the conclusion two triangles are similar.</td>
</tr>
<tr>
<td><strong>SPED Strategies:</strong></td>
</tr>
<tr>
<td>Pre-teach vocabulary using visual and verbal models that are connected to real life situations and ensure that students include these definitions their reference notebook.</td>
</tr>
<tr>
<td>Model the concept, thinking and processes regarding similarity with students in a hands on way and referring to examples in their everyday life.</td>
</tr>
<tr>
<td><strong>ELL Strategies:</strong></td>
</tr>
<tr>
<td>Use the definition of similarity in terms of similarity transformations to decide if two given figures are similar and explain, using similarity transformations, the meaning of triangle similarity.</td>
</tr>
<tr>
<td>Develop definitions orally and in writing for the geometric terms using technical vocabulary in sentences with embedded clauses.</td>
</tr>
<tr>
<td>Justify answers by explaining the similarity transformations in student’s native language and/or use gestures, examples and selected technical words.</td>
</tr>
</tbody>
</table>
**New Jersey Student Learning Standard(s):**
G.SRT.A.3: Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

**Student Learning Objective 3:** Use the properties of similarity transformations to establish the Angle-Angle criterion for two triangles to be similar.

**Modified Student Learning Objectives/Standards:** N/A

<table>
<thead>
<tr>
<th>MPs</th>
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<th>Essential Understandings/Questions (Accountable Talk)</th>
<th>Tasks/Activities</th>
</tr>
</thead>
<tbody>
<tr>
<td>MP 1</td>
<td>HS.C.14.5</td>
<td>Angle-Angle (AA) Similarity Theorem: If two angles of one triangle are congruent to two angles of another triangle, then the two triangles are similar.</td>
<td>How can I use transformations to establish that two triangles are similar using Angle-Angle criterion? Why is it unnecessary to have either an AAS or ASA theorem to prove two triangles are similar?</td>
<td>Type I: Similar Triangles</td>
</tr>
<tr>
<td>MP 1</td>
<td>● Construct, autonomously, chains of reasoning that will justify or refute geometric propositions or conjectures. Content scope: G-SRT.A.</td>
<td>Explain Angle-Angle criterion and its relationship to similarity transformations and properties of triangles. They will also apply this theorem in a real-life application to find the height of an object using indirect measure.</td>
<td>How can you describe the relationship between sides of similar figures? How can you show that two triangles are similar?</td>
<td>Additional Tasks: Theodolite - Civil Engineering</td>
</tr>
<tr>
<td>MP 2</td>
<td></td>
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</tr>
</tbody>
</table>
Use properties of similar triangles to find segment lengths.

**Visual reasoning:** Use colored pencils to show congruent angles. This will help you write similarity statements.

You may find it helpful to redraw the triangles separately.

**SPED Strategies:**

Model how dilations can prove similarity by discussing enlarging a photograph or shape and connect this to triangles.

Review angle-angle criterion with students and encourage students to add this concept to
Provide students with hands on opportunities to explore and extend their understanding of similarity of triangles.

**ELL Strategies:**
Introduce and reinforce geometry concepts and vocabulary.

Provide use of translating dictionary.

Review angle-angle criterion with students.

Explain in writing how the AA criterion can establish similarity in triangles in student’s native language and/or use gestures, examples and selected technical words.
New Jersey Student Learning Standard(s):

**G.CO.C.9:** Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment’s endpoints.

**G.CO.C.10:** Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.

**G.CO.C.11:** Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.

**Student Learning Objective 4:** Construct and explain formal proofs of theorems involving lines, angles, triangles, and parallelograms.

**Modified Student Learning Objectives/Standards:** N/A

<table>
<thead>
<tr>
<th>MPs</th>
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<th>Essential Understandings/Questions (Accountable Talk)</th>
<th>Tasks/Activities</th>
</tr>
</thead>
</table>
| MP 3 MP 6 | G-CO.C  ● About 75% of tasks align to G.CO.9 or G.CO.10.  ● Theorems include but are not limited to the examples listed in standards G.CO.9, 10,11.  ● Multiple types of proofs are allowed (e.g. two-column proof, indirect proof, A formal proof may be represented with a paragraph proof or a two-column proof. Construct and explain proofs of theorems about lines and angles including: vertical angles are congruent; congruence of alternate interior angles; congruence of corresponding angles; and points on a perpendicular bisector of a line segment are exactly those equidistant from the segment’s endpoints. **Right Angles Congruence Theorem:** All right angles are congruent. **Congruent Supplements Theorem:** If two angles are supplementary to the same angle How are proofs used to develop conjectures in Mathematics? **Type I:** Congruent Angles Made by Parallel Lines and a Transverse

**Type II, III:** Classifying Triangles

**Congruence of Parallelograms

**Congruent Angles in Isosceles Triangles
<table>
<thead>
<tr>
<th>Question</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>What relationships among angles, sides, and other segments in parallelograms are always true?</td>
<td>Congruent Angles Made by Parallel Lines</td>
</tr>
<tr>
<td>Why is it important to include every logical step in a proof?</td>
<td>Is this a Parallelogram</td>
</tr>
<tr>
<td>How is a theorem different from an axiom?</td>
<td>Midpoints of the Sides of a Parallelogram</td>
</tr>
<tr>
<td>What is the difference between an axiom and a postulate?</td>
<td>Midpoints of Triangle Sides</td>
</tr>
<tr>
<td>How do you know when a proof is complete and valid?</td>
<td>Parallelograms and Translations</td>
</tr>
<tr>
<td>Why does a line cutting a triangle parallel to one side of the triangle divide the other two proportionally?</td>
<td>Placing a Fire Hydrant</td>
</tr>
<tr>
<td>How many pieces of information do you need in order to solve for the missing measures of a triangle?</td>
<td>Points Equidistant</td>
</tr>
<tr>
<td>How is the concept of similarity used to make scale drawings?</td>
<td>Quadrilaterals</td>
</tr>
<tr>
<td>Sum of Angles in a Triangle</td>
<td>Seven Circles I</td>
</tr>
</tbody>
</table>

**HS.C.14.1**

- Construct, autonomously, chains of reasoning that will justify or refute geometric propositions or conjectures. Content scope: G-CO.9, G-CO.10

Theorems include, but are not limited to, the examples listed in standards G-CO.9 & G-CO.10.

- **Congruent Complements Theorem:** If two angles are complementary to the same angle (or to congruent angles), then they are congruent.

- **Linear Pair Postulate:** If two angles form a linear pair, they are supplementary.

- **Vertical Angles Congruence Theorem:** Vertical angles are congruent.

- **Corresponding Angles Theorem:** If two parallel lines are cut by a transversal, then the pairs of corresponding angles are congruent.

- **Alternate Interior Angles Theorem:** If two parallel lines are cut by a transversal, then the pairs of alternate interior angles are congruent.

- **Alternate Exterior Angles Theorem:** If two parallel lines are cut by a transversal, then the
pairs of alternate exterior angles are congruent.

**Consecutive Interior Angles Theorem:** If two parallel lines are cut by a transversal, then the pairs of consecutive interior angles are supplementary.

Construct and explain proofs of theorems about triangles including: sum of interior angles of a triangle; congruence of base angles of an isosceles triangle; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; and the medians of a triangle meet at a point.

**Triangle Sum Theorem:** The sum of the measures of the interior angles of a triangle is $180^\circ$.

![Triangle Sum Theorem Diagram]

**Base Angle Theorem:** If two sides of a triangle are congruent, then the angles opposite them are congruent.
Construct and explain proofs of theorems about parallelograms including: opposite sides are congruent; opposite angles are congruent; the diagonals of a parallelogram bisect each other; and rectangles are parallelograms with congruent diagonals.

**Parallelogram Opposites Sides Converse:**
If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

**Parallelogram Opposite Angles Converse:**
If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram.
Opposite Sides Parallel and Congruent
**Theorem:** If one pair of opposite sides of a quadrilateral are congruent and parallel, then the quadrilateral is a parallelogram.

Parallelogram Diagnosis Converse: If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.

Use inductive reasoning to identify patterns and make conjectures.

Disprove conjectures using counterexamples.
| Identify, write, and analyze the truth value of a conditional statement. |
| Write the inverse, converse, and contrapositive of a conditional statement. |
| Use deductive reasoning. |
| Write and analyze biconditional statements. A biconditional statement is when a conditional statement and its converse are both true, you can write them as a single biconditional statement and contains the phrase “if and only if.” |
| **Words**  
  \[ p \text{ if and only if } q \]  
| **Symbols**  
  \[ p \leftrightarrow q \] |
| Recognize the Hypothesis and Conclusion of an if-then statement. |
| Apply the properties of algebra and congruence in proofs Use the Midpoint Theorem and Angle Bisector Theorem. |
| **Angle Bisector Theorem:** If a point lies on the bisector of an angle, then it is equidistant
from the two sides of the angle.

Converse of the Angle Bisector Theorem:
If a point is in the interior of an angle and is equidistant from the two sides of the angle, then it lies on the bisector of the triangle.

Apply the definitions of complementary and supplementary angles in proofs and in algebraic solutions.

Apply the definitions and theorems of vertical angles and perpendicular lines in proofs and in algebraic solutions.
### SPED Strategies:

Pre-teach vocabulary using visual and verbal models that are connected to real life situations and ensure that students include these definitions their reference notebook.

Model the thinking and processes involved in constructing a two column or paragraph proof. Provide students with notes and examples to illustrate the concept and skills necessary to demonstrate proficiency.

Encourage students to use their reference notebook when constructing proofs as a tool.

Encourage students to verbalize their thinking while working in small groups by asking assessing and advancing questions. Use this information to tailor instruction to student needs.

### ELL Strategies:

Create and explain orally the proofs of theorems in student’s native language and/or use gestures, examples and selected technical words.

Provide graphic organizers.

Have students work in triads or small groups where they are able to support each other’s learning by giving each other input and filling in gaps in background. Students often work
best when they have defined roles (surrounding the content they are studying) that they are responsible for.

New Jersey Student Learning Standard(s):
G.SRT.B.4: Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.

Student Learning Objective 5: Prove theorems about triangles.

Modified Student Learning Objectives/Standards: N/A

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</thead>
<tbody>
<tr>
<td>MP 2</td>
<td>HS.C.14.6</td>
<td>Construct and explain proofs of theorems about triangles including:</td>
<td>How do I use similar triangles to prove the Pythagorean theorem?</td>
<td>Lesson Seed</td>
</tr>
<tr>
<td></td>
<td></td>
<td>o a line parallel to one side of a triangle divides the other two sides proportionally;</td>
<td>What strategies can I use to determine missing side lengths and areas of similar figures?</td>
<td>Joining Two Midpoints of Sides of a Triangle</td>
</tr>
<tr>
<td></td>
<td></td>
<td>o and the Pythagorean Theorem (using triangle similarity).</td>
<td>How does coordinate geometry allow you to find relationships in triangles?</td>
<td>Pythagorean Theorem</td>
</tr>
<tr>
<td>MP 6</td>
<td></td>
<td>Use the similarity criteria to establish a new and important relationship found in similar triangles, such as the side splitting theorem and the angle bisector theorem.</td>
<td>How do you solve problems that involve measurements of triangles?</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td><strong>Side Splitting Theorem:</strong> A line parallel to one side of a triangle divides the other two proportionally.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Students are able to handle the easy relationships of comparing pieces to pieces, but when the ratio of the full length side is used in a ratio they often compare the wrong things. Many examples should be done to work out this confusion.

**Angle Bisector Theorem:** An angle bisector of an angle of a triangle divides the opposite side in two segments that are proportional to the other two sides of the triangle.

Use student prior connection to similarity with connections to ratios, scale factors, and proportion all connect to the concept of similarity and the non-isometric transformation, dilation.
Prove the Pythagorean Theorem using similarity and the geometric means.

A common error is comparing the proportional pieces cut by the parallel line to the third side that has not been cut into pieces. Students too often relate pieces of sides to whole sides and that causes an error.

<table>
<thead>
<tr>
<th>AB</th>
<th>BE</th>
</tr>
</thead>
<tbody>
<tr>
<td>BC</td>
<td>CF</td>
</tr>
</tbody>
</table>

**SPED Strategies**

Model the thinking and processes involved in constructing a two column or paragraph proof involving triangles. Provide students with notes and examples to illustrate the concept.
and skills necessary to demonstrate proficiency.

Encourage students to verbalize their thinking while working in small groups by asking assessing and advancing questions. Use this information to tailor instruction to student needs.

**ELL Strategies:**
Sequence and explain the steps to prove theorems about triangles in student’s native language and/or use gestures, examples and selected technical words.

Provide students with graphic organizers.

<table>
<thead>
<tr>
<th>New Jersey Student Learning Standard(s):</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>G.SRT.B.5:</strong> Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.</td>
</tr>
</tbody>
</table>

**Student Learning Objective 6:** Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

**Modified Student Learning Objectives/Standards:** N/A

<table>
<thead>
<tr>
<th>MPs</th>
<th>Evidence Statement Key/Clarifications</th>
<th>Skills, Strategies &amp; Concepts</th>
<th>Essential Understandings/Questions (Accountable Talk)</th>
<th>Tasks/Activities</th>
</tr>
</thead>
<tbody>
<tr>
<td>MP 7</td>
<td>G-SRT.5</td>
<td>Corresponding parts of congruent triangles are congruent (CPCTC).</td>
<td>What criteria can you use to explain and prove that corresponding parts of congruent triangles are congruent (CPCTC)?</td>
<td>Type II, III: Blank Shot</td>
</tr>
<tr>
<td><strong>HS.C.14.6</strong></td>
<td><strong>HS.D. 3-2a</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>----------------</td>
<td>----------------</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>- Construct, autonomously, chains of reasoning that will justify or refute geometric propositions or conjectures. Content scope: G-SRT.B</td>
<td>- Micro-models: Autonomously apply a technique from pure mathematics to a real-world situation in which the technique yields valuable results even though it is obviously not applicable in a strict mathematical sense (e.g., profitably applying proportional relationships to a phenomenon that is obviously nonlinear or statistical in nature). Content Scope: Knowledge and skills articulated in the Geometry Type I, Sub-</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>Students will be able to derive the three geometric mean relationships and use them to solve for sides of triangles. <strong>Right Triangle Similarity Theorem:</strong> If the altitude is drawn to the hypotenuse of a right triangle, then the two triangles formed are similar to the original triangle and to each other.</td>
<td>The challenge many students have is correctly naming the pairs of triangles that are similar. One reason for the difficulty is that the triangles are in different orientations. Make physical copies of triangles to manipulate, cut two congruent right scalene triangles. Draw the altitude to the hypotenuse in one of the right triangles. Cut along the altitude, making two smaller triangles.</td>
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</tr>
<tr>
<td>What strategies can I use to determine missing side lengths and areas of similar figures?</td>
<td>What relationships among sides and other segments in a triangle are always true? How does the use of congruency and similarity concepts allow us to model relationships between geometric figures? Similarity unlocks many new powerful relationships such as geometric mean and special right triangles. Two angles locks in the side proportionality of a triangle.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Finding Triangle Coordinates</td>
<td>How Far is the Horizon Points From Directions</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Finding Triangle Coordinates</td>
<td>Tangent Line to Two Circles</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unit Squares and Triangles</td>
<td></td>
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</tbody>
</table>

**Right Triangle Similarity Theorem:**

If the altitude is drawn to the hypotenuse of a right triangle, then the two triangles formed are similar to the original triangle and to each other.

\[ \triangle CBD \sim \triangle ABC, \triangle ACD \sim \triangle ABC, \text{ and } \triangle CBD \sim \triangle ACD. \]
<table>
<thead>
<tr>
<th>Claim A Evidence Statements.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>HS.D. 3-4a</strong></td>
</tr>
<tr>
<td>• Reasoned estimates: Use reasonable estimates of known quantities in a chain of reasoning that yields an estimate of an unknown quantity.</td>
</tr>
<tr>
<td><strong>Content Scope:</strong> Knowledge and skills articulated in the Geometry Type I, Sub-Claim A Evidence Statements.</td>
</tr>
</tbody>
</table>

**Geometric Mean (Altitude) Theorem:** In a right triangle, the altitude from the right angle to the hypotenuse divides the hypotenuse into two segments. The length of the altitude is the geometric mean of the lengths of the two segments of the hypotenuse.

![Diagram of Geometric Mean (Altitude) Theorem](image)

\\( CD^2 = AD \cdot BD \)

**Geometric Mean (Leg) Theorem:** In a right triangle, the altitude from the right angle to the hypotenuse divides the hypotenuse into two segments. The length of each leg of the right triangle is the geometric mean of the lengths of the hypotenuse and the segment of the hypotenuse that is adjacent to the leg.

![Diagram of Geometric Mean (Leg) Theorem](image)

\\( CB^2 = DB \cdot AB \)

\\( AC^2 = AD \cdot AB \)

Prove geometric relationships in figures using criteria for triangle congruence.

Prove geometric relationships in figures using criteria for triangle congruence.
<table>
<thead>
<tr>
<th>Solve problems using triangle congruence criteria (SSS, ASA, SAS, HL).</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solve problems using triangle similarity criteria (AA).</td>
</tr>
<tr>
<td>Use manipulatives, such as 2-D triangles to revisit similar triangles.</td>
</tr>
</tbody>
</table>

**SPED Strategies:**

Pre-teach triangle congruence criteria using visual and verbal models that are connected to real life situations and ensure that students include these definitions in their reference notebook.

Provide students with opportunities to practice applying these concepts and skills by working in pairs/small groups.

**ELL Strategies:**

Explain orally and in writing how to use congruence and similarity criteria for triangles to prove relationships in student’s native language and/or use drawings, examples and selected technical words.

Teacher Modeling

Use of word wall.

Use of graphic organizers.
Integrated Evidence Statements

**HS.C.14.5:** Construct, autonomously, chains of reasoning that will justify or refute geometric propositions or conjectures. Content scope: G-SRT.A - MP.3 Y C

**HS.C.14.6:** Construct, autonomously, chains of reasoning that will justify or refute geometric propositions or conjectures. Content scope: G-SRT.B
G-SRT.1a: Verify experimentally the properties of dilations given by a center and a scale factor. a) A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.

G-SRT.1b: Verify experimentally the properties of dilations given by a center and a scale factor. b) The dilation of a line segment is longer or shorter in the ratio given by the scale factor.

G-SRT.2: Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar.
- The "explain" part of standard G-SRT.2 is not assessed here. See Sub-Claim C for this aspect of the standard.

G-SRT.5: Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.
- For example, find a missing angle or side in a triangle.

G-CO.C: Prove geometric theorems as detailed in G-CO.C.
- About 75% of tasks align to G.CO.9 or G.CO.10.
- Theorems include but are not limited to the examples listed in standards G-CO.9, 10, 11.
- Multiple types of proofs are allowed (e.g., two-column proof, indirect proof, paragraph proof, and flow diagrams).

**HS.C.14.1:** Construct, autonomously, chains of reasoning that will justify or refute geometric propositions or conjectures. Content scope: G-CO.9, G-CO.10.
- Theorems include, but are not limited to, the examples listed in standards G-CO.9 & G-CO.10.

**HS.C.18.2** Use a combination of algebraic and geometric reasoning to construct, autonomously, chains of reasoning that will justify or refute propositions or conjectures about geometric figures. Content scope: Algebra content from Algebra 1 course; geometry content from the Geometry course.
- For the Geometry course, we are reaching back to Algebra 1 to help students synthesize across the two subjects.
# Unit 2 Vocabulary

- adjacent angles
- alternate exterior angles
- Alternate Exterior Angles Theorem
- alternate interior angles
- Alternate Interior Angles Theorem
- altitude
- Angle-Angle Similarity Theorem
- Angle Bisector Theorem
- base angles of an isosceles triangle
- Base Angle Theorem
- biconditional
- bisector
- center of dilation
- complementary angles
- congruent segments
- Congruent Complements Theorem
- Congruent Supplements Theorem
- conjecture
- consecutive interior angles
- Consecutive Interior Angles Theorem
- contrapositive
- converse

- Converse of the Angle Bisector Theorem
- Coordinate Rule for Dilations
- corresponding angles
- Corresponding Angles Theorem
- corresponding parts
- diagonal
- dilation
- equidistant
- Geometric Mean (Altitude) Theorem
- Geometric Mean (Leg) Theorem
- horizontal stretch
- hypotenuse
- interior angles
- line segment
- linear pair
- Linear Pair Postulate
- median
- midpoint
- non-isometric transformation
- nonrigid

- Opposite Sides Parallel and Congruent Theorem
- parallel
- parallelogram
- Parallelogram Diagnosis Converse
- Parallelogram Opposite Angles Converse
- Parallelogram Opposites Sides Converse
- perpendicular bisector
- proof
- proportion
- Pythagorean Theorem
- Right Angles Congruence Theorem
- Right Triangle Similarity Theorem
- Side Splitting Theorem
- supplementary angles
- transversal
- Triangle Sum Theorem
- vertex angle
- vertical angles
- Vertical Angles Congruence Theorem
- vertical stretch
## References & Suggested Instructional Websites

- [https://hcpss.instructure.com/courses/162](https://hcpss.instructure.com/courses/162)
- [https://www.desmos.com/](https://www.desmos.com/)
- [http://www.cpalms.org/Public/ToolkitGradeLevelGroup/Toolkit?id=14](http://www.cpalms.org/Public/ToolkitGradeLevelGroup/Toolkit?id=14)
- [www.corestandards.org](http://www.corestandards.org)
- [www.nctm.org](http://www.nctm.org)
- [https://www.khanacademy.org](https://www.khanacademy.org)
- [http://achievethecore.org](http://achievethecore.org)
- [https://www.illustrativemathematics.org/](https://www.illustrativemathematics.org/)
- [www.insidemathematics.org](http://www.insidemathematics.org)
- [https://learnzillion.com/resources/75114-math](https://learnzillion.com/resources/75114-math)
- [http://maccss.ncdpi.wikispaces.net/](http://maccss.ncdpi.wikispaces.net/) (Choose your grade level on the left.)
- [http://nrich.maths.org](http://nrich.maths.org)
- [https://www.youcubed.org/week-of-inspirational-math/](https://www.youcubed.org/week-of-inspirational-math/)
- [http://illuminations.nctm.org/Lessons-Activities.aspx](http://illuminations.nctm.org/Lessons-Activities.aspx) (choose grade level and connect to search lessons)
- [www.ck12.org](http://www.ck12.org)
Field Trip Ideas

**SIX FLAGS GREAT ADVENTURE** - This educational event includes workbooks and special science and math related shows throughout the day. Your students will leave with a better understanding of real world applications of the material they have learned in the classroom. Each student will have the opportunity to experience different rides and attractions linking mathematical and scientific concepts to what they are experiencing.

[www.sixflags.com](http://www.sixflags.com)

**MUSEUM of MATHEMATICS** - Mathematics illuminates the patterns that abound in our world. The National Museum of Mathematics strives to enhance public understanding and perception of mathematics. Its dynamic exhibits and programs stimulate inquiry, spark curiosity, and reveal the wonders of mathematics. The Museum’s activities lead a broad and diverse audience to understand the evolving, creative, human, and aesthetic nature of mathematics.

[www.momath.org](http://www.momath.org)

**LIBERTY SCIENCE CENTER** - An interactive science museum and learning center located in Liberty State Park. The center, which first opened in 1993 as New Jersey's first major state science museum, has science exhibits, the largest IMAX Dome theater in the United States, numerous educational resources, and the original *Hoberman sphere*.

[http://lsc.org/plan-your-visit/](http://lsc.org/plan-your-visit/)