Geometry Honors: Unit 3
Trigonometric Ratios & Geometric Equations and Probability of Compound Events
Course Philosophy/Description

Geometry Honors is for our accelerated Math students. It stresses the ability to reason logically and to think critically, using spatial sense. In this course, students will engage in activities that allow them to create geometric understanding. A major part of the course will be devoted to teaching the student how to present a formal proof. Geometric properties of both two and three dimensions are emphasized as they apply to points, lines, planes, and solids. In this course students learn to recognize and work with geometric concepts in various contexts. They build on ideas of inductive and deductive reasoning, logic, concepts, and techniques of Euclidean plane and solid geometry and develop an understanding of mathematical structure, method, and applications of Euclidean plane and solid geometry. Students use visualizations, spatial reasoning, and geometric modeling to solve problems. Topics of study include points, lines, and angles; triangles; quadrilaterals and other polygons; circles; coordinate geometry; three-dimensional solids; geometric constructions; symmetry; similarity; and the use of transformations. Students use tools of geometry to develop, verify and prove geometric principles and relationships. Students extend their knowledge and understanding by solving open-ended real-world problems and thinking critically through the use of high level tasks and long-term projects.

Upon successful completion of this course, students will be able to: Use and prove basic theorems involving congruence and similarity of figures; determine how changes in dimensions affect perimeter and area of common geometric figures; apply and use the properties of proportion; perform basic constructions with straight edge and compass; prove the Pythagorean Theorem; use the Pythagorean Theorem to determine distance and find missing dimensions of right triangles; know and use formulas for perimeter, circumference, area, volume, lateral and surface area of common figures; find and use measures of sides, interior and exterior angles of polygons to solve problems; use relationships between angles in polygons, complementary, supplementary, vertical and exterior angle properties; use special angle and side relationships in special right triangles; understand, apply, and solve problems using basic trigonometric functions; prove and use relationships in circles to solve problems; prove and use theorems involving properties of parallel lines cut by a transversal, quadrilaterals and circles; write geometric proofs, including indirect proofs; construct and judge validity of logical arguments; prove theorems using coordinate geometry including the midpoint of a segment and distance formula; understand transformations in the coordinate plane; construct logical verifications to test conjectures and counterexamples; and write basic mathematical arguments in paragraph and statement-reason form.
This ESL framework was designed to be used by bilingual, dual language, ESL and general education teachers. Bilingual and dual language programs use the home language and a second language for instruction. ESL teachers and general education or bilingual teachers may use this document to collaborate on unit and lesson planning to decide who will address certain components of the SLO and language objective. ESL teachers may use the appropriate leveled language objective to build lessons for ELLs which reflects what is covered in the general education program. In this way, whether it is a pull-out or push-in model, all teachers are working on the same Student Learning Objective connected to the New Jersey Student Learning Standards. The design of language objectives are based on the alignment of the World-Class Instructional Design Assessment (WIDA) Consortium’s English Language Development (ELD) standards with the New Jersey Student Learning Standards (NJSLS). WIDA’s ELD standards advance academic language development across content areas ultimately leading to academic achievement for English learners. As English learners are progressing through the six developmental linguistic stages, this framework will assist all teachers who work with English learners to appropriately identify the language needed to meet the requirements of the content standard. At the same time, the language objectives recognize the cognitive demand required to complete educational tasks. Even though listening and reading (receptive) skills differ from speaking and writing (expressive) skills across proficiency levels the cognitive function should not be diminished. For example, an Entering Level One student only has the linguistic ability to respond in single words in English with significant support from their home language. However, they could complete a Venn diagram with single words which demonstrates that they understand how the elements compare and contrast with each other or they could respond with the support of their native language with assistance from a teacher, para-professional, peer or a technology program.

http://www.state.nj.us/education/modelcurriculum/ela/ELLOverview.pdf
# | Student Learning Objective | NJSLS | Big Ideas Math Correlation | Instruction: 8 weeks | Assessment: 1 week |
--- | --- | --- | --- | --- | --- |
1 | Use coordinates to prove simple geometric theorems algebraically. | G.GPE.B.4 | 5.8, 10.7 | | |
2 | Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems. | G.GPE.B.5 | 3.5, 8.3 | | |
3 | Find the point on a directed line segment between two given points that partitions the segment in a given ratio and use coordinates to compute perimeters of polygons and areas of triangles and rectangles. | G.GPE.B.6, G.GPE.B.7 | 1.4, 3.5, 8.4 | | |
4 | Show and explain that definitions for trigonometric ratios derive from similarity of right triangles. | G.SRT.C.6 | 9.4, 9.5 | | |
5 | Explain and use the relationship between the sine and cosine of complementary angles; use trigonometric ratios and the Pythagorean Theorem to compute all angle measures and side lengths of triangles in applied problems. | G.SRT.C.7, G.SRT.C.8 | 9.1, 9.4, 9.5, 9.6 | | |
6 | Derive the equation of a circle of given the center and radius using the Pythagorean Theorem. Given an equation, complete the square to find the center and radius of the circle. | G.GPE.A.1 | 10.7 | | |
7 | Prove that all circles are similar. | G.C.A.1 | 10.2 | | |
8 | Identify and describe relationships among inscribed angles, radii, and chords; use these relationships to solve problems. | G.C.A.2 | 10.1, 10.2, 10.3, 10.4, 10.5, 10.6 | | |
9 | Find arc lengths and areas of sectors of circles; use similarity to show that the length of the arc intercepted by an angle is proportional to the radius. Derive the formula for the area of a sector. | G.C.B.5 | 11.1, 11.2 | |
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<tr>
<th>#</th>
<th>Activity</th>
<th>Standards</th>
<th>Lessons</th>
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<tr>
<td>10</td>
<td>Prove the properties of angles for a quadrilateral inscribed in a circle and construct inscribed and circumscribed circles of a triangle using geometric tools and geometric software.</td>
<td>G.C.A.3.</td>
<td>6.2, 10.4</td>
</tr>
<tr>
<td>11</td>
<td>Find the conditional probability of A given B as the fraction of B’s outcomes that also belong to A and apply the Addition Rule [P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)].</td>
<td>S.CP.B.6, S.CP.B.7</td>
<td>12.2, 12.4, 12.6</td>
</tr>
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</table>
Research about Teaching and Learning Mathematics

Structure teaching of mathematical concepts and skills around problems to be solved (Checkly, 1997; Wood & Sellars, 1996; Wood & Sellars, 1997)

Encourage students to work cooperatively with others (Johnson & Johnson, 1975; Davidson, 1990)

Use group problem-solving to stimulate students to apply their mathematical thinking skills (Artzt & Armour-Thomas, 1992)

Students interact in ways that support and challenge one another’s strategic thinking (Artzt, Armour-Thomas, & Curcio, 2008)

Activities structured in ways allowing students to explore, explain, extend, and evaluate their progress (National Research Council, 1999)

There are three critical components to effective mathematics instruction (Shellard & Moyer, 2002):

- Teaching for conceptual understanding
- Developing children’s procedural literacy
- Promoting strategic competence through meaningful problem-solving investigations

Teachers should be:

- Demonstrating acceptance and recognition of students’ divergent ideas.
- Challenging students to think deeply about the problems they are solving, extending thinking beyond the solutions and algorithms required to solve the problem
- Influencing learning by asking challenging and interesting questions to accelerate students’ innate inquisitiveness and foster them to examine concepts further.
- Projecting a positive attitude about mathematics and about students’ ability to “do” mathematics

Students should be:

- Actively engaging in “doing” mathematics
- Solving challenging problems
- Investigating meaningful real-world problems
- Making interdisciplinary connections
- Developing an understanding of mathematical knowledge required to “do” mathematics and connect the language of mathematical ideas with numerical representations
- Sharing mathematical ideas, discussing mathematics with one another, refining and critiquing each other’s ideas and understandings
- Communicating in pairs, small group, or whole group presentations
- Using multiple representations to communicate mathematical ideas
- Using connections between pictures, oral language, written symbols, manipulative models, and real-world situations
- Using technological resources and other 21st century skills to support and enhance mathematical understanding
Mathematics is not a stagnant field of textbook problems; rather, it is a dynamic way of constructing meaning about the world around us, generating knowledge and understanding about the real world every day. Students should be metaphorically rolling up their sleeves and “doing mathematics” themselves, not watching others do mathematics for them or in front of them. (Protheroe, 2007)

**Balanced Mathematics Instructional Model**

Balanced math consists of three different learning opportunities; guided math, shared math, and independent math. Ensuring a balance of all three approaches will build conceptual understanding, problem solving, computational fluency, and procedural fluency. Building conceptual understanding is the focal point of developing mathematical proficiency. Students should frequently work on rigorous tasks, talk about the math, explain their thinking, justify their answer or process, build models with graphs or charts or manipulatives, and use technology.

When balanced math is used in the classroom it provides students opportunities to:

- solve problems
- make connections between math concepts and real-life situations
- communicate mathematical ideas (orally, visually and in writing)
- choose appropriate materials to solve problems
- reflect and monitor their own understanding of the math concepts
- practice strategies to build procedural and conceptual confidence

Teacher builds conceptual understanding by modeling through demonstration, explicit instruction, and think alouds, as well as guiding students as they practice math strategies and apply problem solving strategies. (whole group or small group instruction)

Teacher and students practice mathematics processes together through interactive activities, problem solving, and discussion. (whole group or small group instruction)

Students practice math strategies independently to build procedural and computational fluency. Teacher assesses learning and reteaches as necessary. (whole group instruction, small group instruction, or centers)
<table>
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<th>Effective Pedagogical Routines/Instructional Strategies</th>
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<td>Collaborative Problem Solving</td>
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<td>Connect Previous Knowledge to New Learning</td>
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<td>Making Thinking Visible</td>
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<td>Develop and Demonstrate Mathematical Practices</td>
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<td>Inquiry-Oriented and Exploratory Approach</td>
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<td>Multiple Solution Paths and Strategies</td>
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<td>Explain the Rationale of your Math Work</td>
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<td>Student Modeling</td>
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<td>Analyze Student Work</td>
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<td>Identify Student’s Mathematical Understanding</td>
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<td>Identify Student’s Mathematical Misunderstandings</td>
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<td>Interviews</td>
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<td>Role Playing</td>
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<tr>
<td>Diagrams, Charts, Tables, and Graphs</td>
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<td>Anticipate Likely and Possible Student Responses</td>
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<td>Collect Different Student Approaches</td>
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<td>Multiple Response Strategies</td>
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<td>Asking Assessing and Advancing Questions</td>
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<td>Marking/Recapping</td>
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<td>Challenging</td>
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<td>Pressing for Accuracy and Reasoning</td>
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<td>Maintain the Cognitive Demand</td>
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## Educational Technology

### Standards

|-------------------------------|

- **Technology Operations and Concepts**
  - Create a report from a relational database consisting of at least two tables and describe the process, and explain the report results.

  **Example:** Students will create tables in Excel to show the angles in right triangles are related to the ratios of the side lengths.

- **Creativity and Innovation**
  - Apply previous content knowledge by creating a piloting a digital learning game or tutorial.

  **Example:** Using video software, students will use their knowledge to create a tutorial that proves all circles are similar.

- **Communication and Collaboration**
  - Develop an innovative solution to a real world problem or issue in collaboration with peers and experts, and present ideas for feedback through social media or in an online community.

  **Example:** Students will be able to explain and prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems on Edmodo. Students can present their solutions and communicate through this platform.
## Career Ready Practices

Career Ready Practices describe the career-ready skills that all educators in all content areas should seek to develop in their students. They are practices that have been linked to increase college, career, and life success. Career Ready Practices should be taught and reinforced in all career exploration and preparation programs with increasingly higher levels of complexity and expectation as a student advances through a program of study.

- **CRP2. Apply appropriate academic and technical skills.**
  Career-ready individuals readily access and use the knowledge and skills acquired through experience and education to be more productive. They make connections between abstract concepts with real-world applications, and they make correct insights about when it is appropriate to apply the use of an academic skill in a workplace situation.

  **Example:** Students will apply prior knowledge when solving real world problems. Students will make sound judgements about the use of specific tools, such as geometric tools and geometric software, to prove the properties of angles for a quadrilateral inscribed circle and construct inscribed and circumscribed circles of triangles.

- **CRP4. Communicate clearly and effectively and with reason.**
  Career-ready individuals communicate thoughts, ideas, and action plans with clarity, whether using written, verbal, and/or visual methods. They communicate in the workplace with clarity and purpose to make maximum use of their own and others’ time. They are excellent writers; they master conventions, word choice, and organization, and use effective tone and presentation skills to articulate ideas. They are skilled at interacting with others; they are active listeners and speak clearly and with purpose. Career-ready individuals think about the audience for their communication and prepare accordingly to ensure the desired outcome.

  **Example:** Students will on a daily basis communicate their reasoning behind their solution paths by making connections to the context and the quantities, using proper vocabulary, along with decontextualizing and/or contextualizing the problem. Students will show and explain that definitions for trigonometric ratios derive from similarity of right triangles. They will also explain the meaning behind the quantities and units involved. Students will also ask probing questions to clarify and improve arguments.
Career Ready Practices

- **CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.**
  Career-ready individuals readily recognize problems in the workplace, understand the nature of the problem and devise effective plans to solve the problem. They are aware of problems when they occur and take action quickly to address the problem; they thoughtfully investigate the root cause of the problem prior to introducing solutions. They carefully consider the options to solve the problem. Once a solution is agreed upon, they follow through to ensure the problem is solved, whether through their own actions or the actions of others.

  **Example:** Throughout their daily lessons, students will understand the meaning of a problem and look for entry points into solving their problems by analyzing the relationships of the quantities, constraints and goals of the task. Plans for solution paths will be made and have meaning. Students will self-monitor, evaluate and critique their process and progress as they are working and make changes as necessary.

- **CRP12. Work productively in teams while using cultural global competence.**
  Career-ready individuals positively contribute to every team, whether formal or informal. They apply an awareness of cultural difference to avoid barriers to productive and positive interaction. They find ways to increase the engagement and contribution of all team members. They plan and facilitate effective team meetings.

  **Example:** Students will work in collaborative and whole group settings to develop various solutions to math tasks that are presented to them. They will work together to understand the terms of the problem, ask clarifying and challenging questions among each other, and develop agreed upon solutions using a variety of strategies and models. Students will listen to, read and discuss arguments with each other with respect and courtesy at all times and will be willing to assist those that may need assistance. Students will prove and explain to a peer or small group the slope criteria for parallel and perpendicular line and use them to solve geometric problems.
WIDA Proficiency Levels

At the given level of English language proficiency, English language learners will process, understand, produce or use

<table>
<thead>
<tr>
<th>Level</th>
<th>Language Proficiency</th>
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</table>
| 6- Reaching | - Specialized or technical language reflective of the content areas at grade level  
- A variety of sentence lengths of varying linguistic complexity in extended oral or written discourse as required by the specified grade level  
- Oral or written communication in English comparable to proficient English peers |
| 5- Bridging | - Specialized or technical language of the content areas  
- A variety of sentence lengths of varying linguistic complexity in extended oral or written discourse, including stories, essays or reports  
- Oral or written language approaching comparability to that of proficient English peers when presented with grade level material. |
| 4- Expanding | - Specific and some technical language of the content areas  
- A variety of sentence lengths of varying linguistic complexity in oral discourse or multiple, related sentences or paragraphs  
- Oral or written language with minimal phonological, syntactic or semantic errors that may impede the communication, but retain much of its meaning, when presented with oral or written connected discourse, with sensory, graphic or interactive support |
| 3- Developing | - General and some specific language of the content areas  
- Expanded sentences in oral interaction or written paragraphs  
- Oral or written language with phonological, syntactic or semantic errors that may impede the communication, but retain much of its meaning, when presented with oral or written, narrative or expository descriptions with sensory, graphic or interactive support |
| 2- Beginning | - General language related to the content area  
- Phrases or short sentences  
- Oral or written language with phonological, syntactic, or semantic errors that often impede of the communication when presented with one to multiple-step commands, directions, or a series of statements with sensory, graphic or interactive support |
| 1- Entering | - Pictorial or graphic representation of the language of the content areas  
- Words, phrases or chunks of language when presented with one-step commands directions, WH-, choice or yes/no questions, or statements with sensory, graphic or interactive support |
# Language Development Supports For English Language Learners

To Increase Comprehension and Communication Skills

## Environment

- Welcoming and stress-free
- Respectful of linguistic and cultural diversity
- Honors students’ background knowledge
- Sets clear and high expectations
- Includes routines and norms
- Is thinking-focused vs. answer-seeking
- Offers multiple modalities to engage in content learning and to demonstrate understanding
- Includes explicit instruction of specific language targets
- Provides participation techniques to include all learners

- Integrates learning centers and games in a meaningful way
- Provides opportunities to practice and refine receptive and productive skills in English as a new language
- Integrates meaningful and purposeful tasks/activities that:
  - Are accessible by all students through multiple entry points
  - Are relevant to students’ lives and cultural experiences
  - Build on prior mathematical learning
  - Demonstrate high cognitive demand
  - Offer multiple strategies for solutions
  - Allow for a language learning experience in addition to content

## Sensory Supports*

- Real-life objects (realia) or concrete objects
- Physical models
- Manipulatives
- Pictures & photographs
- Visual representations or models such as diagrams or drawings
- Videos & films
- Newspapers or magazines
- Gestures
- Physical movements
- Music & songs

## Graphic Supports*

- Graphs
- Charts
- Timelines
- Number lines
- Graphic organizers
- Graphing paper

## Interactive Supports*

- In a whole group
- In a small group
- With a partner such as *Turn-and-Talk*
- In pairs as a group (first, two pairs work independently, then they form a group of four)
- In triads
- Cooperative learning structures such as *Think-Pair-Share*
- Interactive websites or software
- With a mentor or coach

## Verbal and Textual Supports

- Labeling
- Students’ native language
- Modeling
- Repetitions
- Paraphrasing
- Summarizing
- Guiding questions
- Clarifying questions
- Probing questions
- Leveled questions such as *What? When? Where? How? Why?*
- Questioning prompts & cues
- Word Banks
- Sentence starters
- Sentence frames
- Discussion frames
- Talk moves, including *Wait Time*

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BUILDING EQUITY IN YOUR TEACHING PRACTICE

How do the essential questions highlight the connection between the big ideas of the unit and equity in your teaching practice?

**CONTENT INTEGRATION**
Teachers use examples and content from a variety of cultures & groups.

This unit / lesson is connected to other topics explored with students.

There are multiple viewpoints reflected in the content of this unit / lesson.

The materials and resources are reflective of the diverse identities and experiences of students.

The content affirms students, as well as exposes them to experiences other than their own.

**KNOWLEDGE CONSTRUCTION**
Teachers help students understand how knowledge is created and influenced by cultural assumptions, perspectives & biases.

This unit / lesson provides context to the history of privilege and oppression.

This unit / lesson addresses power relationships.

This unit / lesson helps students to develop research and critical thinking skills.

This curriculum creates windows and mirrors for students.

**PREJUDICE REDUCTION**
Teachers implement lessons and activities to assert positive images of ethnic groups & improve intergroup relations.

This unit / lesson helps students question and unpack biases & stereotypes.

This unit / lesson helps students examine, research and question information and sources.

The curriculum encourages discussion and understanding about the groups of people being represented.

This unit / lesson challenges dominant perspectives.

**EQUITABLE PEDAGOGY**
Teachers modify techniques and methods to facilitate the academic achievement of students from diverse backgrounds.

The instruction has been modified to meet the needs of each student.

Students feel respected and their cultural identities are valued.

Additional supports have been provided for students to become successful and independent learners.

Opportunities are provided for student to reflect on their learning and provide feedback.

**EMPOWERING SCHOOL CULTURE**
Using the other four dimensions to create a safe and healthy educational environment for all.

There are opportunities for students to connect with the community.

My classroom is welcoming and supportive for all students?

I am aware of and sensitive to the needs of my students and their families.

There are effective parent communication systems established. Parents can talk to me about issues as they arise in my classroom.
### Culturally Relevant Pedagogy Examples

- **Integrate Relevant Word Problems:** Contextualize equations using word problems that reference student interests and cultures.
  **Example:** When learning about trigonometric ratios and the Pythagorean Theorem, problems that relate to student interests such as music, sports and art enable the students to understand and relate to the concept in a more meaningful way.

- **Everyone has a Voice:** Create a classroom environment where students know that their contributions are expected and valued.
  **Example:** Norms for sharing are established that communicate a growth mindset for mathematics. All students are capable of expressing mathematical thinking and contributing to the classroom community. Students learn new ways of looking at problem solving by working with and listening to each other.

- **Run Problem Based Learning Scenarios:** Encourage mathematical discourse among students by presenting problems that are relevant to them, the school and/or the community.
  **Example:** Using a Place Based Education (PBE) model, students explore math concepts while determining ways to address problems that are pertinent to their neighborhood, school or culture.

- **Encourage Student Leadership:** Create an avenue for students to propose problem solving strategies and potential projects.
  **Example:** Students can deepen their understanding of the slope criteria for parallel and perpendicular lines by creating problems together and deciding if the problems fit the necessary criteria. This experience will allow students to discuss and explore their current level of understanding by applying the concepts to relevant real-life experiences.

- **Present New Concepts Using Student Vocabulary:** Use student diction to capture attention and build understanding before using academic terms.
  **Example:** Teach math vocabulary in various modalities for students to remember. Use multi-modal activities, analogies, realia, visual cues, graphic representations, gestures, pictures and cognates. Directly explain and model the idea of vocabulary words having multiple meanings. Students can create the Word Wall with their definitions and examples to foster ownership.
## Differentiated Instruction

### Accommodate Based on Students Individual Needs: Strategies

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<th>Time/General</th>
<th>Processing</th>
<th>Comprehension</th>
<th>Recall</th>
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<tr>
<td>Extra time for assigned tasks</td>
<td>Extra Response time</td>
<td>Precise processes for balanced math instructional model</td>
<td>Teacher-made checklist</td>
</tr>
<tr>
<td>Adjust length of assignment</td>
<td>Have students verbalize steps</td>
<td>Short manageable tasks</td>
<td>Use visual graphic organizers</td>
</tr>
<tr>
<td>Timeline with due dates for reports and projects</td>
<td>Repeat, clarify or reword directions</td>
<td>Brief and concrete directions</td>
<td>Reference resources to promote independence</td>
</tr>
<tr>
<td>Communication system between home and school</td>
<td>Mini-breaks between tasks</td>
<td>Provide immediate feedback</td>
<td>Visual and verbal reminders</td>
</tr>
<tr>
<td>Provide lecture notes/outline</td>
<td>Provide a warning for transitions</td>
<td>Small group instruction</td>
<td>Graphic organizers</td>
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<td></td>
<td>Partnering</td>
<td>Emphasize multi-sensory learning</td>
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<th>Behavior/Attention</th>
<th>Organization</th>
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<tr>
<td>Computer/whiteboard</td>
<td>Extended time</td>
<td>Consistent daily structured routine</td>
<td>Individual daily planner</td>
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<tr>
<td>Tape recorder</td>
<td>Study guides</td>
<td>Simple and clear classroom rules</td>
<td>Display a written agenda</td>
</tr>
<tr>
<td>Video Tape</td>
<td>Shortened tests</td>
<td>Frequent feedback</td>
<td>Note-taking assistance</td>
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<tr>
<td></td>
<td>Read directions aloud</td>
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<td>Color code materials</td>
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</table>

### Assistive Technology
- Computer/whiteboard
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- Video Tape

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- Extended time
- Study guides
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- Read directions aloud

### Behavior/Attention
- Consistent daily structured routine
- Simple and clear classroom rules
- Frequent feedback

### Organization
- Individual daily planner
- Display a written agenda
- Note-taking assistance
- Color code materials
## Differentiated Instruction

**Accommodate Based on Content Specific Needs**

- Anchor charts to model strategies and use of formulas
- Review Algebra concepts to ensure students have the information needed to progress in understanding
- Pre-teach vocabulary using visual models that are connected to real-life situations
- Reference sheets that list formulas
- Teacher modeling of thinking processes involved in constructing a two column or paragraph proof
- Area models to represent the Pythagorean Theorem
- Record formulas, theorems, and postulates in reference notebooks
- Use string to demonstrate arc length
- Use graph paper to represent the coordinate plane
- Word wall with visual representations of geometric terms
- Calculator to assist with computations
- Have students use a dynamic geometry software package to measure the distances and the lengths of arcs
- Utilize technology through interactive sites to explore Plane Geometry, Constructions, and Coordinate Geometry.

[www.mathopenref.com](http://www.mathopenref.com)  [https://www.geogebra.org/](https://www.geogebra.org/)
Interdisciplinary Connections

*Model interdisciplinary thinking to expose students to other disciplines.*

**Geography Connection:**

*Task Name: Neglecting the Curvature of the Earth (RH.9-10.7)*

- This task takes into consideration the curvature of the earth to find the distance between two locations.

*Task Name: How Far Is The Horizon (RH.9-10.7)*

- The purpose of this modeling task is to have students use mathematics to answer a question in a real-world context using mathematical tools that should be very familiar to them. The task gets at particular aspects of the modeling process, namely, it requires them to make reasonable assumptions and find information that is not provided in the task statement.

**Architecture and Construction Career Cluster Connection:**

*Task name: Access Ramp (9.3.ST-SM-2)*

- You have been commissioned to design an access ramp, which complies with the Americans with Disabilities Act (ADA) requirements, for an entry that is 3 feet above ground level. Your client has asked you to design the ramp and to determine costs, using local pricing, for two types of ramps, wooden and concrete.
## Enrichment

### What is the Purpose of Enrichment?

- The purpose of enrichment is to provide extended learning opportunities and challenges to students who have already mastered, or can quickly master, the basic curriculum. Enrichment gives the student more time to study concepts with greater depth, breadth, and complexity.
- Enrichment also provides opportunities for students to pursue learning in their own areas of interest and strengths.
- Enrichment keeps advanced students engaged and supports their accelerated academic needs.
- Enrichment provides the most appropriate answer to the question, “What do you do when the student already knows it?”

### Enrichment is...

| Planned and purposeful | Just for gifted students (some gifted students may need intervention in some areas just as some other students may need frequent enrichment) |
| Different, or differentiated, work – not just more work | Worksheets that are more of the same (busywork) |
| Responsive to students’ needs and situations | Random assignments, games, or puzzles not connected to the content areas or areas of student interest |
| A promotion of high-level thinking skills and making connections within content | Extra homework |
| The ability to apply different or multiple strategies to the content | A package that is the same for everyone |
| The ability to synthesize concepts and make real world and cross-curricular connections | Thinking skills taught in isolation |
| Elevated contextual complexity | Unstructured free time |
| Sometimes independent activities, sometimes direct instruction | |
| Inquiry based or open-ended assignments and projects | |
| Using supplementary materials in addition to the normal range of resources | |
| Choices for students | |
| Tiered/Multi-level activities with flexible groups (may change daily or weekly) | |
Assessments

Required District/State Assessments
Unit Assessment
PARCC
SGO Assessments

Suggested Formative/Summative Classroom Assessments
Describe Learning Vertically
Identify Key Building Blocks
Make Connections (between and among key building blocks)
Short/Extended Constructed Response Items
Multiple-Choice Items (where multiple answer choices may be correct)
Drag and Drop Items
Use of Equation Editor
Quizzes
Journal Entries/Reflections/Quick-Writes
Accountable talk
Projects
Portfolio
Observation
Graphic Organizers/Concept Mapping
Presentations
Role Playing
Teacher-Student and Student-Student Conferencing
Homework
<table>
<thead>
<tr>
<th>Standard</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>G.GPE.B.4</td>
<td>Use coordinates to prove simple geometric theorems algebraically. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point ((1, \sqrt{3})) lies on the circle centered at the origin and containing the point ((0, 2)).</td>
</tr>
<tr>
<td>G.GPE.B.5</td>
<td>Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).</td>
</tr>
<tr>
<td>G.GPE.B.6</td>
<td>Find the point on a directed line segment between two given points that partitions the segment in a given ratio.</td>
</tr>
<tr>
<td>G.GPE.B.7</td>
<td>Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.</td>
</tr>
<tr>
<td>G.SRT.C.6</td>
<td>Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.</td>
</tr>
<tr>
<td>G.SRT.C.7</td>
<td>Explain and use the relationship between the sine and cosine of complementary angles</td>
</tr>
<tr>
<td>G.SRT.C.8</td>
<td>Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.</td>
</tr>
<tr>
<td>G.GPE.A.1</td>
<td>Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.</td>
</tr>
<tr>
<td>G.C.A.1</td>
<td>Prove that all circles are similar.</td>
</tr>
<tr>
<td>G.C.A.2</td>
<td>Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.</td>
</tr>
<tr>
<td>G.C.B.5</td>
<td>Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.</td>
</tr>
<tr>
<td>G.C.A.3</td>
<td>Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.</td>
</tr>
</tbody>
</table>
# Mathematical Practices

1. Make sense of problems and persevere in solving them.

2. Reason abstractly and quantitatively.

3. Construct viable arguments and critique the reasoning of others.

4. Model with mathematics.

5. Use appropriate tools strategically.

6. Attend to precision.

7. Look for and make use of structure.

8. Look for and express regularity in repeated reasoning.
**Unit Focus:**
- Use coordinates to prove simple geometric theorems
- Define trigonometric ratios and solve problems involving right triangles
- Translate between the geometric description and the equation for a conic section
- Understand and apply theorems about circles
- Find arc lengths and areas of sectors of circles

**New Jersey Student Learning Standard(s):**

G.GPE.B.4: Use coordinates to prove simple geometric theorems algebraically. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1,√3) lies on the circle centered at the origin and containing the point (0, 2).

**Student Learning Objective 1:** Use coordinates to prove simple geometric theorems algebraically.

**Modified Student Learning Objectives/Standards:** N/A

<table>
<thead>
<tr>
<th>MPs</th>
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<th>Tasks/Activities</th>
</tr>
</thead>
<tbody>
<tr>
<td>MP 3</td>
<td>HS.C.13.2</td>
<td>Coordinate geometry is the study of geometry using algebra. Use coordinates to prove geometric theorems including: Coordinate geometry can be used to prove geometric theorems because it is possible to replace specific coordinates with variables to</td>
<td>Coordinate geometry can be used to prove geometric theorems because it is possible to replace specific coordinates with variables to</td>
<td>IFL Sets of Related Lessons “Investigating Coordinate Geometry and Its Use in Solving</td>
</tr>
<tr>
<td>to draw geometric conclusions. Content scope: G-GPE.4</td>
<td>show that a relationship remains true, regardless of the coordinates.</td>
<td></td>
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<td>------------------------------------------------------</td>
<td>---------------------------------------------------------------------</td>
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<tr>
<td>prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle (or other quadrilateral); prove or disprove that a given point lies on a circle of a given center and radius or point on the circle. Students may use geometric simulation software to model figures and prove simple geometric theorems. <strong>Example:</strong> Use slope and distance formula to verify the polygon formed by connecting the points (-3, -2), (5, 3), (9, 9), (1, 4) is a parallelogram. Students will have the ability to explain the connection between algebra and geometry. Prove geometric concepts by using geometric proofs and be able to write coordinate proofs. Students will have the ability to name coordinates of special figures by using their properties. They will also be able to position figures in the coordinate plane for use in coordinate proofs. Use a system of equations to learn about points of intersections and relationships between different geometric shapes. Students should be able to recall previous understandings, establish relationships, and/or show that a relationship remains true, regardless of the coordinates.</td>
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</tr>
</tbody>
</table>

**Mathematical Problems”**

**Type II, III:**

**A Midpoint Miracle**

**Unit Squares and Triangles**
characteristics of geometric shapes using coordinate geometry.

Students should be able to classify a quadrilateral through use of coordinate analysis.

Review of Algebra skills, such as equations of lines, working with slopes, applying the distance formula, determining the solution to a system of equations is helpful.

**SPED Strategies**

Pre-teach vocabulary using visual and verbal models that are connected to real life situations and ensure that students include these definitions their reference notebook.

Review algebra concepts as needed to ensure that students have the information they need to progress in understanding.

Model verbally and visually how to use algebra to prove geometry theorems using contextual problems.

**ELL Strategies:**

Demonstrate comprehension of how to use coordinates to prove simple geometric theorems algebraically by explaining the process in the student’s native language and/or use gestures, examples and selected technical words.

Posters/charts are a great support for ELLs to reference. A chart can contain certain words confirm many geometric truths in an algebraic way. Many geometric concepts, theorems, characteristic and properties through coordinate analysis.
from a unit, definitions, helpful hints, etc. This support provides consistency because the information on a chart/poster is always in the same place.

Students can use bilingual (paper or electronic) dictionaries to clearly define words that they do not know.

Explain the process of simple geometric theorems algebraically using key vocabulary in the student’s native language and/simple sentences.

Utilize Manipulatives and develop hands-on activities.

Have students visualize actual models of shapes and create their own

Utilize Pictures/illustrations and have student write meaning in their Math Journals.
**New Jersey Student Learning Standard(s):**

G.GPE.B.5: Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).

**Student Learning Objective 2:** Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems.

**Modified Student Learning Objectives/Standards:** N/A

<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>MP 3</td>
<td><strong>HS.C.13.3</strong></td>
<td>Prove the slope criteria for parallel lines (parallel lines have congruent slopes and its converse). Prove the slope criteria for perpendicular lines (the product of the slopes of perpendicular lines equals -1) Solve problems using the slope criteria for parallel and perpendicular lines and determine whether two slopes represent parallel or perpendicular relationships. Find the equation of a line parallel and line perpendicular to a given line that passes through a given point. Students should also be able to graph and write linear equations and the slope of a line. Slopes tell us a lot of information about geometric shapes. Parallel and perpendicular relationships occur in many geometric shapes</td>
<td>The set of points that are equidistant from two points A and B lie on the perpendicular bisector of line segment AB, because every point on the perpendicular bisector can be used to construct two triangles that are congruent by definition of triangle congruence, reflection and/or Side-Angle-Side; corresponding parts of congruent triangles are congruent. For any point C that lies on the perpendicular bisector of points A and B, C is equidistant from points A and B because the perpendicular bisector divides triangle ABC into two congruent right triangles.</td>
<td>IFL Sets of Related Lessons “Investigating Coordinate Geometry and Its Use in Solving Mathematical Problems” Type I: Equal Area Triangles on the Same Base I Type II, III: A Midpoint Miracle Equal Area Triangles on the Same Base I Equal Area Triangles on the Same Base II</td>
</tr>
<tr>
<td>MP 8</td>
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</table>
such as the parallelogram family. Determining slope helps us classify shapes more specifically.

Knowing that parallel lines that have equal slopes and perpendicular lines have negative reciprocal slopes allows us to further analyze geometric shapes to determine what they are and what properties they have.

Classify geometric shapes using slopes and/or distances.

The ability to understand and recognize negative reciprocal is difficult to students. So many students use negative slopes or reciprocal slopes as perpendicular slopes instead or negative reciprocal slopes.

The new part of this skill is using two slope relationships to establish or classify various geometric shapes using slope and distance. Students feel comfortable on the grid counting things out but when we use variables for coordinates instead of values they start to struggle.

A midsegment connects the midpoints of two sides of a triangle and divided the side lengths into a 1:2 ratio. The use of coordinate geometry techniques for finding length and slope verify that a midsegment’s length is half of the third side and it is also parallel to the third side.

How do you determine if two lines are parallel, perpendicular or neither?

Given an equation of a line and a point not on the line, how do you write the equation of a line that is parallel to the given line and through the given point? Perpendicular?

How are geometry and algebra related to each other?

How can you use coordinate geometry to prove relationships?

What is the relationship between slopes of perpendicular and parallel lines?
<table>
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<th>SPED Strategies</th>
</tr>
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<tbody>
<tr>
<td>Review how to calculate slope with students.</td>
</tr>
<tr>
<td>Model verbally and visually how to use slope to prove parallel and perpendicular lines using contextual problems.</td>
</tr>
<tr>
<td>Encourage students to add this concept to their reference notebook by providing notes or guiding notetaking.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>ELL Strategies:</th>
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</thead>
<tbody>
<tr>
<td>Explain orally and in writing the slope criteria for parallel and perpendicular lines and use criteria to solve geometric problems in the student’s native language and/or use gestures, examples and selected technical words.</td>
</tr>
<tr>
<td>Explain orally and in writing the criteria to solve geometric problems in the student’s native language and/or use selected technical vocabulary in phrases and short sentences and simple sentences.</td>
</tr>
<tr>
<td>Utilize Manipulatives and develop hands-on activities.</td>
</tr>
<tr>
<td>Provide students with translation dictionary.</td>
</tr>
<tr>
<td>Have students work with partners, small groups.</td>
</tr>
</tbody>
</table>
New Jersey Student Learning Standards (s):
G.GPE.B.6: Find the point on a directed line segment between two given points that partitions the segment in a given ratio.

G.GPE.B.7: Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.

Student Learning Objective 3: Find the point on a directed line segment between two given points that partitions the segment in a given ratio and use coordinates to compute perimeters of polygons and areas of triangles and rectangles.

Modified Student Learning Objectives/Standards:
M.EE.G-GPE.7: Find perimeters and areas of squares and rectangles to solve real-world problems.

<table>
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<th>Tasks/Activities</th>
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<tbody>
<tr>
<td>MP 1</td>
<td>G-GPE.6</td>
<td>Locate the point on a directed line segment that creates two segments of a given ratio.</td>
<td>A midpoint of a line is a point that partitions the segment into two segments that have the same length. Each of these segments is in a 1:2 ratio with the whole segment. For a line segment with endpoints at A(x1,y1) and B(x2,y2), since the midpoint is in the “middle” of segment AB, it is located at , because the midpoint represents the average value for both the x- and y-values of the two coordinates of segment AB.</td>
<td>IFL Sets of Related Lessons “Investigating Coordinate Geometry and Its Use in Solving Mathematical Problems”</td>
</tr>
<tr>
<td>MP 2</td>
<td>G-Int.1</td>
<td>Find perimeters of polygons using coordinates, the Pythagorean theorem and the distance formula. Partition a line segment based on a provided ratio.</td>
<td></td>
<td>Type I: Finding Triangle Coordinates</td>
</tr>
<tr>
<td></td>
<td>HS.C.13.1</td>
<td></td>
<td></td>
<td>Type II, III: Scaling a Triangle in the Coordinate Plane</td>
</tr>
</tbody>
</table>
- Apply geometric reasoning in a coordinate setting, and/or use coordinates to draw geometric conclusions. Content scope: G-GPE.6, G-GPE.7

**HS.D.2-2**

Solve multi-step contextual problems with degree of difficulty appropriate to the course involving perimeter, area, or volume that require finding an approximate solution to a polynomial equation using numerical/graphical means.

- Tasks may have a real world or mathematical context.
- Tasks may involve coordinates (G-GPE.7).
- Refer to A-REI.11 for some of the content knowledge from the previous course relevant to these tasks.
- Cubic polynomials are limited to polynomials

| Sides are parallel to the axes: |
| If the polygon is drawn such that its sides (or needed segments) lie ON the grids of the graph paper, COUNT the lengths and use the appropriate area formula. |

| Sides are NOT parallel to the axes: |
| If the polygon is drawn such that its sides (or needed segments) do NOT lie ON the grids of the graph paper, draw a "BOX" around the polygon to determine the area. |

**Finding the height using perpendicular lines**

**Heron’s Formula:** Allows you to determine the area of a triangle using only the lengths of the sides of a triangle. If you are given the three vertices of a triangle all you need to do is calculate the three distances and then plug in their values into the formula to determine the area.

\[
A = \sqrt{s(s-a)(s-b)(s-c)}
\]

where \(a\), \(b\), and \(c\) are the sides of the triangle and \(s\) is the semi-perimeter \(s = \frac{a+b+c}{2}\).

Describe the effect on perimeter and area when one or more dimensions of a figure are changed is \(m:n\). If the endpoints of segment AB are located at \((x_1,y_1)\) and \((x_2,y_2)\). C is located at the point \((x_3,y_3)\), because C is located \(m:n\) of the distance of the segment from the endpoint in both the horizontal and vertical directions.

The distance between two points on a coordinate plane is the length of the line segment that connects them and is the count of the number of units that form the line.

- The length of a line segment parallel to the x-axis (y-axis) can be determined by subtracting the values of the x-coordinates (y-coordinates) of the points forming the segment because subtraction counts the number of units in the segment.

- The length of a diagonal line segment can be determined by utilizing the line segment as the hypotenuse of a right

---

**Squares on a Coordinate Plane**

**Triangles Perimeters**
in which linear and quadratic factors are available

- To make the tasks involve strategic use of tools (MP.5), calculation and graphing aids are available but tasks do not prompt the student to use them.

**HS.D. 3-2a**

- Micro-models: Autonomously apply a technique from pure mathematics to a real-world situation in which the technique yields valuable results even though it is obviously not applicable in a strict mathematical sense (e.g., profitably applying proportional relationships to a phenomenon that is obviously nonlinear or statistical in nature).

**Content Scope:** Knowledge and skills articulated in the

<table>
<thead>
<tr>
<th>and apply the relationship between perimeter and area in problem solving situations.</th>
<th>triangle and computing its length using the Pythagorean Theorem. The Distance Formula can then be derived by replacing specific coordinates with variables in such a situation.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>SPED Strategies</strong> Use a contextual problem to model the thinking, processes and procedures involved in locating a point on a directed line segment based on a given ratio and finding the perimeter and area of polygons using coordinates.</td>
<td>The perimeter of the triangle formed by connecting points on two sides of a triangle whose side lengths are in an m:n ratio will also be in an m:n ratio because each side length in such a triangle is m/n of the lengths of the sides of the original triangle</td>
</tr>
<tr>
<td>Encourage students to add this concept to their reference notebook by providing notes or guiding notetaking.</td>
<td>How can a segment be partitioned in a given ratio?</td>
</tr>
<tr>
<td>Provide students with hands on opportunities to explore and extend their understanding by working in small groups.</td>
<td>How can areas and perimeters of polygons be found using coordinate geometry?</td>
</tr>
<tr>
<td><strong>ELL Strategies:</strong> Use oral and writing explanations in the student’s native language and/or using gestures, graphs and selected technical words, in order to explain how to find the point on a directed line segment.</td>
<td>How do you find the point on a directed line segment between two given points that partitions the segment in a given ratio?</td>
</tr>
<tr>
<td>Explain orally and in writing how to calculate the area and perimeter of polygons, triangles and rectangles in the student’s native language and/or use gestures, examples and selected technical words and using technical vocabulary in phrases and short simple sentences.</td>
<td></td>
</tr>
<tr>
<td>Geometry Type I, Sub-Claim A Evidence Statements.</td>
<td></td>
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<tr>
<td>------------------------------------------------</td>
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<tr>
<td><strong>HS.D. 3-4a</strong></td>
<td></td>
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<tr>
<td>- Reasoned estimates: Use reasonable estimates of known quantities in a chain of reasoning that yields an estimate of an unknown quantity.</td>
<td></td>
</tr>
<tr>
<td>Content Scope: Knowledge and skills articulated in the Geometry Type I, Sub-Claim A Evidence Statements.</td>
<td></td>
</tr>
</tbody>
</table>

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<thead>
<tr>
<th>HS.D.2-1</th>
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<tbody>
<tr>
<td>Solve multi-step contextual problems with degree of difficulty appropriate to the course involving perimeter, area, or volume that require solving a quadratic equation.</td>
</tr>
<tr>
<td>- Tasks do not cue students to the type of equation or specific</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Utilize Manipulatives and develop hands-on activities.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Have students visualize actual models of shapes and create their own.</td>
</tr>
<tr>
<td>Provide students with translation dictionary.</td>
</tr>
<tr>
<td>Have students work with partners, small groups.</td>
</tr>
<tr>
<td>Develop interactive games and activities to promote retention.</td>
</tr>
</tbody>
</table>
solution method involved in the task. For example: An artist wants to build a right-triangular frame in which one of the legs exceeds the other in length by 1 unit, and in which the hypotenuse exceeds the longer leg in length by 1 unit. Use algebra to show that there is one and only one such right triangle, and determine its side lengths.

**HS.C.18.2**

Use a combination of algebraic and geometric reasoning to construct, autonomously, chains of reasoning that will justify or refute propositions or conjectures about geometric figures. Content scope: Algebra content from Algebra 1 course; geometry content from the Geometry course.

- For the Geometry course, we are reaching back to Algebra 1 to
help students synthesize across the two subjects,

New Jersey Student Learning Standard(s):
G.SRT.C.6: Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.

Student Learning Objective 4: Show and explain that definitions for trigonometric ratios derive from similarity of right triangles.

Modified Student Learning Objectives/Standards: N/A

<table>
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<tbody>
<tr>
<td>MP 2</td>
<td>G-SRT.6</td>
<td>A trigonometric ratio is a ratio of the lengths of two sides in a right triangle. All right triangles with a given acute angle are similar by the AA Similarity Theorem (Theorem 8.3). So, $\triangle JKL \sim \triangle XYZ$, and you can write $\frac{KL}{IL} = \frac{YZ}{XZ}$. This can be rewritten as $\frac{KL}{IL} = \frac{YZ}{XZ}$ which is a trigonometric ratio. So, trigonometric ratios are constant for a given angle measure.</td>
<td>How can I use the similarity of right triangles to derive trigonometric ratios?</td>
<td>Type II, III: Defining Trigonometric Ratios</td>
</tr>
<tr>
<td>MP 4</td>
<td>H.S.C.15.14</td>
<td>The angles in right triangles are related to the ratios of the side lengths.</td>
<td>How does the use of congruency and similarity concepts allow us to model relationships between geometric figures?</td>
<td>Tangent of Acute Angles Hopewell Geometry</td>
</tr>
<tr>
<td>MP 7</td>
<td>Present solutions to multi-step problems in the form of valid chains of reasoning, using symbols such as equals signs appropriately (for example, rubrics award less than full credit for the presence of nonsense statements such as $1 + 4$)</td>
<td>The tangent ratio is a trigonometric ratio for acute angles that involves the lengths of the legs of a right triangle.</td>
<td>How do the ratios of the side lengths of right triangles relate to the angles in the triangle?</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Show and explain that definitions for trigonometric ratios derive from similarity of right triangles.</td>
<td>How to you use trigonometric ratios to find missing side lengths in right triangles?</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>The angles in right triangles are related to the ratios of the side lengths.</td>
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</table>

= 5 + 7 = 12, even if the final answer is correct), or identify or describe errors in solutions to multi-step problems and present corrected solutions. Content scope: G-SRT.C

| **Apply the concept of similarity relationships in right triangles to solve problems.** |
| **Label a triangle in relation to the reference angle (opposite, adjacent, and hypotenuse).** |
| **Determine the most appropriate trigonometric ratio (sine, cosine, and tangent) to use for a given problem based on the information provided.** |
| **Compare common ratios for similar triangles and develop a relationship between the ratio and the acute angle leading to trigonometry ratios.** |
| **Generalize that side ratios from similar triangles are equal and that these relationships lead to the definition of the six trigonometric ratios.** |

**SPED Strategies**
Pre-teach vocabulary using visual and verbal models that are connected to real life situations and ensure that students include these definitions their reference notebook.

Review similarity of right triangles and show how the sides are proportional. Bridge into trigonometric ratios and explain how they are used in real world to solve problems.

**ELL Strategies:**
Identify and explain orally and in writing the trigonometric ratios based on similarity of right triangles in the student’s native language and/or use

The similarity connects angles to sides and sides to angle and that refer to that as Trigonometry. When we work in right triangles and we know one of the complementary angles we have similar triangles and the ratios of the sides are fixed.

\[
\tan \theta = \frac{\text{Opposite}}{\text{Adjacent}}
\]

\[
\sin \theta = \frac{\text{Opposite}}{\text{Hypotenuse}}
\]

\[
\cos \theta = \frac{\text{Adjacent}}{\text{Hypotenuse}}
\]
gestures, examples and selected technical words, phrases and short simple sentences.
Utilize Manipulatives and develop hands-on activities.
Model structure and clarify unfamiliar syntax.
Provide students with translation dictionary.
Have students work with partners, small groups.
Have students visualize and create their own conversion charts, and diagrams with measurement labels.

### New Jersey Student Learning Standard(s):

- **G.SRT.C.7.** Explain and use the relationship between the sine and cosine of complementary angles.
- **G.SRT.C.8.** Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

### Student Learning Objective 5:
Explain and use the relationship between the sine and cosine of complementary angles; use trigonometric ratios and the Pythagorean Theorem to compute all angle measures and side lengths of triangles in applied problems.

### Modified Student Learning Objectives/Standards: N/A

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</thead>
<tbody>
<tr>
<td>MP 1</td>
<td>G.SRT.7-2</td>
<td>Understand the relationship between sine and cosine of complementary angles</td>
<td>How do I determine the relationship between the sine and cosine of complementary angles?</td>
<td>Type II, III:</td>
</tr>
<tr>
<td>MP 2</td>
<td></td>
<td>When algebraic unknowns are put in the angle locations students want to set the values equal</td>
<td></td>
<td>Ask the Pilot</td>
</tr>
<tr>
<td>MP 5</td>
<td></td>
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<tr>
<td>MP 6</td>
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<td>MP 7</td>
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</tbody>
</table>

- The “explain” part of standard G-SRT.7 is not assessed here.
<table>
<thead>
<tr>
<th>G.SRT.8</th>
<th>Sine and Cosine of Complementary Angles:</th>
</tr>
</thead>
<tbody>
<tr>
<td>The task may have a real world or mathematical context. For rational solutions, exact values are required. For irrational solutions, exact or decimal approximations may be required. Simplifying or rewriting radicals is not required; however, students will not be penalized if they simplify the radicals correctly.</td>
<td>The sine and cosine of complementary angles are related.</td>
</tr>
<tr>
<td></td>
<td>Determine and compare sine and cosine ratios of complementary angles in a right triangle.</td>
</tr>
<tr>
<td></td>
<td>What is the relationship of the cosine and the sine of two complementary angles?</td>
</tr>
<tr>
<td></td>
<td>What does it mean to &quot;solve&quot; a triangle?</td>
</tr>
<tr>
<td></td>
<td>What is the relationship between the sine and cosine of complementary angles?</td>
</tr>
<tr>
<td></td>
<td>How can right triangles be used to solve real world problems?</td>
</tr>
<tr>
<td></td>
<td>Why do we need trigonometric ratios to solve some real-world problems involving right triangles?</td>
</tr>
</tbody>
</table>

**HS.C.15.14**

Present solutions to multi-step problems in the form of valid chains of reasoning, using symbols such as equals signs appropriately (for example, rubrics award less than full credit for the presence of nonsense statements such as $1 + 4 = 5 + 7 = 12$, even if the final answer is correct), or identify or describe errors in solutions to multi-step problems and present to each other instead of summing to 90 degrees.

Sine and cosine are co-functions and that simply a horizontal shift of $\pi/2$. Knowing values of sine can help us determine cosine values as well.

Solve right triangles (determine all angle measures and all side lengths) using trigonometric ratios and the Pythagorean Theorem.

The sine and cosine of complementary angles are related.

**Constructing Special Angles**

**Neglecting the Curvature of the Earth**

**Setting Up Sprinklers**

**Sine and Cosine of Complementary Angles**

**Task Access Ramp**

**Trigonometric Function Values**

Let $A$ and $B$ be complementary angles.

\[
\begin{align*}
\sin A &= \cos(90^\circ - A) = \cos B \\
\cos A &= \sin(90^\circ - A) = \sin B \\
\sin B &= \cos(90^\circ - B) = \cos A \\
\cos B &= \sin(90^\circ - B) = \sin A
\end{align*}
\]
Solve multi-step contextual word problems with degree of difficulty appropriate to the course, requiring application of course-level knowledge and skills articulated in G-SRT.8, involving right triangles in an applied setting.

- Tasks may, or may not, require the student to autonomously make an assumption or simplification in order to apply techniques of right triangles. For example, a configuration of three buildings might form a triangle that is nearly, but not quite, a right triangle; then, a good approximate result can be obtained if the student autonomously approximates the

<table>
<thead>
<tr>
<th>HS.D.2-11</th>
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</table>

**Solving a Right Triangle:** To solve a right triangle means to find all unknown side lengths and angle measures. You can solve a right triangle when you know either of the following:
- Two side lengths
- One side length and the measure of one acute angle.

**SPED Strategies**  
Model how trigonometric ratios and the Pythagorean Theorem are used to calculate the measure of angles and sides in right triangles. Ensure that students include this information in their reference notebook.

Provide students with multiple contextually based problems to solve so that they can practice the thinking and skills needed for this work.

Provide students with hands on opportunities to explore and extend their understanding by working in small groups.

**ELL Strategies:**  
Explain orally and in writing the relationship between the sine and cosine of complementary angles in the student’s native language and/or use gestures, examples and selected technical words.

Explain orally and in writing the relationship between the sine and cosine of complementary
<table>
<thead>
<tr>
<th>triangle as a right triangle.</th>
<th>angles in the student’s native language and/or use selected technical vocabulary in phrases, short sentences and simple sentences</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>HS.D. 3-2a</strong></td>
<td>Explain orally and in writing the concept of trigonometric ratios and Pythagorean Theorem by solving right triangles in the student’s native language and/or with gestures, examples and selected technical words, phrases and short simple sentences.</td>
</tr>
<tr>
<td>Micro-models:</td>
<td>Provide students with translation dictionary.</td>
</tr>
<tr>
<td>Autonomously apply a technique from pure mathematics to a real-world situation in which the technique yields valuable results even though it is obviously not applicable in a strict mathematical sense (e.g., profitably applying proportional relationships to a phenomenon that is obviously nonlinear or statistical in nature). Content Scope: Knowledge and skills articulated in the Geometry Type I, Sub-Claim A Evidence Statements.</td>
<td>Develop interactive games and activities to promote retention</td>
</tr>
<tr>
<td><strong>HS.D. 3-4a</strong></td>
<td></td>
</tr>
<tr>
<td>Reasoned estimates: Use reasonable estimates of known quantities in a chain of reasoning that yields an estimate of an unknown quantity. Content Scope: Knowledge and skills</td>
<td></td>
</tr>
</tbody>
</table>
New Jersey Student Learning Standard(s):
G.GPE.A.1: Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

Student Learning Objective 6: Derive the equation of a circle of given the center and radius using the Pythagorean Theorem. Given an equation, complete the square to find the center and radius of the circle.

Modified Student Learning Objectives/Standards: N/A

<table>
<thead>
<tr>
<th>MPs</th>
<th>Evidence Statement Key/Clarifications</th>
<th>Skills, Strategies &amp; Concepts</th>
<th>Essential Understandings/Questions (Accountable Talk)</th>
<th>Tasks/Activities</th>
</tr>
</thead>
<tbody>
<tr>
<td>MP 6 MP 7</td>
<td>G.GPE.1-1</td>
<td>Standard Equation of a Circle: Let ((x,y)) represent any point on a circle with center ((h,k)) and radius (r). The equation of a circle can be derived using the Pythagorean Theorem.</td>
<td>How do you derive the equation of a circle, given a center and a radius?</td>
<td>Type II, III: Explaining the Equation for a Circle</td>
</tr>
<tr>
<td></td>
<td>G.GPE.1-2</td>
<td>[(x - h)^2 + (y - k)^2 = r^2.]</td>
<td>How can you determine the center and radius of a circle from its equation?</td>
<td>Slopes and Circles</td>
</tr>
<tr>
<td></td>
<td>• The &quot;derive&quot; part of standard G-GPE.1 is not assessed here.</td>
<td>How do you derive the equation of a parabola, given a focus and a directrix?</td>
<td></td>
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</tr>
<tr>
<td></td>
<td>• Tasks must go beyond simply finding the center and radius of a circle.</td>
<td>What information do you need to derive the equation of a circle or parabola, and why?</td>
<td></td>
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</tr>
</tbody>
</table>
The center and radius of a circle can be found by completing the square an/or an equation of a circle.

The goal in completing the square is to rewrite the equation in the form $(x + a)^2 = c$.

The equation of a circle has a constant value that represents the square of the radius, not the radius.

$$(x-h)^2 + (y-k)^2 = r^2$$

Lots of students assume that $(x-1)^2 + (y+1)^2 = 4$ has a radius of 4.

The equation of a parabola can be derived given its focus and directrix.

Emphasize the connection between the familiar shape of a circle to the coordinate grid.

The equation of a circle is derived from the definition of a circle and the distance formula.

**SPED Strategies**

Pre-teach vocabulary using visual and verbal models that are connected to real life situations and ensure that students include these definitions in their reference notebook.

Model how to derive the equation of a circle given the center and radius using the
| Pythagorean Theorem | Ensure that students include this information in their reference notebook. 

Model how to use the equation of a circle to determine the radius and center. Provide students with hands on opportunities to explore and extend their understanding by working in small groups, see the application to real life |

**ELL Strategies:**

Explain orally and in writing the equation of a circle of given center and radius in the student’s native language and/or use gestures, examples and selected technical words, and short simple sentences.

Review vocabulary words and provide visual examples for students to reference. |
New Jersey Student Learning Standard(s):
G.C.A.1: Prove that all circles are similar.

**Student Learning Objective 7:** Prove that all circles are similar

**Modified Student Learning Objectives/Standards:** N/A

<table>
<thead>
<tr>
<th>MPs</th>
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</tr>
</thead>
<tbody>
<tr>
<td>MP 3</td>
<td>N/A</td>
<td>Construct a formal proof of the similarity of all circles by determining a single or sequence of similarity transformation between circles.</td>
<td>Why are all circles similar? How are segments within circles (i.e. radii, diameters, and chords) related to each other? What is the relationship of their measurements?</td>
<td>Type II, III: Similar Circles</td>
</tr>
<tr>
<td>MP 5</td>
<td>N/A</td>
<td>Compare the ratio of the circumference of a circle to the diameter of the circle and determine this ratio is constant for all circles.</td>
<td>An essential relationship found in circles is that no matter the size of the circle the circumference divided by the radius is a constant value, pi.</td>
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<td></td>
<td>Emphasize on radians to make them easier to see and understand for students.</td>
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<td></td>
<td>Use geometric simulation software to model transformations and demonstrate a sequence of transformations to show congruence or similarity of figures.</td>
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<tr>
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<td></td>
<td><strong>Similar Circles Theorem:</strong> All circles are similar.</td>
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<tr>
<td></td>
<td></td>
<td>Two arcs are similar arcs if and only if they have the same measure. All congruent arcs are similar, but not all similar arcs are congruent.</td>
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</tbody>
</table>
Arcs that have the same measure are congruent.

Circles with the same radius are congruent.

**SPED Strategies**
Review knowledge of circles (i.e. equation, definition) and model how to prove that all circles are similar.

Provide students with hands on opportunities to explore and extend their understanding by working in small groups to prove that two circles they are given are in fact similar.

**ELL Strategies:**

Explain orally and in writing the sequence of steps to prove that all circles are similar in the student’s native language and/or use gestures, selected technical words and in phrases and short simple sentences.

Have students work with partners, small groups.
New Jersey Student Learning Standard(s):

G.C.A.2: Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.

Student Learning Objective 8: Identify and describe relationships among inscribed angles, radii, and chords; use these relationships to solve problems.

Modified Student Learning Objectives/Standards: N/A

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</tr>
</thead>
<tbody>
<tr>
<td>MP 1 MP 5</td>
<td>G-C.2</td>
<td>Use the relationship between inscribed angles, radii, arcs, and chords to solve problems. Use the relationship between central, inscribed, and circumscribed angles to solve problems. Identify inscribed angles on a diameter as right angles. Identify the radius of a circle as perpendicular to the tangent where the radius intersects the circle. Identify relationships between parts of a circle and angles formed by parts of a circle. Determine angle values for all angles formed in the exterior, interior, and on the circle.</td>
<td>What patterns do you notice between angles formed by radii, chords, secants and tangents? What patterns do you notice about segment lengths involving chords, secants and tangents? How are segments within circles (i.e. radii, diameters, and chords) related to each other? What is the relationship of their measurement?</td>
<td>Type I: Circle and Squares Type II, III: Neglecting the Curvature of the Earth Right Triangles Inscribed in Circles I Tangent Lines and the Radius of a Circle</td>
</tr>
<tr>
<td></td>
<td>● Include the relationship between central, inscribed, and circumscribed angles: inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle. ● This does not include angles and segment relationships with tangents and secants. Tasks will not assess angle relationships formed outside the circle using secants and tangents.</td>
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</table>
- Tasks may involve the degree measure of an arc.

Determine the lengths of intersecting chords and secants.

Solve problems involving angles and arcs formed by any combination of radii, chords, secants, and tangents.

**Examples:**
Given the circle below with radius of 10 and chord length of 12, find the distance from the chords to the center of the circle.

![Circle with radius 10 and chord length 12](image)

Find the unknown length in the picture below:

![Circle with chord](image)

Determine angle values for all angles formed in exterior, interior, and on the circle.

The interior angle formula and the external angle formula are quite simple when both arcs are provided but if one of them is missing the problem seems to get much more difficult.

A chord is a line segment that has both endpoints on the circle. The longest chord possible in a circle is the one that passes
through the center of the circle and is called the diameter of the circle.

A tangent line intersects the circle EXACTLY ONCE at a point known as the point of tangency. A secant line on the other hand intersects the circle EXACTLY TWICE forming a chord inside the circle.

**SPED Strategies**
Pre-teach vocabulary using visual and verbal models that are connected to real life situations and ensure that students include these definitions their reference notebook.

Model how the relationships between chords and angles (inscribed and central) are used to calculate the measure of unknown values. Ensure that students include this information in their reference notebook

Provide students with hands on opportunities to explore and extend their understanding of the relationships between chords and angles by working in small groups.

**ELL Strategies:**
After listening to oral explanation and reading the directions, demonstrate comprehension by identifying and describing relationships among inscribed angles, radii and chords in the student’s native language and/or using gestures and selected technical words.
Identify and describe relationships among inscribed angles, radii, and chords; use these relationships to solve problems.

New Jersey Student Learning Standards (s):
G.C.B.5: Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

Student Learning Objective 9: Find arc lengths and areas of sectors of circles; use similarity to show that the length of the arc intercepted by an angle is proportional to the radius. Derive the formula for the area of a sector.

Modified Student Learning Objectives/Standards: N/A

<table>
<thead>
<tr>
<th>MPs</th>
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<th>Tasks/Activities</th>
</tr>
</thead>
<tbody>
<tr>
<td>MP 2 MP 3</td>
<td>G-C.B</td>
<td>A proportional relationship exists between the length of an arc that is intercepted by an angle and the radius of the circle. Use similarity to derive the fact that the length of the arc intercepted by an angle is proportional to the radius. Define radian measure of an angle as the constant of proportionality when the length of the arc intercepted by an angle is proportional to the radius. Practice estimating the size of things in radians to familiarize students to the new measurement.</td>
<td>How can you prove relationships between angles and arcs in a circle? When lines intersect a circle or within a circle, how do you find the measures of resulting angles, arcs, and segments? How does the area of a circle relate to the area of a sector?</td>
<td>Type II, III: Measures of Arcs and Radians How on Earth? Mutually Tangent Circles</td>
</tr>
</tbody>
</table>
Solve real-world problems using arc length and areas of sectors.

Radians are a very powerful concept for the future and will be used instead of degrees.

![Diagram of a circle with radius r and angle in radians](image)

Derive and use the formula for the area of a sector in terms of radians.

Compute arc lengths and areas of sectors of circles.

Derive and use the formula for arc length in terms of radians.

Convert between degrees and radians.

Students are often intimidated by the use of pi and how they are unable to visualize how big something in radian measure is, whereas they handle degrees so easily.

The length of a given arc is equal to a fractional amount of the circumference.
**Arc length:** In a circle, the ratio of the length of a given arc to the circumference is equal to the ratio of the measure of the arc to 360°.

\[
\text{Arc length of } \overline{AB} = \frac{m\overline{AB}}{360^\circ} \cdot 2\pi r
\]

Converting between Degrees and Radians:

- **Degrees to radians**
  Multiply degree measure by: \[\frac{2\pi \text{ radians}}{360^\circ}, \text{ or } \frac{\pi \text{ radians}}{180^\circ}\]

- **Radians to degrees**
  Multiply radian measure by: \[\frac{360^\circ}{2\pi \text{ radians}}, \text{ or } \frac{180^\circ}{\pi \text{ radians}}\]

A sector of a circle is the region bounded by two radii of the circle and their intercepted arc.
You can divide a circle into congruent sections and rearrange the sections to form a figure that approximates a parallelogram. Increasing the number of congruent sections increases the figure’s resemblance to a parallelogram.

**Area of a sector**: The ratio of the area of a sector of a circle to the area of a whole circle \((\pi^2)\) is equal to the ratio of the measure of the intercepted arc to 360°.

**SPED Strategies**: Pre-teach vocabulary, concepts and formulas related to arc length, sector area and radian measure using visual and verbal models that are connected to real life situations and ensure
that students include these definitions their reference notebook.

Provide students with the formula needed to convert degrees to radians and radians to degrees and explain why it facilitates understanding and is necessary.

Model how to use the concepts and formulas to solve real life problems.

Provide students with hands on opportunities to explore and extend their understanding of the relationships between chords and angles by working in small groups.

**ELL Strategies:**
Describe orally and in writing the use of similarity to show and explain that the length of the arc intercepted by an angle is proportional to the radius in the student’s native language and/or use drawings, examples and selected technical words in phrases and short simple sentences.

Provide students with hands on opportunities to explore and extend their understanding of the relationships between chords and angles by working in small groups.
**New Jersey Student Learning Standards (s):**

G.C.A.3: Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.

**Student Learning Objective 10:** Prove the properties of angles for a quadrilateral inscribed in a circle and construct inscribed and circumscribed circles of a triangle using geometric tools and geometric software.

**Modified Student Learning Objectives/Standards:** N/A

<table>
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<tr>
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<th>Tasks/Activities</th>
</tr>
</thead>
<tbody>
<tr>
<td>MP 3 MP 5</td>
<td>N/A</td>
<td>An inscribed angle is an angle whose vertex is on a circle and whose sides contain chords of the circle. An arc that lies between two lines, rays, or segments is called a intercepted arc. A polygon is an inscribed polygon when all its vertices lie in a circle. The circle that contains the vertices is a circumscribed circle. An arc that lies between two lines, rays, or segments is called and intercepted arc. If the endpoints of a chord or arc lie on the sides of an inscribed angle, then the chord or arc is said to subtend the angle.</td>
<td>How can various figures be inscribed in a circle using various tools? How do the properties of these figures relate to the parts of a circle? Why does the formula for the circumference and area of a circle work?</td>
<td>Type II, III: Circumcenter of a Triangle Circumscribed Triangles Inscribing a Triangle in a Circle Locating Warehouse Orbiting Satellite Inscribing a Circle in a Triangle I Inscribing a Circle in a Triangle II Inscribing a Triangle in a Circle</td>
</tr>
</tbody>
</table>

Construct the inscribed circle of a triangle.
<table>
<thead>
<tr>
<th>Inscribed Angles of a Circle Theorem: If two inscribed angles of a circle intercept the same arc, then the angles are congruent.</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Inscribed Angles" /></td>
</tr>
<tr>
<td><strong>Inscribed Right Triangle Theorem:</strong> If a right triangle is inscribed in a circle, then the hypotenuse is a diameter of the circle. Conversely, if one side of an inscribed triangle is a diameter of the circle, then the triangle is a right triangle and the angle opposite the diameter is the right angle.</td>
</tr>
<tr>
<td><img src="image" alt="Inscribed Right Triangle" /></td>
</tr>
<tr>
<td>Construct the circumscribed circle of a triangle.</td>
</tr>
<tr>
<td>Prove properties of the angles of a quadrilateral that is inscribed in a circle.</td>
</tr>
<tr>
<td>Opposite Angles in a Cyclic Quadrilateral</td>
</tr>
</tbody>
</table>
**Inscribed Quadrilateral Theorem:** A quadrilateral can be inscribed in a circle if and only if its opposite angles are supplementary.

Opposite angles of an inscribed quadrilateral in a circle are supplementary.

Construct the incenter ad the incircle of a triangle. The incenter is the point that is equidistant from all three sides of the triangle.

If \( AP, BP, \) and \( CP \) are angle bisectors of \( \triangle ABC \), then \( PD = PE = PF \).

Construct the circumcenter and the circumcircle of a triangle. The circumcenter is the point that is equidistant from all three vertices of the triangle.
Use concurrence of perpendicular bisectors and angle bisectors for the basis of the construction of the circle.

Circle constructions provide a way to connect circle properties to the physical construction.

**SPED Strategies:**
Pre-teach vocabulary, concepts and formulas related to figures inscribed within and circumscribed by circles using visual and verbal models that are connected to real life situations and ensure that students include these definitions their reference notebook.

Model the thinking process and procedures needed to work proficiently.

Encourage students to verbalize their thinking using assessing and advancing questions. Use student responses to tailor instructional strategies and meet student needs.
Provide students with hands on opportunities to explore and extend their understanding of the relationships between figures inscribed within and circumscribed by circles by working in small groups.

**ELL Strategies:**
Explain orally or in writing the properties of angles for a quadrilateral inscribed in a circle and constructing inscribed and circumscribed circles of a triangle in the student’s native language and/or use gestures, examples and selected technical words in phrases and short simple sentences.

Utilize manipulatives and develop hands-on activities.

**New Jersey Student Learning Standard(s):**

**S.CP.B.6:** Find the conditional probability of $A$ given $B$ as the fraction of $B$’s outcomes that also belong to $A$, and interpret the answer in terms of the model.

**S.CP.B.7:** Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, and interpret the answer in terms of the model.

**Student Learning Objective 11:** Find the conditional probability of $A$ given $B$ as the fraction of $B$’s outcomes that also belong to $A$ and apply the Addition Rule [$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$].

**Modified Student Learning Objectives/Standards:**

**M.EE.S-IC.1–2:** Determine the likelihood of an event occurring when the outcomes are equally likely to occur.
| MP 1 | N/A | Scatter plots of data sets can be used to identify the type of function that best represents the shape of the data (linear, quadratic or exponential). Residuals (lines of regressions) are drawn on scatter plots in order to informally assess the fit of a function to a data set. If a scatter plot has a linear association, then a line of best fit can be drawn to interpret the data set. Mutually exclusive events exist. Analyze event B’s outcomes to determine the proportion of B’s outcomes that also belong to event A. Interpret this proportion as conditional probability of A given B. Identify two events as mutually exclusive (disjoint). Calculate probabilities using the Addition rule 
\[ P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B). \] **SPED Strategies:**
Pre-teach vocabulary using visual and verbal models that are connected to real life situations. Review scatter plots and lines of regression and relate this concept to probability. |
| Type II, III: Coffee at Mom's Diner |
| Rain and Lightning. |
| Venn Diagrams and the Addition Rule |
| How Do You Get to School |
| The Titanic 1 |
| The Titanic 2 |
| The Titanic 3 |
| The Addition Rule |
Model conditional probability scenarios and the mathematical models that correlate to each. Memorialize this learning by creating a reference document that illustrates the types of conditional probability scenarios, the mathematical model that correlates and the thinking process involved in solving these problem types.

**ELL Strategies:**

Demonstrate comprehension of how to find the conditional probability of A given B as the fraction of B’s outcomes that also belong to A and by applying and interpreting the addition rule of probability in a uniform probability model in the student’s native language and/or use gestures, examples and selected technical words.

Use a graphic calculator to simulate, and then estimate conditional probabilities.

Write two examples to define dependent and conditional probability and introduce the notation $P(B|A)$. Ask students to read aloud and describe the formula in their own words.
### Honors Projects (Must complete all)

<table>
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<th>Project 1</th>
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<th>Project 3</th>
</tr>
</thead>
</table>
| **What Can I Learn from My Shadow?**  
*G.SRT.C.8*  
**Essential Question:**  
How can trigonometric ratios be used to solve problems involving indirect measurement?  
**Skills:**  
1. Accurately calculate values using trigonometry.  
2. Develop a more complete understanding of trigonometric principles. |
| **Triangle Artwork**  
*G.SRT.C.6, G.SRT.C.7 G.SRT.C.8 G.SRT.C.10*  
**Essential Question:**  
How can geometric figures such as triangles be used in art and what about their characteristics and position make them artistically pleasing?  
**Skills:**  
1. Accurately draw various triangles.  
2. Analyze their work and provide a written description. |
| **The Geometry of Baseball**  
*G.GPE.B.6 and G.GPE.B.7*  
**Essential Question:**  
How do geometric principles affect our lives and past times?  
**Skills:**  
1. Create a scale model.  
2. Analyze established methods of construction for accuracy. |
Integrated Evidence Statements

G-Int.1: Solve multi-step contextual word problems with degree of difficulty appropriate to the course, requiring application of course-level knowledge and skills articulated in G-MG and G-GPE.7.
   ● MG is the primary content.
   ● See examples at [https://www.illustrativemathematics.org/](https://www.illustrativemathematics.org/) for G-MG.

G-C.2: Identify and describe relationships among inscribed angles, radii, and chords and apply these concepts in problem solving situations.
   ● Include the relationship between central, inscribed, and circumscribed angles: inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.
   ● This does not include angles and segment relationships with tangents and secants. Tasks will not assess angle relationships formed outside the circle using secants and tangents.
   ● Tasks may involve the degree measure of an arc.

G-C.B: Find arc lengths and areas of sectors of circles. i.) Tasks involve computing arc lengths or areas of sectors given the radius and the angle subtended; or vice versa.

G-GPE.1-1: Complete the square to find the center and radius of a circle given by an equation. i.) The "derive" part of standard G-GPE.1 is not assessed here.

G-GPE.1-2 Understand or complete a derivation of the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.
   ● Tasks must go beyond simply finding the center and radius of a circle.

G-GPE.6: Find the point on a directed line segment between two given points that partitions the segment in a given ratio.

G-SRT.6 Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
   ● Trigonometric ratios include sine, cosine, and tangent only.

G-SRT.7-2: Use the relationship between the sine and cosine of complementary angles.
Integrated Evidence Statements

- The "explain" part of standard G-SRT.7 is not assessed here; See Sub-Claim C for this aspect of the standard.

G-SRT.8 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.
- The task may have a real world or mathematical context. For rational solutions, exact values are required. For irrational solutions, exact or decimal approximations may be required. Simplifying or rewriting radicals is not required; however, students will not be penalized if they simplify the radicals correctly.

HS.C.13.1 Apply geometric reasoning in a coordinate setting, and/or use coordinates to draw geometric conclusions. Content scope: G-GPE.6, G-GPE.7.

HS.C.13.2 Apply geometric reasoning in a coordinate setting, and/or use coordinates to draw geometric conclusions. Content scope: G-GPE.4.

HS.C.13.3 Apply geometric reasoning in a coordinate setting, and/or use coordinates to draw geometric conclusions. Content scope: G-GPE.5.

HS.C.15.14: Present solutions to multi-step problems in the form of valid chains of reasoning, using symbols such as equals signs appropriately (for example, rubrics award less than full credit for the presence of nonsense statements such as $1 + 4 = 5 + 7 = 12$, even if the final answer is correct), or identify or describe errors in solutions to multi-step problems and present corrected solutions. Content scope: G-SRT.C

HS.C.18.2 Use a combination of algebraic and geometric reasoning to construct, autonomously, chains of reasoning that will justify or refute propositions or conjectures about geometric figures. Content scope: Algebra content from Algebra 1 course; geometry content from the Geometry course.
- For the Geometry course, we are reaching back to Algebra 1 to help students synthesize across the two subjects.

HS.D.1-2: Solve multi-step contextual problems with degree of difficulty appropriate to the course, requiring application of knowledge and skills articulated in 6.G, 7.G, and/or 8.G.
Integrated Evidence Statements

HS.D.2-1 Solve multi-step contextual problems with degree of difficulty appropriate to the course involving perimeter, area, or volume that require solving a quadratic equation.

- Tasks do not cue students to the type of equation or specific solution method involved in the task.
  - For example: An artist wants to build a right-triangular frame in which one of the legs exceeds the other in length by 1 unit, and in which the hypotenuse exceeds the longer leg in length by 1 unit. Use algebra to show that there is one and only one such right triangle, and determine its side lengths.

HS.D.2-2: Solve multi-step contextual problems with degree of difficulty appropriate to the course involving perimeter, area, or volume that require finding an approximate solution to a polynomial equation using numerical/graphical means.

- Tasks may have a real world or mathematical context.
- Tasks may involve coordinates (G-GPE.7).
- Refer to A-REI.11 for some of the content knowledge from the previous course relevant to these tasks.
- Cubic polynomials are limited to polynomials in which linear and quadratic factors are available.
- To make the tasks involve strategic use of tools (MP.5), calculation and graphing aids are available but tasks do not prompt the student to use them.

HS.D.2-11: Solve multi-step contextual word problems with degree of difficulty appropriate to the course, requiring application of course-level knowledge and skills articulated in G-SRT.8, involving right triangles in an applied setting.

- Tasks may, or may not, require the student to autonomously make an assumption or simplification in order to apply techniques of right triangles.
  - For example, a configuration of three buildings might form a triangle that is nearly, but not quite, a right triangle; then, a good approximate result can be obtained if the student autonomously approximates the triangle as a right triangle.

HS.D.3-2a: Micro-models: Autonomously apply a technique from pure mathematics to a real-world situation in which the technique yields valuable results even though it is obviously not applicable in a strict mathematical sense (e.g., profitably applying proportional relationships to a phenomenon that is obviously nonlinear or statistical in nature). Content Scope: Knowledge and skills articulated in the Geometry Type I, Sub-Claim A Evidence Statements.

HS.D.3-4a: Reasoned estimates: Use reasonable estimates of known quantities in a chain of reasoning that yields an estimate of an unknown quantity. Content Scope: Knowledge and skills articulated in the Geometry Type I, Sub-Claim A Evidence Statement
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# References & Suggested Instructional Websites

- [https://hcpss.instructure.com/courses/162](https://hcpss.instructure.com/courses/162)
- [https://www.desmos.com/](https://www.desmos.com/)
- [http://www.cpalms.org/Public/ToolkitGradeLevelGroup/Toolkit?id=14](http://www.cpalms.org/Public/ToolkitGradeLevelGroup/Toolkit?id=14)
- [www.corestandards.org](http://www.corestandards.org)
- [www.nctm.org](http://www.nctm.org)
- [https://www.khanacademy.org](https://www.khanacademy.org)
- [http://achievethecore.org](http://achievethecore.org)
- [https://www.illustrativemathematics.org/](https://www.illustrativemathematics.org/)
- [www.insidemathematics.org](http://www.insidemathematics.org)
- [https://learnzillion.com/resources/75114-math](https://learnzillion.com/resources/75114-math)
- [http://maccss.ncdpi.wikispaces.net/](http://maccss.ncdpi.wikispaces.net/) (Choose your grade level on the left.)
- [http://nrich.maths.org](http://nrich.maths.org)
- [https://www.youcubed.org/week-of-inspirational-math/](https://www.youcubed.org/week-of-inspirational-math/)
- [http://illuminations.nctm.org/Lessons-Activities.aspx](http://illuminations.nctm.org/Lessons-Activities.aspx) (choose grade level and connect to search lessons)
- [www.ck12.org](http://www.ck12.org)

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Field Trip Ideas

**SIX FLAGS GREAT ADVENTURE**- This educational event includes workbooks and special science and math related shows throughout the day. Your students will leave with a better understanding of real world applications of the material they have learned in the classroom. Each student will have the opportunity to experience different rides and attractions linking mathematical and scientific concepts to what they are experiencing.

www.sixflags.com

**MUSEUM of MATHEMATICS**- Mathematics illuminates the patterns that abound in our world. The National Museum of Mathematics strives to enhance public understanding and perception of mathematics. Its dynamic exhibits and programs stimulate inquiry, spark curiosity, and reveal the wonders of mathematics. The Museum’s activities lead a broad and diverse audience to understand the evolving, creative, human, and aesthetic nature of mathematics.

www.momath.org

**LIBERTY SCIENCE CENTER** - An interactive science museum and learning center located in Liberty State Park. The center, which first opened in 1993 as New Jersey's first major state science museum, has science exhibits, the largest IMAX Dome theater in the United States, numerous educational resources, and the original *Hoberman sphere*.

http://lsc.org/plan-your-visit/