Business Math: Unit 3
Overview of Math Proficiencies
Course Philosophy/Description

Business Mathematics is an elective Mathematics course of which students learn to use mathematics effectively as a tool in their personal and business lives. After students have completed this course, they will be able to apply mathematical concepts in various personal and business situations. All standards are aligned to New Jersey Student Learning Standards of Mathematics and the New Jersey Personal Financial Literacy Standards.

Students will review and apply mathematical concepts that they learned in four of the conceptual categories, namely Number and Quantity, Algebra, Functions, and Statistics and Probability. They will understand terminology relating to personal and business mathematics applications and apply basic math skills to the solution of both personal and business applications. They will use common mathematical formulas to solve a variety of personal and business mathematics as well as apply knowledge of computer and calculator use. Students will also learn strategies for critical thinking and problem solving both in finance and business ethics.
ESL Framework

This ESL framework was designed to be used by bilingual, dual language, ESL and general education teachers. Bilingual and dual language programs use the home language and a second language for instruction. ESL teachers and general education or bilingual teachers may use this document to collaborate on unit and lesson planning to decide who will address certain components of the SLO and language objective. ESL teachers may use the appropriate leveled language objective to build lessons for ELLs which reflects what is covered in the general education program. In this way, whether it is a pull-out or push-in model, all teachers are working on the same Student Learning Objective connected to the Common Core standard. The design of language objectives is based on the alignment of the World-Class Instructional Design Assessment (WIDA) Consortium’s English Language Development (ELD) standards with the Common Core State Standards (CCSS). WIDA’s ELD standards advance academic language development across content areas ultimately leading to academic achievement for English learners. As English learners are progressing through the six developmental linguistic stages, this framework will assist all teachers who work with English learners to appropriately identify the language needed to meet the requirements of the content standard. At the same time, the language objectives recognize the cognitive demand required to complete educational tasks. Even though listening and reading (receptive) skills differ from speaking and writing (expressive) skills across proficiency levels the cognitive function should not be diminished. For example, an Entering Level One student only has the linguistic ability to respond in single words in English with significant support from their home language. However, they could complete a Venn diagram with single words which demonstrates that they understand how the elements compare and contrast with each other or they could respond with the support of their home language (L1) with assistance from a teacher, para-professional, peer or a technology program.

http://www.state.nj.us/education/modelcurriculum/ela/ELLOverview.pdf
<table>
<thead>
<tr>
<th>#</th>
<th>Student Learning Objective: Number and Quantity</th>
<th>NJSLS</th>
<th>NJ-Personal Financial Literacy Standards</th>
<th>Marking Period 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>Explain and justify conclusions about sums and products of rational and irrational numbers.</td>
<td>N.RN.A.1, N.RN.B.3, N.RN.A.2, N-RN-B.1</td>
<td></td>
<td>9.1.12.E.2</td>
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<tr>
<td>3</td>
<td>Distinguish between and explain situations modeled with linear functions and exponential functions.</td>
<td>F-LE.A.1a, F-LE.A.1b, F-LE.A.1c</td>
<td></td>
<td>9.1.12.E.2</td>
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<td>4</td>
<td>Solve quadratic equations with real coefficients that have complex solutions by taking square roots, completing the square and factoring.</td>
<td>N.CN.A.1, N.CN.A.2, N.CN.C.7, A.REI.B.4, 4b</td>
<td></td>
<td>9.1.12.B.1</td>
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<td></td>
<td>Pacing Chart – Unit 3</td>
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<td>6</td>
<td>Construct chains of reasoning that will justify or refute proposition or conjectures about algebraic equations or systems.</td>
<td>A.REI.D.11, A.REI.B.4, A.CED.A.1, A.APR.A.1, A.SSE.B.3, A.SSE.A.2.</td>
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<td>9.1.12.B.1, 9.1.12.E.2</td>
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<td>7</td>
<td>Sketch graphs of linear, radical and exponential functions expressed symbolically or from a verbal description. Show key features and interpret parameters in context. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). Calculate and interpret the average rate of change of a function presented symbolically or as a table; estimate the rate of change from a graph</td>
<td>A.APR.B.3, F.IF.C.9, F.LE.A.3, F.IF.B.6, F.IF.B.4, F.IF.B.5, F.LE.B.5</td>
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<tr>
<td>8</td>
<td>Construct chains of reasoning that will justify or refute proposition or conjectures about Functions</td>
<td>F.IF.C.9, F.LE.A.3, F.IF.B.6, F.IF.B.4</td>
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<td></td>
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<td>9.1.12.B.1</td>
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<tr>
<td>9</td>
<td>Represent linear and non-linear (quadratic, exponential, and trigonometric) data for two variables on a scatter plot, fit a function to the data, analyze slope, intercept (in case of linear) and residuals (in order to informally assess fit), and use the function to solve problems. Use given functions or choose a function suggested by the context;</td>
<td>S.ID.B.6, S.ID.C.7, S.ID.C.9, S.ID.C.8</td>
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<td>10</td>
<td>Identify and evaluate random sampling methods. Identify the differences among and purposes of sample surveys, experiments, and observational studies, explaining how randomization relates to each. Use data from a randomized experiment to compare two treatments and use simulations to decide if differences between parameters are significant; evaluate reports based on data.</td>
<td>S.IC.B.3, S.IC.A.2, S.IC.B.6, S.IC.B.5</td>
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<td></td>
<td></td>
<td>9.1.12.B.1, 9.1.12.E.2</td>
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</table>
Research about Teaching and Learning Mathematics

Structure teaching of mathematical concepts and skills around problems to be solved (Checkly, 1997; Wood & Sellars, 1996; Wood & Sellars, 1997)

Encourage students to work cooperatively with others (Johnson & Johnson, 1975; Davidson, 1990)

Use group problem-solving to stimulate students to apply their mathematical thinking skills (Artzt & Armour-Thomas, 1992)

Students interact in ways that support and challenge one another’s strategic thinking (Artzt, Armour-Thomas, & Curcio, 2008)

Activities structured in ways allowing students to explore, explain, extend, and evaluate their progress (National Research Council, 1999)

There are three critical components to effective mathematics instruction (Shellard & Moyer, 2002):

- Teaching for conceptual understanding
- Developing children’s procedural literacy
- Promoting strategic competence through meaningful problem-solving investigations

Teachers should be:

- Demonstrating acceptance and recognition of students’ divergent ideas.
- Challenging students to think deeply about the problems they are solving, extending thinking beyond the solutions and algorithms required to solve the problem
- Influencing learning by asking challenging and interesting questions to accelerate students’ innate inquisitiveness and foster them to examine concepts further.
- Projecting a positive attitude about mathematics and about students’ ability to “do” mathematics

Students should be:

- Actively engaging in “doing” mathematics
- Solving challenging problems
- Investigating meaningful real-world problems
- Making interdisciplinary connections
- Developing an understanding of mathematical knowledge required to “do” mathematics and connect the language of mathematical ideas with numerical representations
- Sharing mathematical ideas, discussing mathematics with one another, refining and critiquing each other’s ideas and understandings
- Communicating in pairs, small group, or whole group presentations
- Using multiple representations to communicate mathematical ideas
- Using connections between pictures, oral language, written symbols, manipulative models, and real-world situations
- Using technological resources and other 21st century skills to support and enhance mathematical understanding
Mathematics is not a stagnate field of textbook problems; rather, it is a dynamic way of constructing meaning about the world around us, generating knowledge and understanding about the real world every day. Students should be metaphorically rolling up their sleeves and “doing mathematics” themselves, not watching others do mathematics for them or in front of them. (Protheroe, 2007)

Balanced Mathematics Instructional Model

Balanced math consists of three different learning opportunities: guided math, shared math, and independent math. Ensuring a balance of all three approaches will build conceptual understanding, problem solving, computational fluency, and procedural fluency. Building conceptual understanding is the focal point of developing mathematical proficiency. Students should frequently work on rigorous tasks, talk about the math, explain their thinking, justify their answer or process, build models with graphs or charts or manipulatives, and use technology.

When balanced math is used in the classroom it provides students opportunities to:

- solve problems
- make connections between math concepts and real-life situations
- communicate mathematical ideas (orally, visually and in writing)
- choose appropriate materials to solve problems
- reflect and monitor their own understanding of the math concepts
- practice strategies to build procedural and conceptual confidence

Teacher builds conceptual understanding by modeling through demonstration, explicit instruction, and think alouds, as well as guiding students as they practice math strategies and apply problem solving strategies. (whole group or small group instruction)

Teacher and students practice mathematics processes together through interactive activities, problem solving, and discussion. (whole group or small group instruction)

Students practice math strategies independently to build procedural and computational fluency. Teacher assesses learning and reteaches as necessary. (whole group instruction, small group instruction, or centers)
<table>
<thead>
<tr>
<th>Effective Pedagogical Routines/Instructional Strategies</th>
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<tr>
<td><strong>Collaborative Problem Solving</strong></td>
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<tr>
<td>Connect Previous Knowledge to New Learning</td>
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<td>Making Thinking Visible</td>
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<td>Develop and Demonstrate Mathematical Practices</td>
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<tr>
<td>Inquiry-Oriented and Exploratory Approach</td>
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<td>Multiple Solution Paths and Strategies</td>
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<tr>
<td>Use of Multiple Representations</td>
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<tr>
<td>Explain the Rationale of your Math Work</td>
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<tr>
<td>Quick Writes</td>
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<tr>
<td>Pair/Trio Sharing</td>
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<td>Turn and Talk</td>
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<td>Charting</td>
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<td>Gallery Walks</td>
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<td>Small Group and Whole Class Discussions</td>
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<td>Student Modeling</td>
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<tr>
<td><strong>Analyze Student Work</strong></td>
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<tr>
<td>Identify Student’s Mathematical Understanding</td>
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<tr>
<td>Identify Student’s Mathematical Misunderstandings</td>
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<tr>
<td>Interviews</td>
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<tr>
<td>Role Playing</td>
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<tr>
<td>Diagrams, Charts, Tables, and Graphs</td>
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<tr>
<td>Anticipate Likely and Possible Student Responses</td>
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<tr>
<td>Collect Different Student Approaches</td>
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<tr>
<td>Multiple Response Strategies</td>
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<td>Asking Assessing and Advancing Questions</td>
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<td>Revoicing</td>
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<td>Marking</td>
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<tr>
<td>Recapping</td>
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<td>Challenging</td>
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<tr>
<td>Pressing for Accuracy and Reasoning</td>
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<td>Maintain the Cognitive Demand</td>
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## Educational Technology Standards


- **Technology Operations and Concepts**
  - Create professional documents (e.g., newsletter, personalized learning plan, business letter or flyer) using advanced features of a word processing program.
  - Select and use appropriate tools and digital resources to accomplish a variety of tasks and to solve problems.

- **Digital Citizenship**
  - Model appropriate online behaviors related to cyber safety, cyber bullying, cyber security, and cyber ethics.

- **Research and Information Literacy**
  - Gather and analyze findings to produce a possible solution for a content-related or real world problem using data collection technology.

- **Design: Critical Thinking, Problem Solving, and Decision Making**
  - Design and create a product using the design process that addresses a real world problem with specific criteria and constraints.
Career Ready Practices describe the career-ready skills that all educators in all content areas should seek to develop in their students. They are practices that have been linked to increase college, career, and life success. Career Ready Practices should be taught and reinforced in all career exploration and preparation programs with increasingly higher levels of complexity and expectation as a student advances through a program of study.

- **CRP1. Act as a responsible and contributing citizen and employee**
  Career-ready individuals understand the obligations and responsibilities of being a member of a community, and they demonstrate this understanding every day through their interactions with others. They are conscientious of the impacts of their decisions on others and the environment around them. They think about the near-term and long-term consequences of their actions and seek to act in ways that contribute to the betterment of their teams, families, community and workplace. They are reliable and consistent in going beyond the minimum expectation and in participating in activities that serve the greater good.

- **CRP2. Apply appropriate academic and technical skills.**
  Career-ready individuals readily access and use the knowledge and skills acquired through experience and education to be more productive. They make connections between abstract concepts with real-world applications, and they make correct insights about when it is appropriate to apply the use of an academic skill in a workplace situation

- **CRP4. Communicate clearly and effectively and with reason.**
  Career-ready individuals communicate thoughts, ideas, and action plans with clarity, whether using written, verbal, and/or visual methods. They communicate in the workplace with clarity and purpose to make maximum use of their own and others’ time. They are excellent writers; they master conventions, word choice, and organization, and use effective tone and presentation skills to articulate ideas. They are skilled at interacting with others; they are active listeners and speak clearly and with purpose. Career-ready individuals think about the audience for their communication and prepare accordingly to ensure the desired outcome.

- **CRP6. Demonstrate creativity and innovation.**
  Career-ready individuals regularly think of ideas that solve problems in new and different ways, and they contribute those ideas in a useful and productive manner to improve their organization. They can consider unconventional ideas and suggestions as solutions to issues, tasks or problems, and they discern which ideas and suggestions will add greatest value. They seek new methods, practices, and ideas from a variety of sources and seek to apply those ideas to their own workplace. They take action on their ideas and understand how to bring innovation to an organization.

- **CRP7. Employ valid and reliable research strategies.**
  Career-ready individuals are discerning in accepting and using new information to make decisions, change practices or inform strategies. They use reliable research process to search for new information. They evaluate the validity of sources when considering the use and adoption of external information or practices in their workplace situation.
Career Ready Practices

- **CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.**
  Career-ready individuals readily recognize problems in the workplace, understand the nature of the problem, and devise effective plans to solve the problem. They are aware of problems when they occur and take action quickly to address the problem; they thoughtfully investigate the root cause of the problem prior to introducing solutions. They carefully consider the options to solve the problem. Once a solution is agreed upon, they follow through to ensure the problem is solved, whether through their own actions or the actions of others.

- **CRP11. Use technology to enhance productivity.**
  Career-ready individuals find and maximize the productive value of existing and new technology to accomplish workplace tasks and solve workplace problems. They are flexible and adaptive in acquiring new technology. They are proficient with ubiquitous technology applications. They understand the inherent risks-personal and organizational-of technology applications, and they take actions to prevent or mitigate these risks.

- **CRP12. Work productively in teams while using cultural global competence.**
  Career-ready individuals positively contribute to every team, whether formal or informal. They apply an awareness of cultural difference to avoid barriers to productive and positive interaction. They find ways to increase the engagement and contribution of all team members. They plan and facilitate effective team meetings.
### WIDA Proficiency Levels

At the given level of English language proficiency, English language learners will process, understand, produce or use:

<table>
<thead>
<tr>
<th>Level</th>
<th>Description</th>
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</table>
| 6- Reaching | - Specialized or technical language reflective of the content areas at grade level  
- A variety of sentence lengths of varying linguistic complexity in extended oral or written discourse as required by the specified grade level  
- Oral or written communication in English comparable to proficient English peers |
| 5- Bridging | - Specialized or technical language of the content areas  
- A variety of sentence lengths of varying linguistic complexity in extended oral or written discourse, including stories, essays or reports  
- Oral or written language approaching comparability to that of proficient English peers when presented with grade level material. |
| 4- Expanding | - Specific and some technical language of the content areas  
- A variety of sentence lengths of varying linguistic complexity in oral discourse or multiple, related sentences or paragraphs  
- Oral or written language with minimal phonological, syntactic or semantic errors that may impede the communication, but retain much of its meaning, when presented with oral or written connected discourse, with sensory, graphic or interactive support |
| 3- Developing | - General and some specific language of the content areas  
- Expanded sentences in oral interaction or written paragraphs  
- Oral or written language with phonological, syntactic or semantic errors that may impede the communication, but retain much of its meaning, when presented with oral or written, narrative or expository descriptions with sensory, graphic or interactive support |
| 2- Beginning | - General language related to the content area  
- Phrases or short sentences  
- Oral or written language with phonological, syntactic, or semantic errors that often impede the communication when presented with one to multiple-step commands, directions, or a series of statements with sensory, graphic or interactive support |
| 1- Entering | - Pictorial or graphic representation of the language of the content areas  
- Words, phrases or chunks of language when presented with one-step commands directions, WH-, choice or yes/no questions, or statements with sensory, graphic or interactive support |
# Language Development Supports for English Language Learners

To Increase Comprehension and Communication Skills

## Environment

- Welcoming and stress-free
- Respectful of linguistic and cultural diversity
- Honors students' background knowledge
- Sets clear and high expectations
- Includes routines and norms
- Is thinking-focused vs. answer-seeking
- Offers multiple modalities to engage in content learning and to demonstrate understanding
- Includes explicit instruction of specific language targets
- Provides participation techniques to include all learners
- Integrates learning centers and games in a meaningful way
- Provides opportunities to practice and refine receptive and productive skills in English as a new language
- Integrates meaning and purposeful tasks/activities that:
  - Are accessible by all students through multiple entry points
  - Are relevant to students' lives and cultural experiences
  - Build on prior mathematical learning
  - Demonstrate high cognitive demand
  - Offer multiple strategies for solutions
  - Allow for a language learning experience in addition to content

## Sensory Supports*

- Real-life objects (realia) or concrete objects
- Physical models
- Manipulatives
- Pictures & photographs
- Visual representations or models such as diagrams or drawings
- Videos & films
- Newspapers or magazines
- Gestures
- Physical movements
- Music & songs

## Graphic Supports*

- Graphs
- Charts
- Timelines
- Number lines
- Graphic organizers
- Graphing paper

## Interactive Supports*

- In a whole group
- In a small group
- With a partner such as *Turn-and-Talk*
- In pairs as a group (first, two pairs work independently, then they form a group of four)
- In triads
- Cooperative learning structures such as *Think-Pair-Share*
- Interactive websites or software
- With a mentor or coach

## Verbal and Textual Supports

- Labeling
- Students' native language
- Modeling
- Repetitions
- Paraphrasing
- Summarizing
- Guiding questions
- Clarifying questions
- Probing questions
- Leveled questions such as *What? When? Where? How? Why?*
- Questioning prompts & cues
- Word Banks
- Sentence starters
- Sentence frames
- Discussion frames
- Talk moves, including *Wait Time*

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BUILDING EQUITY IN YOUR TEACHING PRACTICE

How do the essential questions highlight the connection between the big ideas of the unit and equity in your teaching practice?

**CONTENT INTEGRATION**
Teachers use examples and content from a variety of cultures & groups.

- This unit / lesson is connected to other topics explored with students.
- There are multiple viewpoints reflected in the content of this unit / lesson.
- The materials and resources are reflective of the diverse identities and experiences of students.
- The content affirms students, as well as exposes them to experiences other than their own.

**KNOWLEDGE CONSTRUCTION**
Teachers help students understand how knowledge is created and influenced by cultural assumptions, perspectives & biases.

- This unit / lesson provides context to the history of privilege and oppression.
- This unit / lesson addresses power relationships.
- This unit / lesson helps students with research and critical thinking skills.
- This curriculum creates windows and mirrors for students.

**PREJUDICE REDUCTION**
Teachers implement lessons and activities to assert positive images of ethnic groups & improve intergroup relations.

- This unit / lesson helps students question and unpack biases & stereotypes.
- This unit / lesson helps students examine, research and question information and sources.
- The curriculum encourages discussion and understanding about the groups of people being represented.
- This unit / lesson challenges dominant perspectives.

**EQUITABLE PEDAGOGY**
Teachers modify techniques and methods to facilitate the academic achievement of students from diverse backgrounds.

- The instruction has been modified to meet the needs of each student.
- Students feel respected and their cultural identities are valued.
- Additional supports have been provided for students to become successful and independent learners.
- Opportunities are provided for students to reflect on their learning and provide feedback.

**EMPOWERING SCHOOL CULTURE**
Using the other four dimensions to create a safe and healthy educational environment for all.

- There are opportunities for students to connect with the community.
- My classroom is welcoming and supportive for all students?
- I am aware of and sensitive to the needs of my students and their families.
- There are effective parent-communication systems established. Parents can talk to me about issues as they arise in my classroom.

Culturally Relevant Pedagogy Examples

- **Call on Each Student:** Encourage each student to share his or her thoughts through call-and-response, keeping the class’s attention in the process.
  
  **Example:** Foster confidence. Make the assessment process less intimidating by offering different ways to demonstrate skills and understanding. For example, avoid handing out quizzes that are purely multiple choice or fill-in-the-blank. Mix in problems that involve explaining the step necessary to get to the answer. Then give students time to monitor their performance and assess their own progress, helping them focus on growth.

- **Run Problem Based Learning Scenarios:** Encourage mathematical discourse among students by presenting problems that are relevant to them, the school and/or the community.
  
  **Example:** Using a Place Based Education (PBE) model, students explore math concepts such as systems of equations while determining ways to address problems that are pertinent to their neighborhood, school or culture.

- **Run Problem-Based Learning Scenarios:** Present relatable real-world problems for your students to solve, explicitly referencing cultures and communities when applicable.
  
  **Example:** This allows you to bridge two cultural connections. First, include cultural links in the questions, whether they are explicit or students make it themselves. Second, allow students to apply different approaches to solve the question, using unique cultural perspectives.

- **Encourage Student Leadership:** Create an avenue for students to propose problem solving strategies and potential projects.
  
  **Example:** Students can learn to construct and compare non-linear functions by creating problems together and deciding if the problems fit the necessary criteria. This experience will allow students to discuss and explore their current level of understanding by applying the concepts to relevant real-life experiences.
### Differentiated Instruction

Accommodate Based on Students Individual Needs: Strategies

<table>
<thead>
<tr>
<th>Time/General</th>
<th>Processing</th>
<th>Comprehension</th>
<th>Recall</th>
</tr>
</thead>
<tbody>
<tr>
<td>- Extra time for assigned tasks</td>
<td>- Extra Response time</td>
<td>- Precise processes for balanced math instructional model</td>
<td>- Teacher-made checklist</td>
</tr>
<tr>
<td>- Adjust length of assignment</td>
<td>- Have students verbalize steps</td>
<td>- Short manageable tasks</td>
<td>- Use visual graphic organizers</td>
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<tr>
<td>- Timeline with due dates for reports and projects</td>
<td>- Repeat, clarify or reword directions</td>
<td>- Brief and concrete directions</td>
<td>- Reference resources to promote independence</td>
</tr>
<tr>
<td>- Communication system between home and school</td>
<td>- Mini-breaks between tasks</td>
<td>- Provide immediate feedback</td>
<td>- Visual and verbal reminders</td>
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<tr>
<td>- Provide lecture notes/outline</td>
<td>- Provide a warning for transitions</td>
<td>- Small group instruction</td>
<td>- Graphic organizers</td>
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<tr>
<td>- Partnering</td>
<td>- Partnering</td>
<td>- Emphasize multi-sensory learning</td>
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<thead>
<tr>
<th>Assistive Technology</th>
<th>Tests/Quizzes/Grading</th>
<th>Behavior/Attention</th>
<th>Organization</th>
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</thead>
<tbody>
<tr>
<td>- Computer/whiteboard</td>
<td>- Extended time</td>
<td>- Consistent daily structured routine</td>
<td>- Individual daily planner</td>
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<tr>
<td>- Tape recorder</td>
<td>- Study guides</td>
<td>- Simple and clear classroom rules</td>
<td>- Display a written agenda</td>
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<tr>
<td>- Video Tape</td>
<td>- Shortened tests</td>
<td>- Frequent feedback</td>
<td>- Note-taking assistance</td>
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<td>- Read directions aloud</td>
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<td>- Color code materials</td>
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Interdisciplinary Connections

*Model interdisciplinary thinking to expose students to other disciplines.*

**Social Studies and ELA Literacy Connection:**

- **Name of Task:** Americans’ spending: NJSLS: 9.1.12.A.9; W.11-12.1

From July 1998 to July 1999, Americans’ spending rose from 5.82 trillion dollars to 6.20 trillion dollars

a. Let \( x = 0 \) represent July 1998, \( x = 1 \) represent August 1998, ..., and \( x = 12 \) represent July 1999. Write a linear equation for Americans' spending in terms of the month \( x \)

b. Use the equation in (a) to predict Americans' spending in July 2002.

c. Based on the model created in (a) when would the aggregate expenditure exceed 10 trillion dollars?

d. What part of the US GDP is spent by the Americans in 2013?

**Federal Income Tax**

- Tax reform can be approached in a variety of ways. Some people worry that new tax plans may only help the rich and not the poor or the middle class. To understand proposed changes in tax plans, students must first understand our current tax plan. That is what they will discover in this project.

**Science Connection:**

**Counting Trees**

- Students will determine how scientists make estimates about a population sizing using a representative sample by applying ratio and proportion, and organizing data in a frequency table.

**Predicting Population**

- In this task, students will use populations, samples, and proportions in order to make predictions about total population sizes.
# Enrichment

## What is the Purpose of Enrichment?

- The purpose of enrichment is to provide extended learning opportunities and challenges to students who have already mastered, or can quickly master, the basic curriculum. Enrichment gives the student more time to study concepts with greater depth, breadth, and complexity.
- Enrichment also provides opportunities for students to pursue learning in their own areas of interest and strengths.
- Enrichment keeps advanced students engaged and supports their accelerated academic needs.
- Enrichment provides the most appropriate answer to the question, “What do you do when the student already knows it?”

### Enrichment is…

- Planned and purposeful
- *Different,* or differentiated, work – not just more work
- Responsive to students’ needs and situations
- A promotion of high-level thinking skills and making connections within content
- The ability to apply different or multiple strategies to the content
- The ability to synthesize concepts and make real world and cross-curricular connections
- Elevated contextual complexity
- Sometimes independent activities, sometimes direct instruction
- Inquiry based or open ended assignments and projects
- Using supplementary materials in addition to the normal range of resources
- Choices for students
- Tiered/Multi-level activities with flexible groups (may change daily or weekly)

### Enrichment is not…

- Just for gifted students (some gifted students may need intervention in some areas just as some other students may need frequent enrichment)
- Worksheets that are more of the same (busywork)
- Random assignments, games, or puzzles not connected to the content areas or areas of student interest
- Extra homework
- A package that is the same for everyone
- Thinking skills taught in isolation
- Unstructured free time
Assessments

Suggested Formative/Summative Classroom Assessments

- Describe Learning Vertically
- Identify Key Building Blocks
- Make Connections (between and among key building blocks)
- Short/Extended Constructed Response Items
- Multiple-Choice Items (where multiple answer choices may be correct)
- Drag and Drop Items
- Use of Equation Editor
- Quizzes
- Journal Entries/Reflections/Quick-Writes
- Accountable talk
- Projects
- Portfolio
- Observation
- Graphic Organizers/ Concept Mapping
- Presentations
- Role Playing
- Teacher-Student and Student-Student Conferencing
- Homework
New Jersey Student Learning Standards

**N.Q.A.1:** Use units as a way to understand problems and to guide the solution of multi-step problems; Choose and interpret units consistently in formulas; Choose and interpret the scale and the origin in graphs and data displays.

**N.Q.A.2:** Define appropriate quantities for the purpose of descriptive modeling.

**N.Q.A.3:** Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

**N.RN.A.1:** Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define* $5^{1/3}$ *to be the cube root of 5 because we want* $(5^{1/3})^3 = 5^{(1/3)\cdot 3}$ *to hold, so* $(5^{1/3})^3$ *must equal 5.*

**N.RN.A.2:** Rewrite expressions involving radicals and rational exponents using the properties of exponents.

**N.RN.B.3:** Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

**N.CN.A.1:** Know there is a complex number $i$ such that $i^2 = -1$, and every complex number has the form $a + bi$ with $a$ and $b$ real.

**N.CN.A.2:** Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

**A.REI.B.3:** Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

**A.REI.A.1:** Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

**A.CED.A.4:** Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law $V = IR$ to highlight resistance $R$.

**A.SSE.A.1:** Interpret expressions that represent a quantity in terms of its context.
New Jersey Student Learning Standards

A.SSE.A.1a. Interpret parts of an expression, such as terms, factors, and coefficients.

A.CED.A.1: Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear functions and quadratic functions, and simple rational and exponential functions.

A.CED.A.2: Create equations in two or more variables to represent relationships between quantities; Graph equations on coordinate axes with labels and scales.

A.REI.D.10: Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). [Focus on linear equations.]

A.REI.D.12: Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

A.REI.C.6: Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

A.REI.C.5: Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

A.CED.A.3: Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.

A.REI.D.11: Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.* [Focus on linear equations.]

A.REI.B.4: Solve quadratic equations in one variable.
# New Jersey Student Learning Standards

**A.REI.B.4a:** Use the method of completing the square to transform any quadratic equation in $x$ into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.

**A.REI.B.4b:** Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers $a$ and $b$.

**A.APR.A.1:** Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

**A.SSE.A.2:** Use the structure of an expression to identify ways to rewrite it. *For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.*

**A.SSE.B.3:** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

**A.SSE.B.3a:** Factor a quadratic expression to reveal the zeros of the function it defines.

**A.SSE.B.3b:** Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.

**A.SSE.B.4:** Derive and/or explain the derivation of the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.*

**F.IF.A.1:** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If $f$ is a function and $x$ is an element of its domain, then $f(x)$ denotes the output of $f$ corresponding to the input $x$. The graph of $f$ is the graph of the equation $y = f(x)$.

**F.IF.A.2:** Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
New Jersey Student Learning Standards

F.IF.B.4: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

F.IF.B.5: Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.

F.IF.B.6: Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

F.IF.C.9: Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

F.LE.A.1: Distinguish between situations that can be modeled with linear functions and with exponential functions.

F.LE.A.1a: Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.

F.LE.A.1b: Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

F.LE.A.1c: Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

F.LE.A.2: Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

F.LE.A.3: Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

F.LE.B.5: Interpret the parameters in a linear or exponential function in terms of a context.

F.BF.A.1. Write a function that describes a relationship between two quantities.
# New Jersey Student Learning Standards

<table>
<thead>
<tr>
<th>Standard</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>F.BF.A.1a</td>
<td>Determine an explicit expression, a recursive process, or steps for calculation from a context</td>
</tr>
<tr>
<td><strong>F.BF.A.1b</strong></td>
<td>Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.</td>
</tr>
<tr>
<td>F.BF.A.2</td>
<td>Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.</td>
</tr>
<tr>
<td><strong>F.BF.B.3</strong></td>
<td>Identify the effect on the graph of replacing ( f(x) ) by ( f(x) + k ), ( k f(x) ), and ( f(x + k) ) for specific values of ( k ) (both positive and negative); find the value of ( k ) given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.</td>
</tr>
<tr>
<td>S.IC.A.2</td>
<td>Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?</td>
</tr>
<tr>
<td>S.IC.B.3</td>
<td>Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.</td>
</tr>
<tr>
<td>S.IC.B.4</td>
<td>Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.</td>
</tr>
<tr>
<td>S.IC.B.5</td>
<td>Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.</td>
</tr>
<tr>
<td>S.IC.B.6</td>
<td>Evaluate reports based on data.</td>
</tr>
<tr>
<td>S.ID.A.4</td>
<td>Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.</td>
</tr>
<tr>
<td>S.ID.B.6</td>
<td>Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.</td>
</tr>
</tbody>
</table>
New Jersey Student Learning Standards

| S.ID.B.6a | Fit a function to the data (including the use of technology); use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models. |
| S.ID.B.6c | Fit a linear function for a scatter plot that suggests a linear association. |
| S.ID.C.7 | Interpolate the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. |
| S.ID.C.8 | Compute (using technology) and interpret the correlation coefficient of a linear fit. |
| S.ID.C.9 | Distinguish between correlation and causation. |
| G.CO.A.1 | Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc. |
| G.CO.A.2 | Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch). |
| G.CO.A.3 | Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself. |
| G.CO.A.4 | Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments. |
| G.CO.A.5 | Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another. |
| G.CO.B.6 | Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent. |
New Jersey Student Learning Standards

**G.CO.B.8:** Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.

**G.CO.B.7:** Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.

**G.CO.C.9:** Prove theorems about lines and angles. *Theorems include:* vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment’s endpoints.

**G.CO.C.10:** Prove theorems about triangles. *Theorems include:* measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.

**G.SRT.C.8:** Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

**G.GPE.B.6:** Find the point on a directed line segment between two given points that partitions the segment in a given ratio.

**G.GPE.B.7:** Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.

**G.GPE.B.4:** Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2).*

**G.GPE.B.5:** Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).
### New Jersey Personal Financial Literacy Standards

**9.1.12.B.1:** Prioritize financial decisions by systematically considering alternatives and possible consequences.

**9.1.12.E.2:** Analyze and apply multiple sources of financial information when prioritizing financial decisions.

**9.1.12.E.3:** Determine how objective, accurate, and current financial information affects the prioritization of financial decisions.
## Mathematical Practices

1. **Make sense of problems and persevere in solving them.**

2. **Reason abstractly and quantitatively.**

3. **Construct viable arguments and critique the reasoning of others.**

4. **Model with mathematics.**

5. **Use appropriate tools strategically.**

6. **Attend to precision.**

7. **Look for and make use of structure.**

8. **Look for and express regularity in repeated reasoning.**
<table>
<thead>
<tr>
<th>Grade: Business Math</th>
<th>Unit: 3 (Three)</th>
<th>Topic: Overview of Math Proficiencies</th>
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</thead>
</table>


**Unit Focus:**
- Perform arithmetic operations on polynomials
- Understand the relationship between zeros and factors
- Interpret the structure of expressions
- Solve equations and inequalities in one variable
- Create equations that describe numbers or relationships
- Interpret functions that arise in applications in terms of the context
- Represent and solve equations and inequalities graphically
- Build a function that models a relationship between two quantities
- Construct & compare linear, quadratic, & exponential models
- Build new functions from existing functions
- Analyze functions using different representations
- Use properties of rational and irrational numbers
- Analyze descriptive and inferential statistical models
New Jersey Student Learning Standard(s): Number and Quantity

**N-RN.A.1:** Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for notation for radicals in terms of rational exponents. *For example, we define* $5^{1/3}$ *to be the cube root of 5 because we want* $5^{1/3} = 5^{(1/3)^3}$ *to hold, so* $5^{(1/3)^3}$ *must equal 5.*

**N-RN.A.2:** Rewrite expressions involving radicals and rational exponents using the properties of exponents.

**9.1.12.B.1:** Prioritize financial decisions by systematically considering alternatives and possible consequences.

**9.1.12.E.2:** Analyze and apply multiple sources of financial information when prioritizing financial decisions.

**Student Learning Objective 1:** Use properties of integer exponents to explain and convert between expressions involving radicals and rational exponents.

<table>
<thead>
<tr>
<th>MPs</th>
<th>Evidence Statement Key/Clarifications</th>
<th>Skills, Strategies &amp; Concepts</th>
<th>Essential Understandings/Questions (Accountable Talk)</th>
<th>Tasks/Activities</th>
</tr>
</thead>
<tbody>
<tr>
<td>MP 2</td>
<td><strong>N-RN.2</strong></td>
<td>Properties of integer exponents extends to rational exponents <em>(for example, we define</em> $5^{1/3}$ <em>to be the cube root of 5 because we want</em> $5^{1/3} = 5^{(1/3)^3}$ <em>to hold, so</em> $5^{(1/3)^3}$ <em>must equal 5)</em></td>
<td>Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values allowing for radicals in terms of rational exponents.</td>
<td>DOER HS.C.2.1</td>
</tr>
<tr>
<td>MP 7</td>
<td><strong>HS.C.2.1</strong></td>
<td>Radical notation is a representation of rational exponents.</td>
<td>Rewrite expressions involving radicals and rational exponents using the properties of exponents.</td>
<td></td>
</tr>
<tr>
<td>MP 6</td>
<td></td>
<td>Students are able to: Rewrite expressions containing rational exponents into radical form.</td>
<td>How can the properties of exponents be used to simplify products and quotients of radicals?</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Given an expression with a rational exponent, how do you...</td>
<td></td>
</tr>
</tbody>
</table>
Rewrite expressions containing radical notation into exponential expressions containing rational exponents.

write the equivalent radical expression?

Why are rational exponents and radicals related to each other?

### New Jersey Student Learning Standard(s): Number and Quantity

**N.RN.A.2:** Rewrite expressions involving radicals and rational exponents using the properties of exponents.

**N-RN-B.1:** Apply properties of rational and irrational numbers to identify rational and irrational numbers.

**N-RN-B.3:** Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

**9.1.12.E.2:** Analyze and apply multiple sources of financial information when prioritizing financial decisions.

### Student Learning Objective 2: Explain and justify conclusions about sums and products of rational and irrational numbers.

<table>
<thead>
<tr>
<th>MPs</th>
<th>Evidence Statement Key/Clarifications</th>
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<th>Essential Understandings/Questions (Accountable Talk)</th>
<th>Tasks/Activities</th>
</tr>
</thead>
<tbody>
<tr>
<td>MP 1</td>
<td>HS.C.2.1 Base explanations/reasoning on the properties of rational and irrational numbers Content scope: N-RN.3.</td>
<td>Identify rational and irrational numbers.</td>
<td>How can I differentiate between rational and irrational numbers?</td>
<td>Int Math 2 HS.C.2.1</td>
</tr>
<tr>
<td>MP 3</td>
<td></td>
<td>Perform arithmetic operations and analyze the solution type.</td>
<td>Do arithmetic operations affect the nature of the numbers?</td>
<td></td>
</tr>
<tr>
<td>MP 5</td>
<td></td>
<td>Simplify radical expressions</td>
<td>How do mathematical operations affect the nature of the relation between rational and irrational numbers?</td>
<td></td>
</tr>
<tr>
<td>MP 7</td>
<td></td>
<td>The sum or product of two rational numbers is rational.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
required; however, students will not be penalized if they simplify the radicals correctly.

**HS.C.3.1**
Construct, autonomously, chains of reasoning that will justify or refute propositions or conjectures about numbers or number systems. Content Scope: N-RN, N-CN.

**HS.C.3.2**
Base explanations/reasoning on the properties of exponents. Content Scope: N-RN.A.

**HS.C.18.4**
Construct, autonomously, chains of reasoning that will justify or refute propositions or conjectures about polynomials, rational expressions, or rational exponents. Content scope: N-RN, A-APR.(2, 3, 4, 6)

| The sum of a rational number and an irrational number is irrational. | Why is the sum or product of rational numbers rational? |
| The product of a nonzero rational number and an irrational number is irrational. | Why is the sum of a rational number and irrational number irrational? |
| Why is the product of a nonzero rational number and an irrational number irrational? |
**New Jersey Student Learning Standard (s):**

F.LE.A.1: Distinguish between situations that can be modeled with linear functions and with exponential functions.

- **F.LE.A.1a:** Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.

- **F.LE.A.1b:** Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

- **F.LE.A.1c:** Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another


**Student Learning Objective 3:** Distinguish between and explain situations modeled with linear functions and with exponential functions.

<table>
<thead>
<tr>
<th>MPs</th>
<th>Evidence Statement Key/ Clarifications</th>
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<th>Tasks/Activities</th>
</tr>
</thead>
<tbody>
<tr>
<td>MP 1 MP 3 MP 5 MP 7</td>
<td><strong>F.LE.A.Int. 1</strong> Understand the concept of a function and use function notation.</td>
<td>Identify and describe situations in which one quantity changes at a constant rate.</td>
<td>What are the differentiating characteristics of linear vs. exponential functions?</td>
<td>DOER HS.D.2.6</td>
</tr>
<tr>
<td></td>
<td><strong>F.LE.2-1</strong> Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table)</td>
<td>Identify and describe situations in which a quantity grows or decays by a constant percent.</td>
<td>Linear and exponential functions can be used to draw conclusions, make predictions and support reasoning.</td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>F.LE.2-2</strong> Solve multi-step contextual problems with degree of difficulty</td>
<td>Show that linear functions grow by equal differences over equal intervals.</td>
<td>Linear and exponential functions can be modeled in many ways including: verbally, graphically and as a table.</td>
<td></td>
</tr>
</tbody>
</table>
appropriate to the course by constructing linear and/or exponential functions models, where exponents are limited to integer exponents.

**HS.C.2.1**
Base explanations/reasoning on the properties of rational and irrational numbers Content scope: N-RN.3.

For rational solutions, exact values are required. For irrational solutions, exact or decimal approximations may be required. Simplifying or rewriting radicals is not required; however, students will not be penalized if they simplify the radicals correctly.

Use real life situations to illustrate both linear and exponential functions.
New Jersey Student Learning Standard(s):

N.CN.A.1: Know there is a complex number \(i\) such that \(i^2 = -1\), and every complex number has the form \(a + bi\) with \(a\) and \(b\) real.

N.CN.A.2: Use the relation \(i^2 = -1\) and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

N.CN.C.7: Solve quadratic equations with real coefficients that have complex solutions.

A.REI.B.4: Solve quadratic equations in one variable.

A.REI.B.4b: Solve quadratic equations by inspection (e.g., for \(x^2 = 49\)), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as \(a \pm bi\) for real numbers \(a\) and \(b\).

Student Learning Objective 4: Solve quadratic equations with real coefficients that have complex solutions by taking square roots, completing the square and factoring.

<table>
<thead>
<tr>
<th>MPs</th>
<th>Evidence Statement Key/Clarifications</th>
<th>Skills, Strategies &amp; Concepts</th>
<th>Essential Understandings/Questions (Accountable Talk)</th>
<th>Tasks/Activities</th>
</tr>
</thead>
<tbody>
<tr>
<td>MP 1, MP 3, MP 5, MP 7</td>
<td>HS.C.5.5 Given an equation or system of equations, reason about the number or nature of the solutions.</td>
<td>Complex numbers (i) is defined such that (i^2 = -1). Every complex number has the form (a + bi) with (a) and (b) real. (i^2 = -1) and the commutative, associative properties to add and subtract complex numbers are to be used. Determine that (i^2 = -1) and the commutative, associative, and distributive properties to multiply complex numbers.</td>
<td>What are the subsets of the set of complex numbers? How are the powers of (i) derived and how are they cyclic? Why is it when simplifying imaginary and complex numbers that the highest power of (i) is one? What are the subsets of the set of complex numbers?</td>
<td>Type I: Computations with Complex Numbers Type II, III: Complex number patterns Powers of a complex number Complex Square Roots</td>
</tr>
</tbody>
</table>
As with real solutions, complex solutions to quadratic equations may be determined by taking square roots, factoring, and completing the square.

Evaluate powers of $i$. Solve quadratic equations with complex solutions.

Solve quadratic equations using the square root method.

Solve quadratic equations by factoring and using the zero product property.

Solve quadratic equations in one variable that have complex solutions by taking square roots.

Solve a quadratic equation in one variable that have complex solutions by completing the square.

<table>
<thead>
<tr>
<th>How are the powers of $i$ derived and how are they cyclic?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Why is it when simplifying imaginary and complex numbers that the highest power of $i$ is one?</td>
</tr>
</tbody>
</table>
New Jersey Student Learning Standard(s): Algebra

A.REI.B.3: Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

A.REI.A.1: Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

A.CED.A.4: Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law $V = IR$ to highlight resistance $R$.

A.SSE.A.1: Interpret expressions that represent a quantity in terms of its context.

A.SSE.A.1a: Interpret parts of an expression, such as terms, factors, and coefficients.

A.CED.A.1: Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear functions and quadratic functions, and simple rational and exponential functions.

A.REI.C.6: Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

A.REI.C.5: Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

A.CED.A.3: Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.

A.REI.D.12: Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

A.CED.A.2: Create equations in two or more variables to represent relationships between quantities; Graph equations on coordinate axes with labels and scales.

A.REI.D.10: Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). [Focus on linear equations.]
N.Q.A.1: Use units as a way to understand problems and to guide the solution of multi-step problems; Choose and interpret units consistently in formulas; Choose and interpret the scale and the origin in graphs and data displays.

A.REI.B.4: Solve quadratic equations in one variable.

A.REI.B.4a: Use the method of completing the square to transform any quadratic equation in $x$ into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.

A.REI.B.4b: Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers $a$ and $b$.

A.SSE.B.3: Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression


9.1.12.E.3: Determine how objective, accurate, and current financial information affects the prioritization of financial decisions.

**Student Learning Objective 5:** Create linear equations and inequalities in one variable and use them in contextual situations to solve single or multistep problems. Create linear and quadratic equations and inequalities in two variables and graph them. Solve systems of linear equations or inequalities in two variables algebraically and graphically.

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<tr>
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</table>
| Solve multi-step contextual word problems with degree of difficulty appropriate to the course, requiring application of course-level knowledge and skills articulated in A-CED, N-Q, A-SSE.3, AREI.6, A-REI.12, A-REI.11-1, limited to linear equations and exponential equations with integer exponents. | Explain each step in the solution process. Create linear equations in two variables, including those from a context. Select appropriate scales for constructing a graph. Interpret the origin in graphs. Graph equations on coordinate axes, including labels and scales. Identify and describe the solutions in the graph of an equation. | HS.D.2.6  
HS.D.3-3A  
HS.D.2.5 |
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<tbody>
<tr>
<td>• A-CED is the primary content; other listed content elements may be involved in tasks as well. <strong>HS.D.2-6</strong></td>
<td>Solves multi-step contextual word problems with degree of difficulty appropriate to the course, requiring application of course-level knowledge and skills articulated in A-CED, N-Q.2, A-SSE.3, A-REI.6, A-REI.12, A-REI.11-1, limited to linear and quadratic equations.</td>
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</tr>
</tbody>
</table>
**HS.D.1-1** |
New Jersey Student Learning Standards (s): Algebra
A.REI.D.11: Explain why the x-coordinates of the points where the graphs of the equations \( y = f(x) \) and \( y = g(x) \) intersect are the solutions of the equation \( f(x) = g(x) \); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where \( f(x) \) and/or \( g(x) \) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.* [Focus on linear equations.]

A.REI.B.4: Solve quadratic equations in one variable.

A.REI.B.4a: Use the method of completing the square to transform any quadratic equation in \( x \) into an equation of the form \((x - p)^2 = q\) that has the same solutions. Derive the quadratic formula from this form.

A.REI.B.4b: Solve quadratic equations by inspection (e.g., for \( x^2 = 49 \)), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as \( a \pm bi \) for real numbers \( a \) and \( b \).

A.CED.A.1: Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear functions and quadratic functions, and simple rational and exponential functions.

A.APR.A.1: Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

A.SSE.A.2: Use the structure of an expression to identify ways to rewrite it. *For example, see \( x^4 - y^4 \) as \((x^2)^2 - (y^2)^2\), thus recognizing it as a difference of squares that can be factored as \((x^2 - y^2)(x^2 + y^2)\).

A.SSE.B.3: Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

A.SSE.B.3a: Factor a quadratic expression to reveal the zeros of the function it defines.
A.SSE.B.3b: Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.

9.1.12.B.1: Prioritize financial decisions by systematically considering alternatives and possible consequences.


**Student Learning Objective 6:** Construct chains of reasoning that will justify or refute proposition or conjectures about algebraic equations or systems.

<table>
<thead>
<tr>
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<th>Essential Understandings/Questions (Accountable Talk)</th>
<th>Tasks/Activities</th>
</tr>
</thead>
<tbody>
<tr>
<td>MP 1</td>
<td><strong>HS.C.5.10-1</strong> Given an equation or system of equations, reason about the number or nature of the solutions. Content scope: A-REI.11, limited to equations of the form f(x) = g(x) where f and g are linear or quadratic</td>
<td>Explain the relationship between the x-coordinate of a point of intersection and the solution to the equation f(x) = g(x) for linear equations y = f(x) and y = g(x). Find approximate solutions to the system by making a table of values, graphing, and finding successive approximations.</td>
<td>y = f(x), y = g(x) represent a system of equations. Systems of equations can be solved graphically</td>
<td>DOER HS.C.16.2</td>
</tr>
<tr>
<td>MP 3</td>
<td><strong>HS.C.8.1</strong> Construct, autonomously, chains of reasoning that will justify or refute algebraic propositions or conjectures. Content scope: A-APR.1</td>
<td>Add and subtract polynomials. Multiply polynomials. Recognize numerical expressions as a difference of squares and rewrite the expression as the product of sums/differences.</td>
<td></td>
<td>DOER HS.C.8.3</td>
</tr>
<tr>
<td>MP 5</td>
<td><strong>HS.C.8.3</strong> Construct, autonomously, chains of reasoning that will justify or refute algebraic propositions or conjectures.</td>
<td>Recognize polynomial expressions in one variable as a difference of squares and rewrite the expression as the product of sums/differences.</td>
<td></td>
<td>HS.C.16.2-2</td>
</tr>
<tr>
<td>MP 6</td>
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<td>HS.C.16.2</td>
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<tr>
<td>MP 8</td>
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</table>
conjectures. Content Scope: A-APR.

**HS.C.5.5**
Given an equation or system of equations, reason about the number or nature of the solutions. Content scope: A-REI.4a, A-REI.4b, limited to real solutions only.

**HS.C.16.2**
Given an equation or system of equations, present the solution steps as a logical argument that concludes with the set of solutions (if any). Tasks are limited to quadratic equations. Content scope: A-REI.1, A-REI.4a, A-REI.4b, limited to real solutions only.

**HS.C.5.10-1**
Given an equation or system of equations, reason about the number or nature of the solutions. Content scope: A-REI.11, limited to equations of the form f(x) = g(x) where f and g are linear or quadratic.
New Jersey Student Learning Standard(s) : Functions

A.APR.B.3: Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial. *[Algebra 1: limit to quadratic and cubic functions in which linear and quadratic factors are available]

F.IF.C.9: Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

F.LE.A.3: Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

F.IF.B.6: Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

F.IF.B.4: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

F.IF.B.5: Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.

F.LE.B.5. Interpret the parameters in a linear or exponential function in terms of a context.


9.1.12.E.3: Determine how objective, accurate, and current financial information affects the prioritization of financial decisions.

Student Learning Objective 7: Sketch graphs of linear, radical and exponential functions expressed symbolically or from a verbal description. Show key features and interpret parameters in context. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). Calculate and interpret the average rate of change of a function presented symbolically or as a table; estimate the rate of change from a graph.
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</table>
| MP 7 | **HS.D.1-1**  
Solve multi-step contextual problems with degree of difficulty appropriate to the course, requiring application of knowledge and skills articulated in 7.RP.A, 7.NS.3, 7.EE, and/or 8.EE. | Find the zeros of a polynomial (quadratic and cubic).  
Test domain intervals to determine where f(x) is greater than or less than zero.  
Use zeros of a function to sketch a graph. | How can we determine the general shape(s) and end behavior of functions and identify their main characteristics? | DOER HS.D.2-10  
DOER HS.D.2.8  
ALG1 PBA HS.D.1.1 |
| | **HS.D.2-10**  
Solve multi-step contextual word problems with degree of difficulty appropriate to the course, requiring application of course-level knowledge and skills articulated in F-BF.A, F-BF.3, F-IF.3, ACED.1, A-SSE.3, F-IF.B, F-IF.7. |  |  |  |
| | **HS.D.2-8**  
Solve multi-step contextual word problems with degree of difficulty appropriate to the course, requiring application of course-level knowledge and skills articulated in F-BF.1a, F-BF.3, ACED.1, A-SSE.3, F-IF.B, F-IF.7, limited to linear functions and exponential functions with domains in the integers. |  |  |  |
New Jersey Student Learning Standard(s): Functions

**F.IF.C.9:** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

**F.IF.B.6:** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

**F.IF.B.4:** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*

**9.1.12.B.1:** Prioritize financial decisions by systematically considering alternatives and possible consequences.

**Student Learning Objective 8:** Construct chains of reasoning that will justify or refute propositions or conjectures about functions.

<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>MP 7</td>
<td><strong>HS.C.12.2</strong> Construct, autonomously, chains of reasoning that will justify or refute propositions or conjectures about functions. <strong>HS.C.CCR</strong> Solve multi-step mathematical problems requiring extended chains of reasoning and drawing on a synthesis of the knowledge and skills articulated across: 7-RP.A.3, 7-NS.A.3, 7-</td>
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<tr>
<td></td>
<td>Find the zeros of a polynomial (quadratic and cubic). Test domain intervals to determine where f(x) is greater than or less than zero. Use zeros of a function to sketch a graph. Interpret the parameters in a linear or exponential function in terms of a context.</td>
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<td></td>
<td>How can we determine the general shape(s) and end behavior of functions and identify their main characteristics?</td>
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<td></td>
<td><strong>DOER HS.C.12.2</strong> <strong>ALG 1 PBA HS.C.18.1</strong> <strong>INT MATH 3</strong> <strong>HS.C.CCR</strong> <strong>HS.C.CCR</strong></td>
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</table>

**HS.C.18.1**  
Construct, autonomously, chains of reasoning that will justify or refute propositions or conjectures about linear equations in one or two variables. Content scope: 8.EE.B
New Jersey Student Learning Standard(s): Probability and Statistics

S.ID.B.6: Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.

S.ID.B.6a: Fit a function to the data (including the use of technology); use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.

S.ID.B.6c: Fit a linear function for a scatter plot that suggests a linear association.

S.ID.C.7: Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.

S.ID.C.8: Compute (using technology) and interpret the correlation coefficient of a linear fit.

S.ID.C.9: Distinguish between correlation and causation.

9.1.12.B.1: Prioritize financial decisions by systematically considering alternatives and possible consequences.


9.1.12.E.3: Determine how objective, accurate, and current financial information affects the prioritization of financial decisions.

Student Learning Objective 9: Represent linear and non-linear (quadratic, exponential, and trigonometric) data for two variables on a scatter plot, fit a function to the data, analyze slope, intercept (in case of linear) and residuals (in order to informally assess fit), and use the function to solve problems. Use given functions or choose a function suggested by the context.

<table>
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</thead>
<tbody>
<tr>
<td>MP 3</td>
<td>HS.D.2-13</td>
<td>Create scatter plots for linear and non linear data relationships.</td>
<td>How can we identify correlation between variables given a set of data?</td>
<td>DOER HS.D.2-13</td>
</tr>
<tr>
<td>MP 6</td>
<td></td>
<td>Create functions of best fit.</td>
<td>How does the interpretation of data deepen understanding of relationships?</td>
<td>DOER HS.D.2-13-2</td>
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<tr>
<td>MP 4</td>
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<td>HS.D.2-13</td>
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</table>
If the content is only S-ID, the task must include Algebra 2 / Math 3 content (S-ID.4 or S-ID.6).

- Longer tasks may require some or all of the steps of the modeling cycle (CCSSM, pp. 72, 73); for example, see ITN Appendix F, "Karnataka" task (Section A "Illustrations of innovative task characteristics," subsection 7 "Modeling/Application," subsection f "Full Models"). As in the Karnataka example, algebra and function skills may be used.
- Predictions should not extrapolate far beyond the set of data provided.
- Line of best fit is always based on the equation of the least squares regression line either provided or calculated through the use of technology. Tasks may involve linear, exponential, or quadratic regressions. If the linear regression is in the task, the task must be written to allow students to choose the regression.
- To investigate associations, students may be asked to evaluate scatterplots that may be provided or created using technology. Evaluation includes shape, direction, strength, presence of outliers, and gaps.

| Interpret slope, rate of change, intercepts and key coefficients of the function. |  |  |
- Analysis of residuals may include the identification of a pattern in a residual plot as an indication of a poor fit.

- Models may assess key features of the graph of the fitted model.

- Tasks that involve S-IC.2 might ask the students to look at the results of a simulation and decide how plausible the observed value is with respect to the simulation. For an example, see question 7 on the calculator section of the online practice test (http://practice.parcc.testnav.com/#).

- Tasks that involve S-ID.4, may require finding the area associated with a z-score using technology. Use of a z-score table will not be required. Tasks may involve finding a value at a given percentile based on a normal distribution.
New Jersey Student Learning Standard(s):  Probability and Statistics
S.IC.B.3: Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.

S.IC.A.2: Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?

S.IC.B.5: Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.

S.IC.B.6. Evaluate reports based on data.

9.1.12.B.1: Prioritize financial decisions by systematically considering alternatives and possible consequences.


Student Learning Objective 10: Identify and evaluate random sampling methods. Identify the differences among and purposes of sample surveys, experiments, and observational studies, explaining how randomization relates to each. Use data from a randomized experiment to compare two treatments and use simulations to decide if differences between parameters are significant; evaluate reports based on data.

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</thead>
<tbody>
<tr>
<td>MP 1 MP 3</td>
<td>*HS.C.17.2 Make inferences and justify conclusions from data. Content scope: S-IC. • For tasks that address simple random sample: A simple random sample requires that every possible group of the given sample size has an equal chance of being selected, not that every unit in the population has an equal chance of being selected.</td>
<td>Create scatter plots for linear and non linear data relationships. Create functions of best fit. Interpret slope, rate of change, intercepts and key coefficients of the function.</td>
<td>How can we identify correlation between variables given a set of data?</td>
<td>INT MATH 3 HS.C.17.5</td>
</tr>
</tbody>
</table>
• For tasks that address comparing two data distributions: Comparisons of center, shape, and spread are required.

**HS.C.17.3**
Make inferences and justify conclusions from data. Content scope: S-IC.3.

• For tasks that address simple random sample: A simple random sample requires that every possible group of the given sample size has an equal chance of being selected, not that every unit in the population has an equal chance of being selected.

**HS.C.17.5**
Make inferences and justify conclusions from data. Content scope: S-IC.6.

• Reports should be based on content from S-IC.
• For tasks that address simple random sample: A simple random sample requires that every possible group of the given sample size has an equal chance of being selected, not that every unit in the population has an equal chance of being selected.
For tasks that address comparing two data distributions: Comparisons of center, shape, and spread are required.

**New Jersey Student Learning Standard(s): Geometry**

**G.CO.A.1:** Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

**G.CO.A.2:** Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).

**G.CO.A.3:** Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.

**G.CO.A.4:** Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

**G.CO.A.5:** Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

**G.CO.B.6:** Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.

**G.CO.B.8:** Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.

**G.CO.B.7:** Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.
G.CO.C.9: Prove theorems about lines and angles. *Theorems include:* vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment’s endpoints.

G.CO.C.10: Prove theorems about triangles. *Theorems include:* measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.

G.SRT.C.8: Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

G.GPE.B.6: Find the point on a directed line segment between two given points that partitions the segment in a given ratio.

G.GPE.B.7: Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.

G.GPE.B.4: Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2).*

G.GPE.B.5: Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).


**Student Learning Objective 11:** Perform rigid transformation on various shapes. Calculate perimeter, area and volume of various objects. Use the properties of triangles to find distances of missing segments.

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</thead>
<tbody>
<tr>
<td>MP 7</td>
<td>HS.D.2-2 Solve multi-step contextual problems with degree of difficulty appropriate to the course involving perimeter, area, or volume that require finding an approximate</td>
<td>Find perimeters of polygons using coordinates, the Pythagorean theorem and the distance formula. Perform formal constructions using a variety of tools and methods.</td>
<td>How can we use geometric tools to construct objects and find relations between figures?</td>
<td>DOER HS.D.2.2 INT MATH 2 HS.D.2-11 GEO PBA HS.D.1-2</td>
</tr>
</tbody>
</table>
solution to a polynomial equation using numerical/graphical means.

**HS.D.1-2**
Solve multi-step contextual problems with degree of difficulty appropriate to the course, requiring application of knowledge and skills articulated in 6.G, 7.G, and/or 8.G.

**HS.D.2-11**
Solve multi-step contextual word problems with degree of difficulty appropriate to the course, requiring application of course-level knowledge and skills articulated in G-SRT.8, involving right triangles in an applied setting.

Develop formal mathematical definitions of a rotation, reflection, and translation.

Solve problems using volume formulas for cylinders, pyramids, cones, and spheres.
New Jersey Student Learning Standard(s): Functions

G.CO.A.1: Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

G.CO.A.2: Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).

G.CO.A.3: Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.

G.CO.A.4: Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

G.CO.A.5: Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

G.CO.B.6: Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.

G.CO.B.7: Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.

G.CO.B.8: Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.

G.CO.B.9: Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment’s endpoints.

G.CO.B.10: Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.
G.SRT.C.8: Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

G.GPE.B.6: Find the point on a directed line segment between two given points that partitions the segment in a given ratio.

G.GPE.B.7: Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.

G.GPE.B.4: Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point \( (1, \sqrt{3}) \) lies on the circle centered at the origin and containing the point \( (0, 2) \).*

G.GPE.B.5: Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).


9.1.12.E.3: Determine how objective, accurate, and current financial information affects the prioritization of financial decisions.

Student Learning Objective 12: Construct chains of reasoning that justify or refute propositions or conjectures about geometric figures and relations?

<table>
<thead>
<tr>
<th>MPs</th>
<th>Evidence Statement Key/Clarifications</th>
<th>Skills, Strategies &amp; Concepts</th>
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| MP 7 | *HS.C.13.1 Apply geometric reasoning in a coordinate setting, and/or use coordinates to draw geometric conclusions. Content scope: G-GPE.6, G-GPE.7  
*HS.C.13.2 Apply geometric reasoning in a coordinate setting, and/or use coordinates to draw geometric conclusions. Content scope: G-GPE.4 | Find perimeters of polygons using coordinates, the Pythagorean theorem and the distance formula.  
Perform formal constructions using a variety of tools and methods.  
Develop formal mathematical definitions of a rotation, reflection, and translation. | How can we use geometric tools to construct objects and find relations between figures? | DOER HS.C.14.2  
INT MATH 3 HS.C.13.3  
HS.C.14-2  
HS.C.14.1 |
| *HS.C.13.3 Apply geometric reasoning in a coordinate setting, and/or use coordinates to draw geometric conclusions. Content scope: G-GPE.5 | Solve problems using volume formulas for cylinders, pyramids, cones, and spheres. |
| *HS.C.14.1 Construct, autonomously, chains of reasoning that will justify or refute geometric propositions or conjectures. Content scope: G-CO.9, G-CO.10 | |
| *HS.C.14.2 Construct, autonomously, chains of reasoning that will justify or refute geometric propositions or conjectures. Content scope: G-CO.A, G-CO.B | |
Unit 3 Vocabulary

Average rate of change
Axis of symmetry
Completing the square
Conjugates
Counterexample
Discriminant
Even function
Intercept form
Irrational
Like radicals
Maximum value
Minimum value
Odd function
Parabola
Quadratic equation
Quadratic function
Radical expression
Rational
Rationalizing the denominator
Simplest form
System of nonlinear equations
Vertex form of a quadratic function
Vertex of a parabola
Zero of a function
## References & Suggested Instructional Websites

- [http://illuminations.nctm.org/index](http://illuminations.nctm.org/index)
- [www.illustrativemathematics.org/](www.illustrativemathematics.org/)
- [http://www.njcore.org/](http://www.njcore.org/)
- [https://www.edcite.com/apps/QuestionsLib?subject=math&grades=9,10,11,12&showpub=true](https://www.edcite.com/apps/QuestionsLib?subject=math&grades=9,10,11,12&showpub=true)
- [http://mathforum.org/pow/financialed/](http://mathforum.org/pow/financialed/)
- [https://portal.ct.gov/SDE/Services/K-12-Education](https://portal.ct.gov/SDE/Services/K-12-Education)
- [http://www.bizmove.com/marketing/m2y3.htm](http://www.bizmove.com/marketing/m2y3.htm)
Field Trip Ideas

MUSEUM OF AMERICAN FINANCE (New York, NY) – For more than 20 years, educators from around the country have been bringing students to the Museum to help them understand how finance impacts their daily lives. The Museum offers discounted admission for pre-booked groups of eight or more, as well as a variety of classes for students in middle school through college.

http://www.moaf.org/index

MUSEUM of MATHEMATICS (New York) Mathematics illuminates the patterns that abound in our world. The National Museum of Mathematics strives to enhance public understanding and perception of mathematics. Its dynamic exhibits and programs stimulate inquiry, spark curiosity, and reveal the wonders of mathematics. The Museum’s activities lead a broad and diverse audience to understand the evolving, creative, human, and aesthetic nature of mathematics.

www.momath.org


https://www.newyorkfed.org/aboutthefed/visiting.html