MATHEMATICS

Algebra II: Unit 1
Complex Solutions and Modeling with Rational Exponent
Course Philosophy/Description

Algebra II continues the students’ study of advanced algebraic concepts including functions, polynomials, rational expressions, systems of functions and inequalities, and matrices. Students will be expected to describe and translate among graphic, algebraic, numeric, tabular, and verbal representations of relations and use those representations to solve problems. Emphasis will be placed on practical applications and modeling. Students extend their knowledge and understanding by solving open-ended real-world problems and thinking critically through the use of high level tasks.

Students will be expected to demonstrate their knowledge in: utilizing essential algebraic concepts to perform calculations on polynomial expression; performing operations with complex numbers and graphing complex numbers; solving and graphing linear equations/inequalities and systems of linear equations/inequalities; solving, graphing, and interpreting the solutions of quadratic functions; solving, graphing, and analyzing solutions of polynomial functions, including complex solutions; manipulating rational expressions, solving rational equations, and graphing rational functions; solving logarithmic and exponential equations; and performing operations on matrices and solving matrix equations.
ESL Framework

This ESL framework was designed to be used by bilingual, dual language, ESL and general education teachers. Bilingual and dual language programs use the home language and a second language for instruction. ESL teachers and general education or bilingual teachers may use this document to collaborate on unit and lesson planning to decide who will address certain components of the SLO and language objective. ESL teachers may use the appropriate leveled language objective to build lessons for ELLs which reflects what is covered in the general education program. In this way, whether it is a pull-out or push-in model, all teachers are working on the same Student Learning Objective connected to the Common Core standard. The design of language objectives are based on the alignment of the World-Class Instructional Design Assessment (WIDA) Consortium’s English Language Development (ELD) standards with the Common Core State Standards (CCSS). WIDA’s ELD standards advance academic language development across content areas ultimately leading to academic achievement for English learners. As English learners are progressing through the six developmental linguistic stages, this framework will assist all teachers who work with English learners to appropriately identify the language needed to meet the requirements of the content standard. At the same time, the language objectives recognize the cognitive demand required to complete educational tasks. Even though listening and reading (receptive) skills differ from speaking and writing (expressive) skills across proficiency levels the cognitive function should not be diminished. For example, an Entering Level One student only has the linguistic ability to respond in single words in English with significant support from their home language. However, they could complete a Venn diagram with single words which demonstrates that they understand how the elements compare and contrast with each other or they could respond with the support of their native language with assistance from a teacher, para-professional, peer or a technology program.

http://www.state.nj.us/education/modelcurriculum/ela/ELLOverview.pdf
<table>
<thead>
<tr>
<th>#</th>
<th>Student Learning Objective</th>
<th>NJSLS</th>
<th>Big Ideas Math Correlation</th>
<th>Instruction: 8 weeks Assessment: 1 week</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Add, subtract, and multiply complex numbers using the commutative, associative and distributive properties.</td>
<td>N.CN.A.1, N.CN.A.2</td>
<td>3.2</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Solve quadratic equations with real coefficients that have complex solutions by taking square roots, completing the square and factoring.</td>
<td>N.CN.C.7, A.REI.B.4</td>
<td>3.1, 3.2, 3.3, 3.4</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Solve simple systems consisting of a linear and quadratic equation in two variables algebraically and graphically.</td>
<td>A.REI.C.7</td>
<td>3.5</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Solve algebraically a system of three linear equations.</td>
<td>A.REI.C.6</td>
<td>1.4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.</td>
<td>F.BF.A.2, F.LE.A.2, F.LE.B.5</td>
<td>1.3, 6.1, 6.2, 6.7, 8.2, 8.3, 8.5</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Use the formula for the sum of a finite geometric series to solve problems [for example, calculate mortgage payments; derive the formula for the sum of a finite geometric series (when the common ratio is not 1)].</td>
<td>A.SSE.B.4</td>
<td>8.3, 8.4</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Use properties of integer exponents to explain and convert between expressions involving radicals and rational exponents.</td>
<td>N.RN.A.1, N.RN.A.2</td>
<td>5.1, 5.2</td>
<td></td>
</tr>
</tbody>
</table>
| 8 | Use the properties of exponents to transform expressions for exponential functions, explain properties of the quantity revealed in the transformed expression or different properties of the function. | A.SSE.B.3  
F.IF.C.8 | 6.1 |
<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>9</td>
<td>Express as a logarithm the solution to $ab^c = d$ where $a$, $c$, and $d$ are numbers and the base $b$ is 2, 10, or $e$; evaluate the logarithm using technology.</td>
<td>F.LE.A.4</td>
<td>6.3, 6.5, 6.6</td>
</tr>
</tbody>
</table>
Research about Teaching and Learning Mathematics

Structure teaching of mathematical concepts and skills around problems to be solved (Checkly, 1997; Wood & Sellars, 1996; Wood & Sellars, 1997)
Encourage students to work cooperatively with others (Johnson & Johnson, 1975; Davidson, 1990)
Use group problem-solving to stimulate students to apply their mathematical thinking skills (Artzt & Armour-Thomas, 1992)
Students interact in ways that support and challenge one another’s strategic thinking (Artzt, Armour-Thomas, & Curcio, 2008)
Activities structured in ways allowing students to explore, explain, extend, and evaluate their progress (National Research Council, 1999)

There are three critical components to effective mathematics instruction (Shellard & Moyer, 2002):
- Teaching for balanced mathematical understanding
- Developing children’s procedural literacy
- Promoting strategic competence through meaningful problem-solving investigations

Teachers should be:
- Demonstrating acceptance and recognition of students’ divergent ideas.
- Challenging students to think deeply about the problems they are solving, extending thinking beyond the solutions and algorithms required to solve the problem
- Influencing learning by asking challenging and interesting questions to accelerate students’ innate inquisitiveness and foster them to examine concepts further
- Projecting a positive attitude about mathematics and about students’ ability to “do” mathematics

Students should be:
- Actively engaging in “doing” mathematics
- Solving challenging problems
- Investigating meaningful real-world problems
- Making interdisciplinary connections
- Developing an understanding of mathematical knowledge required to “do” mathematics and connect the language of mathematical ideas with numerical representations
- Sharing mathematical ideas, discussing mathematics with one another, refining and critiquing each other’s ideas and understandings
- Communicating in pairs, small group, or whole group presentations
- Using multiple representations to communicate mathematical ideas
- Using connections between pictures, oral language, written symbols, manipulative models, and real-world situations
- Using technological resources and other 21st century skills to support and enhance mathematical understanding
Mathematics is not a stagnant field of textbook problems; rather, it is a dynamic way of constructing meaning about the world around us, generating knowledge and understanding about the real world every day. Students should be metaphorically rolling up their sleeves and “doing mathematics” themselves, not watching others do mathematics for them or in front of them. (Protheroe, 2007)

Balanced Mathematics Instructional Model

Balanced math consists of three different learning opportunities; guided math, shared math, and independent math. Ensuring a balance of all three approaches will build conceptual understanding, problem solving, computational fluency, and procedural fluency. Building a balanced mathematical understanding is the focal point of developing mathematical proficiency. Students should frequently work on rigorous tasks, talk about the math, explain their thinking, justify their answer or process, build models with graphs or charts or manipulatives, and use technology.

When balanced math is used in the classroom it provides students opportunities to:

- solve problems
- make connections between math concepts and real-life situations
- communicate mathematical ideas (orally, visually and in writing)
- choose appropriate materials to solve problems
- reflect and monitor their own understanding of the math concepts
- practice strategies to build procedural and conceptual confidence

Teacher builds conceptual understanding by modeling through demonstration, explicit instruction, and think alouds, as well as guiding students as they practice math strategies and apply problem solving strategies. (whole group or small group instruction)

Teacher and students practice mathematics processes together through interactive activities, problem solving, and discussion. (whole group or small group instruction)

Students practice math strategies independently to build procedural and computational fluency. Teacher assesses learning and reteaches as necessary. (whole group instruction, small group instruction, or centers)
## Effective Pedagogical Routines/Instructional Strategies

<table>
<thead>
<tr>
<th>Collaborative Problem Solving</th>
<th>Analyze Student Work</th>
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<tbody>
<tr>
<td>Connect Previous Knowledge to New Learning</td>
<td>Identify Student’s Mathematical Understanding</td>
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<tr>
<td>Making Thinking Visible</td>
<td>Identify Student’s Mathematical Misunderstandings</td>
</tr>
<tr>
<td>Develop and Demonstrate Mathematical Practices</td>
<td>Interviews</td>
</tr>
<tr>
<td>Inquiry-Oriented and Exploratory Approach</td>
<td>Role Playing</td>
</tr>
<tr>
<td>Multiple Solution Paths and Strategies</td>
<td>Diagrams, Charts, Tables, and Graphs</td>
</tr>
<tr>
<td>Use of Multiple Representations</td>
<td>Anticipate Likely and Possible Student Responses</td>
</tr>
<tr>
<td>Explain the Rationale of your Math Work</td>
<td>Collect Different Student Approaches</td>
</tr>
<tr>
<td>Quick Writes</td>
<td>Multiple Response Strategies</td>
</tr>
<tr>
<td>Pair/Trio Sharing</td>
<td>Asking Assessing and Advancing Questions</td>
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<tr>
<td>Turn and Talk</td>
<td>Revoicing</td>
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<tr>
<td>Charting</td>
<td>Marking</td>
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<tr>
<td>Gallery Walks</td>
<td>Recapping</td>
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<tr>
<td>Small Group and Whole Class Discussions</td>
<td>Challenging</td>
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<tr>
<td>Student Modeling</td>
<td>Pressing for Accuracy and Reasoning</td>
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<td></td>
<td>Maintain the Cognitive Demand</td>
</tr>
</tbody>
</table>
## Educational Technology

### Standards

| 8.1.8.A.1, 8.1.8.A.5, 8.1.8.D.1, 8.1.8.E.1, 8.2.8.B.1 |

- **Technology Operations and Concepts**
  - Create a personal digital portfolio which reflects personal and academic interests, achievements, and career aspirations by using a variety of digital tools and resources.
  - Select and use appropriate tools and digital resources to accomplish a variety of tasks and to solve problems.
  
  **Example:** Students create personal digital portfolios for coursework using Google Sites

- **Communication and Collaboration**
  - Develop an innovative solution to a real world problem or issue in collaboration with peers and experts, and present ideas for feedback through social media or in an online community.
  
  **Example:** Use Google Classroom for real-time communication between teachers, students, and peers to complete assignments and discuss strategies for solving systems of equations.

- **Critical Thinking, Problem Solving, and Decision Making**
  - Evaluate the strengths and limitations of emerging technologies and their impact on educational, career, personal or social needs.
  
  **Example:** Students use graphing calculators and graph paper to reveal the strengths and weaknesses of technology associated with solving simple systems of linear and quadratic equations in two variables.

**Link:**
- [http://www.state.nj.us/education/cccs/2014/tech/](http://www.state.nj.us/education/cccs/2014/tech/)
- [http://www.khanacademy.org/math](http://www.khanacademy.org/math)
- [http://www.desmos.com/calculator](http://www.desmos.com/calculator)
Career Ready Practices

Career Ready Practices describe the career-ready skills that all educators in all content areas should seek to develop in their students. They are practices that have been linked to increase college, career, and life success. Career Ready Practices should be taught and reinforced in all career exploration and preparation programs with increasingly higher levels of complexity and expectation as a student advances through a program of study.

- **CRP2. Apply appropriate academic and technical skills.**
  Career-ready individuals readily access and use the knowledge and skills acquired through experience and education to be more productive. They make connections between abstract concepts with real-world applications, and they make correct insights about when it is appropriate to apply the use of an academic skill in a workplace situation.
  
  **Example:** Students will apply prior knowledge when solving real world problems. Students will make sound judgments about the use of specific tools, such as algebra tiles, graphing calculators and technology to deepen their understanding of solving quadric equations.

- **CRP4. Communicate clearly and effectively and with reason.**
  Career-ready individuals communicate thoughts, ideas, and action plans with clarity, whether using written, verbal, and/or visual methods. They communicate in the workplace with clarity and purpose to make maximum use of their own and others’ time. They are excellent writers; they master conventions, word choice, and organization, and use effective tone and presentation skills to articulate ideas. They are skilled at interacting with others; they are active listeners and speak clearly and with purpose. Career-ready individuals think about the audience for their communication and prepare accordingly to ensure the desired outcome.
  
  **Example:** Students will communicate precisely using clear definitions and provide carefully formulated explanations when constructing arguments. Students will communicate and defend mathematical reasoning using objects, drawings, diagrams, and/or actions. Students will ask probing questions to clarify or improve arguments.
## Career Ready Practices

- **CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.**
  Career-ready individuals readily recognize problems in the workplace, understand the nature of the problem, and devise effective plans to solve the problem. They are aware of problems when they occur and take action quickly to address the problem; they thoughtfully investigate the root cause of the problem prior to introducing solutions. They carefully consider the options to solve the problem. Once a solution is agreed upon, they follow through to ensure the problem is solved, whether through their own actions or the actions of others.  
  **Example:** Students will understand the meaning of a problem and look for entry points to its solution. They will analyze information, make conjectures, and plan a solution pathway to solve linear and quadratic equations in two variables.

- **CRP12. Work productively in teams while using cultural global competence.**
  Career-ready individuals positively contribute to every team, whether formal or informal. They apply an awareness of cultural difference to avoid barriers to productive and positive interaction. They find ways to increase the engagement and contribution of all team members. They plan and facilitate effective team meetings.  
  **Example:** Students will work collaboratively in groups to solve mathematical tasks. Students will listen to or read the arguments of others and ask probing questions to clarify or improve arguments. They will be able to explain how to perform operations with complex numbers.

### WIDA Proficiency Levels

At the given level of English language proficiency, English language learners will process, understand, produce or use:

<table>
<thead>
<tr>
<th>Level</th>
<th>Description</th>
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</table>
| 6- Reaching | - Specialized or technical language reflective of the content areas at grade level  
- A variety of sentence lengths of varying linguistic complexity in extended oral or written discourse as required by the specified grade level  
- Oral or written communication in English comparable to proficient English peers |
| 5- Bridging | - Specialized or technical language of the content areas  
- A variety of sentence lengths of varying linguistic complexity in extended oral or written discourse, including stories, essays or reports  
- Oral or written language approaching comparability to that of proficient English peers when presented with grade level material. |
| 4- Expanding | - Specific and some technical language of the content areas  
- A variety of sentence lengths of varying linguistic complexity in oral discourse or multiple, related sentences or paragraphs  
- Oral or written language with minimal phonological, syntactic or semantic errors that may impede the communication, but retain much of its meaning, when presented with oral or written connected discourse, with sensory, graphic or interactive support |
| 3- Developing | - General and some specific language of the content areas  
- Expanded sentences in oral interaction or written paragraphs  
- Oral or written language with phonological, syntactic or semantic errors that may impede the communication, but retain much of its meaning, when presented with oral or written, narrative or expository descriptions with sensory, graphic or interactive support |
| 2- Beginning | - General language related to the content area  
- Phrases or short sentences  
- Oral or written language with phonological, syntactic, or semantic errors that often impede the communication when presented with one to multiple-step commands, directions, or a series of statements with sensory, graphic or interactive support |
| 1- Entering | - Pictorial or graphic representation of the language of the content areas  
- Words, phrases or chunks of language when presented with one-step commands directions, WH-, choice or yes/no questions, or statements with sensory, graphic or interactive support |
# Language Development Supports for English Language Learners

## To Increase Comprehension and Communication Skills

### Environment

- Welcoming and stress-free
- Respectful of linguistic and cultural diversity
- Honors students’ background knowledge
- Sets clear and high expectations
- Includes routines and norms
- Is thinking-focused vs. answer-seeking
- Offers multiple modalities to engage in content learning and to demonstrate understanding
- Includes explicit instruction of specific language targets
- Provides participation techniques to include all learners
- Integrates learning centers and games in a meaningful way
- Provides opportunities to practice and refine receptive and productive skills in English as a new language
- Integrates meaning and purposeful tasks/activities that:
  - Are accessible by all students through multiple entry points
  - Build on prior mathematical learning
  - Demonstrate high cognitive demand
  - Offer multiple strategies for solutions
  - Allow for a language learning experience in addition to content

### Sensory Supports*

- Real-life objects (realia) or concrete objects
- Physical models
- Manipulatives
- Pictures & photographs
- Visual representations or models such as diagrams or drawings
- Videos & films
- Newspapers or magazines
- Gestures
- Physical movements
- Music & songs

### Graphic Supports*

- Graphs
- Charts
- Timelines
- Number lines
- Graphic organizers
- Graphing paper

### Interactive Supports*

- In a whole group
- In a small group
- With a partner such as Turn-and-Talk
- In pairs as a group (first, two pairs work independently, then they form a group of four)
- In triads
- Cooperative learning structures such as Think-Pair-Share
- Interactive websites or software
- With a mentor or coach

### Verbal and Textual Supports

- Labeling
- Students’ native language
- Modeling
- Repetitions
- Paraphrasing
- Summarizing
- Guiding questions
- Clarifying questions
- Probing questions
- Leveled questions such as What? When? Where? How? Why?
- Questioning prompts & cues
- Word Banks
- Sentence starters
- Sentence frames
- Discussion frames
- Talk moves, including Wait Time

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*Galina (Halla) Jmourko, ESOL Coach, PGCPS; 2015, Rvst. 2016*
BUILDING EQUITY IN YOUR TEACHING PRACTICE

How do the essential questions highlight the connection between the big ideas of the unit and equity in your teaching practice?

**CONTENT INTEGRATION**
Teachers use examples and content from a variety of cultures & groups.

This unit / lesson is connected to other topics explored with students.

There are multiple viewpoints reflected in the content of this unit / lesson.

The materials and resources are reflective of the diverse identities and experiences of students.

The content affirms students, as well as exposes them to experiences other than their own.

**KNOWLEDGE CONSTRUCTION**
Teachers help students understand how knowledge is created and influenced by cultural assumptions, perspectives & biases.

This unit / lesson provides context to the history of privilege and oppression.

This unit / lesson addresses power relationships.

This unit / lesson help students to develop research and critical thinking skills.

This curriculum creates windows and mirrors* for students.

**PREJUDICE REDUCTION**
Teachers implement lessons and activities to assert positive images of ethnic groups & improve intergroup relations.

This unit / lesson help students question and unpack biases & stereotypes.

This unit / lesson help students examine, research and question information and sources.

The curriculum encourage discussion and understanding about the groups of people being represented.

This unit / lesson challenges dominant perspectives.

**EQUITABLE PEDAGOGY**
The instruction has been modified to meet the needs of each student.

Students feel respected and their cultural identities are valued.

Additional supports have been provided for students to become successful and independent learners.

Opportunities are provided for student to reflect on their learning and provide feedback.

**EMPOWERING SCHOOL CULTURE**
Using the other four dimensions to create a safe and healthy educational environment for all.

There are opportunities for students to connect with the community.

My classroom is welcoming and supportive for all students?

I am aware of and sensitive to the needs of my students and their families.

There are effective parent communication systems established. Parents can talk to me about issues as they arise in my classroom.

Culturally Relevant Pedagogy Examples

- **Integrate Relevant Word Problems**: Contextualize equations using word problems that reference student interests and cultures.
  
  **Example**: When learning about building functions in two variables, problems that relate to student interests such as music, sports and art enable the students to understand and relate to the concept in a more meaningful way.

- **Everyone has a Voice**: Create a classroom environment where students know that their contributions are expected and valued.
  
  **Example**: Norms for sharing are established that communicate a growth mindset for mathematics. All students are capable of expressing mathematical thinking and contributing to the classroom community. Students learn new ways of looking at problem solving by working with and listening to each other.

- **Run Problem Based Learning Scenarios**: Encourage mathematical discourse among students by presenting problems that are relevant to them, the school and/or the community.
  
  **Example**: Using a Place Based Education (PBE) model, students explore math concepts such as systems of equations while determining ways to address problems that are pertinent to their neighborhood, school or culture.

- **Encourage Student Leadership**: Create an avenue for students to propose problem solving strategies and potential projects.
  
  **Example**: Students can learn to construct and compare linear, quadratic and exponential models by creating problems together and deciding if the problems fit the necessary criteria. This experience will allow students to discuss and explore their current level of understanding by applying the concepts to relevant real-life experiences.

- **Present New Concepts Using Student Vocabulary**: Use student diction to capture attention and build understanding before using academic terms.
  
  **Example**: Teach math vocabulary in various modalities for students to remember. Use multi-modal activities, analogies, realia, visual cues, graphic representations, gestures, pictures and cognates. Directly explain and model the idea of vocabulary words having multiple meanings. Students can create the Word Wall with their definitions and examples to foster ownership.
## Differentiated Instruction

Accommodate Based on Students Individual Needs: Strategies

<table>
<thead>
<tr>
<th>Time/General</th>
<th>Processing</th>
<th>Comprehension</th>
<th>Recall</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Extra time for assigned tasks</td>
<td>• Extra Response time</td>
<td>• Precise processes for balanced mathematics instructional model</td>
<td></td>
</tr>
<tr>
<td>• Adjust length of assignment</td>
<td>• Have students verbalize steps</td>
<td>• Short manageable tasks</td>
<td>• Teacher-made checklist</td>
</tr>
<tr>
<td>• Timeline with due dates for reports and projects</td>
<td>• Repeat, clarify or reword directions</td>
<td>• Brief and concrete directions</td>
<td>• Use visual graphic organizers</td>
</tr>
<tr>
<td>• Communication system between home and school</td>
<td>• Mini-breaks between tasks</td>
<td>• Provide immediate feedback</td>
<td>• Reference resources to promote independence</td>
</tr>
<tr>
<td>• Provide lecture notes/outline</td>
<td>• Provide a warning for transitions</td>
<td>• Small group instruction</td>
<td>• Visual and verbal reminders</td>
</tr>
<tr>
<td></td>
<td>• Partnering</td>
<td>• Emphasize multi-sensory learning</td>
<td>• Graphic organizers</td>
</tr>
</tbody>
</table>

### Assistive Technology
- Computer/whiteboard
- Tape recorder
- Video Tape

### Tests/Quizzes/Grading
- Extended time
- Study guides
- Shortened tests
- Read directions aloud

### Behavior/Attention
- Consistent daily structured routine
- Simple and clear classroom rules
- Frequent feedback

### Organization
- Individual daily planner
- Display a written agenda
- Note-taking assistance
- Color code materials
<table>
<thead>
<tr>
<th>Differentiated Instruction</th>
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<tbody>
<tr>
<td><strong>Accommodate Based on Content Needs: Strategies</strong></td>
</tr>
</tbody>
</table>

- Anchor charts to model strategies for finding the length of the arc of a circle
- Review Algebra concepts to ensure students have the information needed to progress in understanding
- Pre-teach pertinent vocabulary
- Provide reference sheets that list formulas, step-by-step procedures, theorems, and modeling of strategies
- Word wall with visual representations of mathematical terms
- Teacher modeling of thinking processes involved in solving, graphing, and writing equations
- Introduce concepts embedded in real-life context to help students relate to the mathematics involved
- Record formulas, processes, and mathematical rules in reference notebooks
- Graphing calculator to assist with computations and graphing of trigonometric functions
- Utilize technology through interactive sites to represent nonlinear data
- Graphic organizers to help students interpret the meaning of terms in an expression or equation in context
- Translation dictionary
- Sentence stems to provide additional language support for ELL students.
**Interdisciplinary Connections**

*Model interdisciplinary thinking to expose students to other disciplines.*

<table>
<thead>
<tr>
<th><strong>Social Studies Connection:</strong></th>
<th>Social Studies Standard 6.2.12.B.1.a; 6.2.12.B.1.b</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Name of Task:</strong></td>
<td>Carbon 14 Dating</td>
</tr>
<tr>
<td></td>
<td>The task requires the student to use logarithms to solve an exponential equation in the realistic context of carbon dating, important in archaeology and geology, among other places</td>
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</table>

<table>
<thead>
<tr>
<th><strong>Science Connection:</strong></th>
<th>Science Standard HS-PS-1-6</th>
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<tbody>
<tr>
<td><strong>Name of Task:</strong></td>
<td>Course of Antibiotics</td>
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<tr>
<td></td>
<td>This task presents a real-world application of finite geometric series. The context can lead into several interesting follow-up questions and projects. Many drugs only become effective after the amount in the body builds up to a certain level. This can be modeled very well with geometric series.</td>
</tr>
</tbody>
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## Enrichment

### What is the purpose of Enrichment?

- The purpose of enrichment is to provide extended learning opportunities and challenges to students who have already mastered, or can quickly master, the basic curriculum. Enrichment gives the student more time to study concepts with greater depth, breadth, and complexity.
- Enrichment also provides opportunities for students to pursue learning in their own areas of interest and strengths.
- Enrichment keeps advanced students engaged and supports their accelerated academic needs.
- Enrichment provides the most appropriate answer to the question, “What do you do when the student already knows it?”

### Enrichment is...

- Planned and purposeful
- *Different,* or differentiated, work – not just *more* work
- Responsive to students’ needs and situations
- A promotion of high-level thinking skills and making connections within content
- The ability to apply different or multiple strategies to the content
- The ability to synthesize concepts and make real world and cross-curricular connections.
- Elevated contextual complexity
- Sometimes independent activities, sometimes direct instruction
- Inquiry based or open-ended assignments and projects
- Using supplementary materials in addition to the normal range of resources.
- Choices for students
- Tiered/Multi-level activities with flexible groups (may change daily or weekly)

### Enrichment is not...

- Just for gifted students (some gifted students may need intervention in some areas just as some other students may need frequent enrichment)
- Worksheets that are more of the same (busywork)
- Random assignments, games, or puzzles not connected to the content areas or areas of student interest
- Extra homework
- A package that is the same for everyone
- Thinking skills taught in isolation
- Unstructured free time
# Assessments

## Required District/State Assessments
- Unit Assessment
- PARCC
- SGO Assessments

## Suggested Formative/Summative Classroom Assessments
- Describe Learning Vertically
- Identify Key Building Blocks
- Make Connections (between and among key building blocks)
- Short/Extended Constructed Response Items
- Multiple-Choice Items (where multiple answer choices may be correct)
- Drag and Drop Items
- Use of Equation Editor
- Quizzes
- Journal Entries/Reflections/Quick-Writes
- Accountable talk
- Projects
- Portfolio
- Observation
- Graphic Organizers/Concept Mapping
- Presentations
- Role Playing
- Teacher-Student and Student-Student Conferencing
- Homework
# New Jersey Student Learning Standards

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<thead>
<tr>
<th>Standard</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>N.CN.A.1.</td>
<td>Know there is a complex number $i$ such that $i^2 = -1$, and every complex number has the form $a + bi$ with $a$ and $b$ real.</td>
</tr>
<tr>
<td>N.CN.A.2.</td>
<td>Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers</td>
</tr>
<tr>
<td>N.CN.C.7.</td>
<td>Solve quadratic equations with real coefficients that have complex solutions.</td>
</tr>
<tr>
<td>A.REI.B.4.</td>
<td>Solve quadratic equations in one variable.</td>
</tr>
<tr>
<td>A.REI.B.4b.</td>
<td>Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers $a$ and $b$.</td>
</tr>
<tr>
<td>A.REI.C.7.</td>
<td>Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$.</td>
</tr>
<tr>
<td>A.REI.C.6.</td>
<td>Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.</td>
</tr>
<tr>
<td>F.BF.A.2.</td>
<td>Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.</td>
</tr>
<tr>
<td>F.LE.A.2</td>
<td>Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).</td>
</tr>
</tbody>
</table>
New Jersey Student Learning Standards

F.LE.B.5. Interpret the parameters in a linear or exponential function in terms of a context.

A.SSE.B.4. Derive and/or explain the derivation of the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. For example, calculate mortgage payments.

N.RN.A.1. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. For example, we define \(5^{1/3}\) to be the cube root of 5 because we want \((5^{1/3})^3 = 5\) to hold, so \((5^{1/3})^3\) must equal 5.

N.RN.A.2. Rewrite expressions involving radicals and rational exponents using the properties of exponents.

A.SSE.B.3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression

A.SSE.B.3c: Use the properties of exponents to transform expressions for exponential functions. For example the expression \(1.15^t\) can be rewritten as \((1.15^{1/12})^{12t} \approx 1.012^{12t}\) to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.

F.IF.C.8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function

F.IF.C.8b: Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as \(y = (1.02)^t, y = (0.97)^t, y = (1.01)^{12t}, y = (1.2)^{10},\) and classify them as representing exponential growth or decay.

F.LE.A.4. Understand the inverse relationship between exponents and logarithms. For exponential models, express as a logarithm the solution to \(ab^c = d\) where \(a, c,\) and \(d\) are numbers and the base \(b\) is 2, 10, or \(e\); evaluate the logarithm using technology.
Mathematical Practices

1. Make sense of problems and persevere in solving them.

2. Reason abstractly and quantitatively.

3. Construct viable arguments and critique the reasoning of others.

4. Model with mathematics.

5. Use appropriate tools strategically.

6. Attend to precision.

7. Look for and make use of structure.

8. Look for and express regularity in repeated reasoning.
<table>
<thead>
<tr>
<th>Course: Algebra II</th>
<th>Unit: 1 (One)</th>
<th>Topic: Complex Solutions and Modeling with Rational Exponents</th>
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**Unit Focus:**
- Perform arithmetic operations with complex numbers
- Use complex numbers in polynomial identities and equations
- Build a function that models a relationship between two quantities
- Construct & compare linear, quadratic, & exponential models
- Write expressions in equivalent forms to solve problems
- Extend the properties of exponents to rational exponents
- Analyze functions using different representations

**New Jersey Student Learning Standard(s):**

N.CN.A.1: Know there is a complex number \( i \) such that \( i^2 = -1 \), and every complex number has the form \( a + bi \) with \( a \) and \( b \) real.

N.CN.A.2: Use the relation \( i^2 = -1 \) and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers

**Student Learning Objective 1:** Add, subtract, and multiply complex numbers using the commutative, associative and distributive properties.

**Modified Student Learning Objectives/Standards:**

M.EE.N-CN.2.a. Use the commutative, associative, and distributive properties to add, subtract, and multiply whole numbers.
M.EE.N-CN.2.b. Solve real-world problems involving addition and subtraction of decimals, using models when needed.
M.EE.N-CN.2.c. Solve real-world problems involving multiplication of decimals and whole numbers, using models when needed.

<table>
<thead>
<tr>
<th>MP 6</th>
<th>Evidence Statement Key/ Clarifications</th>
<th>Skills, Strategies &amp; Concepts</th>
<th>Essential Understandings/ Questions (Accountable Talk)</th>
<th>Tasks/Activities</th>
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<tbody>
<tr>
<td>MP 7</td>
<td>N-CN.1</td>
<td>Complex number ( i ) is defined such that ( i^2 = -1 ).</td>
<td>What are the subsets of the set of complex numbers?</td>
<td>Type I: Computations with Complex Numbers</td>
</tr>
</tbody>
</table>
Every complex number has the form $a + bi$ with $a$ and $b$ real.

$i^2 = -1$ and the commutative, associative properties to add and subtract complex numbers are to be used.

Determine that $i^2 = -1$ and the commutative, associative, and distributive properties to multiply complex numbers.

**SPED Strategies:**
Relate the idea of adding, subtracting and multiplying complex numbers to whole numbers.

Explain the background of complex numbers and connect to real life by explaining how they are used in electrical circuits.

Use the example of the cyclical nature of the ones digit in the powers of 3 and connect it to the cyclical nature of the powers of $i$.

Develop a reference document with students with verbal and pictorial descriptions.

**ELL Strategies:**
Describe and explain orally and in writing how to use properties of operations to add, subtract, and multiply complex numbers in the student’s native language and/or use selected technical vocabulary in phrases and sentences.

| How are the powers of $i$ derived and how are they cyclic?
| Why is it when simplifying imaginary and complex numbers that the highest power of $i$ is one?
| Type II, III:
| Complex number patterns
| Powers of a complex number
| Complex Square Roots
| Additional Tasks: Complex Cube and Fourth Root of 1
| Vertex of a parabola with complex roots |
short sentences with equations to explain the solution.

Encourage students to highlight like terms - use one color for the real parts and another color for the imaginary parts.

Make a table with the words: Imaginary Unit, Complex Number and Imaginary Number, then write an example for each word in the column.

New Jersey Student Learning Standard(s):
N.CN.C.7: Solve quadratic equations with real coefficients that have complex solutions.

A.REI.B.4: Solve quadratic equations in one variable.

A.REI.B.4b: Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers $a$ and $b$.

Student Learning Objective 2: Solve quadratic equations with real coefficients that have complex solutions by taking square roots, completing the square and factoring.

Modified Student Learning Objectives/Standards: N/A

<table>
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<tr>
<th>MPs</th>
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<tbody>
<tr>
<td>MP 5 MP 7</td>
<td>- N.CN.C.7 - Tasks are limited to equations with non-real solutions. - A-REI.4b-2</td>
<td>- As with real solutions, complex solutions to quadratic equations may be determined by taking square roots, factoring, and completing the square.</td>
<td>- What are the subsets of the set of complex numbers? - How can you complete the square for a quadratic expression?</td>
<td>- Type II, III: Two Squares are Equal - Braking Distance - Additional Tasks:</td>
</tr>
</tbody>
</table>
• Writing solutions in the form $a \pm bi$ is not assessed here (assessed under N-CN.7).

Solve quadratic equations using the square root method.

Solve quadratic equations by factoring and using the zero product property.

Solve quadratic equations in one variable that have complex solutions by taking square roots.

Solve a quadratic equation in one variable that have complex solutions by completing the square.

Solve quadratic equations in one variable that have complex solutions by factoring.

Write complex solutions in $a \pm bi$ form.

**SPED Strategies**
Model the thinking and procedure involved in solving a quadratic equation with complex solutions.

Provide students with a graphic organizer that outlines the possible solution paths, formulas and sample problems to facilitate independence.

Encourage students to verbalize their thinking while solving quadratic equations.

How can you derive a general formula for solving a quadratic equation?

How can you determine whether a polynomial equation has imaginary solutions?

How do you determine which method is best for solving a quadratic equation?

Why do some quadratic equations have extraneous and/or complex solutions?

Why is it when simplifying imaginary and complex numbers that the highest power of $i$ is one?

How does the concept of the zero product property allow you to find the roots of a quadratic function?

**Completing the square**

**Springboard Dive**

**Vertex of a parabola with complex roots**

**Visualizing completing the square**

**Zero Property 4**
equations by asking assessing and advancing questions.

**ELL Strategies:**
Describe and explain how to solve quadratic equations with real coefficients that have complex solutions using in the student’s native language and/or use selected technical vocabulary in phrases and short sentences with equations to explain the solution.

Use a Venn diagram to represent the solution of a quadratic equation that expresses the relationship between Complex numbers and real numbers.

Let students discuss the possible form of the solution of quadratic equations. Indicate if students can obtain a real solution or an imaginary solution.
New Jersey Student Learning Standard(s):
A.REI.C.7: Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$.

**Student Learning Objective 3:** Solve simple systems consisting of a linear and quadratic equation in two variables algebraically and graphically.

Modified Student Learning Objectives/Standards:
M.EE.A-REI.10–12: Interpret the meaning of a point on the graph of a line. For example, on a graph of pizza purchases, trace the graph to a point and tell the number of pizzas purchased and the total cost of the pizzas.

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<tr>
<th>MPs</th>
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<tbody>
<tr>
<td>MP 1</td>
<td>A.REI.C.7 - Tasks have thin context or no context.</td>
<td>Solutions of linear systems contain different function types. Solve a system containing one linear equation and one quadratic equation algebraically. Graph a system containing one linear equation and one quadratic equation to determine a solution. <strong>SPED Strategies</strong> Link the concept of solving a system of equations with one linear and one quadratic equation to solving a system of linear equations. Model the thinking and processes necessary to decide on a solution path and solve a system with one linear equation and one quadratic equation accurately.</td>
<td>How can you solve a nonlinear system of equations? How can you solve a system of two equations when one is linear and the other is quadratic? How do you determine the number of solutions that a system of equations will have? Why does graphing a system of equation yield an approximate solution as opposed to an exact solution?</td>
<td>Type II, III: The Circle and The Line Pythagorean Triples Linear and Quadratic</td>
</tr>
</tbody>
</table>
Provide students with reference sheets/notes to encourage confidence and independence.

**ELL Strategies:**
Demonstrate understanding of solving systems of linear and quadratic equations; then explain orally how to solve the equations in two variables in the student’s native language and/or use gestures, equations and selected, technical words.

Create an outline that allows students to organize and follow information that they are receiving. Outlines can be blank or partially filled in to vary difficulty.

Use a graphic calculator to solve a linear system of equations to help students understand what various type of solutions might look like.

Provide students with construction paper, scissors and tape to build models of linear systems that have one, infinitely many solutions and no solutions.
New Jersey Student Learning Standard(s):
A.REI.C.6: Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

**Student Learning Objective 4:** Solve algebraically a system of three linear equations.

**Modified Student Learning Objectives/Standards:** See EE.A-REI.10–12.

EE.A-REI.10–12. Interpret the meaning of a point on the graph of a line. For example, on a graph of pizza purchases, trace the graph to a point and tell the number of pizzas purchased and the total cost of the pizzas.

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</thead>
</table>
| MP 1  | A.REI.C.6                              | Solving a system of linear equations containing $n$ variables requires $n$ equations. Use the substitution method and/or elimination method to find the solution of a system containing three linear equations. **SPED Strategies:**
Model the thinking and processes necessary to decide on a solution path and solve a system of three linear equations accurately. Provide students with reference sheets/notes to encourage confidence and independence. **ELL Strategies:**
Provide students with a visual representation of a linear system of three equations that students can recognize where the three planes intercept. Let students verbalize the process of solving a system of three variables, in contract with |
| MP 7  |                                        | How can you determine the number of solutions of a linear system? How can you use substitution to solve a system of linear equations? How can you use elimination to solve a system of linear equations? Can a system of equations have no solution or infinitely many solutions? How do you determine the best method for solving a given system of equation? | **Type II, III:** Cash Box Pairs of Whole Numbers Find A System |

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**Type II, III:**
Cash Box
Pairs of Whole Numbers
Find A System
the process of solving a two variables system.

**New Jersey Student Learning Standard(s):**
F.BF.A.2: Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

F.LE.A.2: Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

F.LE.B.5: Interpret the parameters in a linear or exponential function in terms of a context.

**Student Learning Objective 5:** Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

**Modified Student Learning Objectives/Standards:**
M.EE.F-BF.2: Determine an arithmetic sequence with whole numbers when provided a recursive rule.

M.EE.F-LE.1–3: Model a simple linear function such as \( y = mx \) to show that these functions increase by equal amounts over equal intervals.

<table>
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<tbody>
<tr>
<td>MP 1</td>
<td>F-BF.2</td>
<td>Distinguish between recursive and explicit formulas.</td>
<td>How can you recognize an arithmetic sequence from its graph?</td>
<td>Type II, III:</td>
</tr>
<tr>
<td>MP 2</td>
<td>F-BF.2-3</td>
<td>Represent geometric and arithmetic sequences recursively.</td>
<td>How can you recognize a geometric sequence from its graph?</td>
<td>Rumors</td>
</tr>
<tr>
<td>MP 4</td>
<td>Solve multi-step contextual problems with degree of difficulty appropriate to the course by constructing linear and/or exponential function models.</td>
<td>Represent geometric and arithmetic sequences with explicit formulas.</td>
<td>How can you define a sequence recursively?</td>
<td>Taxi!</td>
</tr>
<tr>
<td>MP 6</td>
<td>Prompts describe a scenario using</td>
<td>Translate between recursive form and explicit form of geometric and arithmetic sequences.</td>
<td></td>
<td>Illegal Fish</td>
</tr>
<tr>
<td>MP 7</td>
<td></td>
<td></td>
<td></td>
<td>Additional Tasks:</td>
</tr>
<tr>
<td>MP 8</td>
<td></td>
<td></td>
<td></td>
<td>Exponential Parameters</td>
</tr>
</tbody>
</table>
| everyday language. Mathematical language such as "function," "exponential," etc. is not used. | Recognize explicit formula for geometric sequences as exponential functions containing a domain in the integers only. Interpret the parameters of an exponential function representing a geometric sequence. Interpret the parameters of a linear function representing an arithmetic sequence. **SPED Strategies:** Review the differences between geometric and arithmetic sequences by giving students examples and illustrating the characteristics that distinguish them. Pre-teach the vocabulary and provide verbal and pictorial descriptions. (i.e. recursive and explicit formulas). Model the thinking and procedure involved in writing geometric and arithmetic sequences in recursive and explicit form. Provide students with a graphic organizer/reference sheet/Google Doc that highlights the thinking and procedure involved in writing geometric and arithmetic sequences in recursive and explicit form. **ELL Strategies:** After listening to an oral explanation in the student’s native language, demonstrate comprehension of arithmetic and geometric
| What are some of the characteristics of the graph of an exponential function? How can you recognize polynomial, exponential, and logarithmic models? What data do you need to write a function to model a given situation? How do you determine if a given situation is modeled by a linear or exponential function? Create an example of a linear or exponential situation and give the function that can be used to model the situation. What does each part of the function represent in the context of the problem and what are the parameters? | Kimi and Jordan Population and Food Supply Snake on a Plane Algae Blooms Basketball Rebounds Snail Invasions Susita’s Account Canoe Trip variation 1 Canoe Trip variation 2 US Population 1982-1988 |
sequences both recursively and with an explicit formula and/or an explanation which uses drawings and selected technical words.

Interpret orally and in writing the parameters in a linear or exponential function in terms of a context the parameters in a linear or exponential function in terms of a context in the student’s native language and/or use gestures, pictures and selected, technical words.

Explain the classic tale “The Story of Gauss” that helps students to make sense of arithmetic sequences.

Model an example and do “Think Aloud” in pair reasoning: How do I get from $a_1$ to $a_{19}$?

**New Jersey Student Learning Standard(s):**

**A.SSE.B.4:** Derive and/or explain the derivation of the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.*

**Student Learning Objective 6:** Use the formula for the sum of a finite geometric series to solve problems (*for example, calculate the mortgage payments*; derive the formula for the sum of a finite geometric series (when the common ratio is not 1)).

**Modified Student Learning Objectives/Standards:**

**M.EE.A-SSE.4.** Determine the successive term in a geometric sequence given the common ratio.

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<tbody>
<tr>
<td>MP 1</td>
<td>A-SSE.4-2</td>
<td>Series as a sum of a sequence.</td>
<td>How can students recognize a geometric sequence from its graph?</td>
<td>Type II, III: Course of Antibiotics</td>
</tr>
<tr>
<td>MP 7</td>
<td>• Use the formula for the sum of a finite</td>
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</tr>
</tbody>
</table>
| geometric series to solve multi-step contextual problems.  
| In a multistep task, students may be expected to calculate the value of a single term as well as the sum. | Derive or explain the derivation of the formula for the sum of a finite geometric series.  
| Use the formula for the sum of a finite geometric series to solve problems.  
| **SPED Strategies:** Pre-teach vocabulary and provide verbal and pictorial descriptions to maximize understanding and interest.  
| Introduce the concept imbedded in a real-life context to help students relate to and internalize the mathematics involved.  
| Model the thinking and processes involved in solving problems involving finite geometric series.  
| Provide students with a graphic organizer/reference sheet/Google Doc that highlights the thinking and procedure involved solving problems involving finite geometric series.  
| **ELL Strategies:** Read and write the formula for the sum of a finite geometric series and use the formula to solve the problems in the student’s native language and/or use gestures, examples and selected, technical words.  
| Build on past knowledge by reminding students that a ratio is a comparison of the | How can you find the sum of an infinite geometric series?  
| **YouTube Explosion**  
| **Additional Tasks:**  
| **Triangle Series**  
| **A Lifetime of Savings**  
| **Cantor Set** |
two quantities and can be written as a fraction.

Encourage students to organize what they know about arithmetic and geometric sequences in a Chart, including formulas.

New Jersey Student Learning Standard(s):  
N.RN.A.1: Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define* \(5^{1/3}\) *to be the cube root of 5 because we want* \((5^{1/3})^3\) *to hold, so* \((5^{1/3})^3\) *must equal 5.*

N.RN.A.2: Rewrite expressions involving radicals and rational exponents using the properties of exponents.

Student Learning Objective 7: Use properties of integer exponents to explain and convert between expressions involving radicals and rational exponents.

Modified Student Learning Objectives/Standards:  
M.EE.N-RN.1: Determine the value of a quantity that is squared or cubed.

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</thead>
</table>
| MP 7 | N-RN.2 | Properties of integer exponents extends to rational exponents (*for example, we define* \(5^{1/3}\) *to be the cube root of 5 because we want* \((5^{1/3})^3\) *to hold, so* \((5^{1/3})^3\) *must equal 5*)  
Radical notation is a representation of rational exponents.  
Rewrite expressions containing rational exponents into radical form. | How can students use properties of exponents to simplify products and quotients of radicals?  
How can students write general rules involving properties of exponents? | IFL Sets of Related Lessons “Investigating Rational Exponents”  
Type II, III:  
Evaluating Exponential Expression |
Rewrite expressions containing radical notation into exponential expressions containing rational exponents.

Rational exponents are exponents that are fractions.

Properties of integer exponents extend to properties of rational exponents.

Properties of rational exponents are used to simplify and create equivalent forms of numerical expressions.

Rational exponents can be written as radicals, and radicals can be written as rational exponents.

**SPED Strategies:**
Pre-teach vocabulary and provide verbal and pictorial descriptions to maximize understanding and interest.

Review the rules of exponents with students and provide them with a reference document highlighting rules and examples.

Model the thinking and processes involved in converting expressions involving radicals and rational exponents.

Provide students with opportunity to practice with peers and encourage them to verbalize and justify their thinking process.

How can students write and evaluate the nth root of a number?

How do students use properties of rational exponents to simplify and create equivalent forms of numerical expressions?

Why are rational exponents and radicals related to each other?

Given an expression with a rational exponent, how do you write the equivalent radical expression?

Certain properties govern operations with terms involving exponents. They include:

- the multiplication property of exponents which states that $a^m \times a^n = a^{m+n}$
- when $a \neq 0$, the division property of exponents which states that $\frac{a^m}{a^n} = a^{m-n}$

### Extending the Definitions of Exponents, Variation 2

**Evaluating a Special Exponent**

**Rational or Irrational**

**Checking a calculation of a decimal exponent**

**Rational or Irrational**
**ELL Strategies:**

Demonstrate understanding of the properties of integer exponents; then explain orally how to use properties of integer exponents using correct notation in the student’s native language and/or use gestures, examples and selected, technical words.

Pre-teach vocabulary and provide verbal and pictorial descriptions to review the language and notation of radicals and roots.

Model a real application as Johannes Kepler third law, and make a diagram to conclude about the value of the variables.

Provide students with a visual example of each point of a radical expression in simplest form.

- the power property of exponents which states that \((a^m)^n = a^{m \times n}\); and
- when \(b \neq 0\), the regrouping property of exponents which states that \((ab)^n = a^n \times b^n\) and \(\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}\).

These properties apply to all real number exponents.
New Jersey Student Learning Standard(s):
A.SSE.B.3: Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression

A.SSE.B.3c: Use the properties of exponents to transform expressions for exponential functions. *For example the expression* $1.15^t$ *can be rewritten as* $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ *to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.

F.IF.C.8: Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function

F.IF.C.8b: Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as* $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{10t}$, *and classify them as representing exponential growth or decay.*

Student Learning Objective 8: Use the properties of exponents to transform expressions for exponential functions, explain properties of the quantity revealed in the transformed expression or different properties of the function.

Modified Student Learning Objectives/Standards:
M.EE.A-SSE.3: Solve simple algebraic equations with one variable using multiplication and division.

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<tr>
<td>MP 1</td>
<td>A-SSE.3c-2</td>
<td>Alternate, equivalent forms of an exponential expression containing rational exponents may reveal specific attributes of the function that it defines. Use properties of exponent transform/rewrite an exponential expression for an exponential function. Explain the properties of the quantity or the function.</td>
<td>What are some of the characteristics of exponential growth and exponential decay functions? How are properties of exponents used to transform expressions for exponential functions? Why would you want to transform an expression for an exponential function?</td>
<td>Type II, III: Forms of exponential expressions Additional Tasks: Carbon 14 Dating in Practice I Ice Cream Profit of a Company</td>
</tr>
<tr>
<td>MP 2</td>
<td>Tasks have a real-world context. The equivalent form must reveal something about the real-world context. Tasks require students to make the connection between the equivalent forms of the expression.</td>
<td></td>
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</tr>
<tr>
<td>MP 4</td>
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**SPED Strategies:**
Pre-teach vocabulary and provide verbal and...
| Pictorial descriptions to maximize understanding and interest.  
Review the rules of exponents with students and provide them with a reference document highlighting rules and examples.  
Model the thinking and processes involved in rewriting exponential functions in equivalent forms.  
Explain and illustrate why rewriting exponential functions in equivalent forms helps solve problems more effectively.  
**ELL Strategies:**  
Demonstrate comprehension by rewriting a function in different but equivalent forms to identify and explain different properties of the function in the student’s native language and/or use gestures, examples and selected technical words.  
Create Index cards with the properties of exponents, let students work with a partner with a different set of cards and identify the corresponding property of exponent. Verbalize each step when evaluating expression with rational exponents. | How do different forms of the function help you to identify key features? |
New Jersey Student Learning Standard(s):
F.LE.A.4: Understand the inverse relationship between exponents and logarithms. For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where $a$, $c$, and $d$ are numbers and the base $b$ is 2, 10, or $e$; evaluate the logarithm using technology.

**Student Learning Objective 9:** Express as a logarithm the solution to $ab^{ct} = d$ where $a$, $c$, and $d$ are numbers and the base $b$ is 2, 10, or $e$; evaluate the logarithm using technology.

**Modified Student Learning Objectives/Standards:** N/A

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<th>MPs</th>
<th>Evidence Statement Key/Clarifications</th>
<th>Skills, Strategies &amp; Concepts</th>
<th>Essential Understandings/Questions (Accountable Talk)</th>
<th>Tasks/Activities</th>
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<tr>
<td>MP 2</td>
<td>N/A</td>
<td>Exponents and logarithms have an inverse relationship. Solutions to an exponential equation in one variable can be written as a logarithm. Transform an exponential model represented by $ab^{ct} = d$ where $a$, $c$, and $d$ are numbers and the base $b$ is 2, 10, or $e$. Write the solution to $ab^{ct} = d$ as a logarithm. Use technology to evaluate logarithms having base 2, 10, or $e$.</td>
<td>What is the natural base $e$? What are some of the characteristics of the graph of a logarithmic function? How can you use properties of exponents to derive properties of logarithms? How can you solve exponential and logarithmic equations? How do you evaluate a logarithm using technology? How do logarithms help you to solve exponential functions?</td>
<td>Type II, III: Carbon 14 dating in practice I Additional Tasks: Bacteria Populations Snail Invasion</td>
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**SPED Strategies:**
Pre-teach vocabulary and provide verbal and pictorial descriptions to maximize understanding and interest.
Explain and illustrate the relationship between exponents and logarithms side by side.

Provide students with a reference sheet that illustrates the relationship between exponents and logarithms side by side to encourage confidence and independence.

**ELL Strategies:**
Describe and explain as a logarithm the solution to $a^{bc} = d$ where $a$, $c$, and $d$ are numbers and the base $b$ is 2, 10, or $e$; evaluate the logarithm using key, technical vocabulary in a series of simple sentences.

Provide some pairs of powers and ask students to describe verbally how to rewrite them with the same base.

Connect the equation for Newton’s Law of cooling by identifying the value of each variable.
Integrated Evidence Statements

A.Int.1: Solve equations that require seeing structure in expressions

- Tasks do not have a context.
- Equations simplify considerably after appropriate algebraic manipulations are performed. For example, $x^4 - 17x^2 + 16 = 0$, $2^{3x} = 7(2^{2x}) + 2^{2x}$, $x - \sqrt{x} = 3\sqrt{x}$
- Tasks should be course level appropriate.

F-BF.Int.2: Find inverse functions to solve contextual problems. Solve an equation of the form $f(x) = c$ for a simple function $f$ that has an inverse and write an expression for the inverse. For example, $f(x) = 2x^3$ or $f(x) = \frac{x+1}{x-1}$ for $x \neq 1$.

- For example, see http://illustrativemathematics.org/illustrations/234.
- As another example, given a function $C(L) = 750L^2$ for the cost $C(L)$ of planting seeds in a square field of edge length $L$, write a function for the edge length $L(C)$ of a square field that can be planted for a given amount of money $C$; graph the function, labeling the axes.
- This is an integrated evidence statement because it adds solving contextual problems to standard F-BF.4a.

F-Int.1-2: Given a verbal description of a polynomial, exponential, trigonometric, or logarithmic functional dependence, write an expression for the function and demonstrate various knowledge and skills articulated in the Functions category in relation to this function.

- Given a verbal description of a functional dependence, the student would be required to write an expression for the function and then, e.g., identify a natural domain for the function given the situation; use a graphing tool to graph several input-output pairs; select applicable features of the function, such as linear, increasing, decreasing, quadratic, periodic, nonlinear; and find an input value leading to a given output value.

HS-Int.3-3: Solve multi-step contextual word problems with degree of difficulty appropriate to the course, requiring application of course-level knowledge and skills articulated in F-LE, A-CED.1, A-SSE.3, F-IF.B, F-IF.7★

- F-LE.A, Construct and compare linear, quadratic, and exponential models and solve problems, is the primary content and at least one of the other listed content elements will be involved in tasks as well.
Integrated Evidence Statements

HS.C.3.1: Construct, autonomously, chains of reasoning that will justify or refute propositions or conjectures about numbers or number systems. Content Scope: N-RN, N-CN

HS.C.3.2: Base explanations/reasoning on the properties of exponents. Content Scope: N-RN.A

HS.C.4.1: Derive and use a formula. Content Scope: A-SSE.4

HS.C.12.2: Construct, autonomously, chains of reasoning that will justify or refute propositions or conjectures about functions. Content scope: F-IF.B.8.

HS.C.18.4: Construct, autonomously, chains of reasoning that will justify or refute propositions or conjectures about polynomials, rational expressions, or rational exponents. Content scope: N-RN, A-APR.(2, 3, 4, 6)


- Tasks will draw on securely held content from previous grades and courses, including down to Grade 7, but that are at the Algebra II/ Mathematics III level of rigor.
- Tasks will synthesize multiple aspects of the content listed in the evidence statement text, but need not be comprehensive.
- Tasks should address at least A-SSE.A.1b, A-REI.A.1, and F-IF.A.2 and either F-IF.C.7a or F-IF.C.7e (excluding trigonometric and logarithmic functions). Tasks should also draw upon additional content listed for grades 7 and 8 and from the remaining standards in the Evidence Statement Text.


- A-CED is the primary content; other listed content elements may be involved in tasks as well.
Integrated Evidence Statements

**HS.D.3-5:** Decisions from data: Identify relevant data in a data source, analyze it, and draw reasonable conclusions from it. Content scope: Knowledge and skills articulated in Algebra 2.

- Tasks may result in an evaluation or recommendation. ii.) The purpose of tasks is not to provide a setting for the student to demonstrate breadth in data analysis skills (such as box-and-whisker plots and the like). Rather, the purpose is for the student to draw conclusions in a realistic setting using elementary techniques.

**HS.D.CCR:** Solve problems using modeling: Identify variables in a situation, select those that represent essential features, formulate a mathematical representation of the situation using those variables, analyze the representation and perform operations to obtain a result, interpret the result in terms of the original situation, validate the result by comparing it to the situation, and either improve the model or briefly report the conclusions. Content scope: Knowledge and skills articulated in the Standards as described in previous courses and grades, with a particular emphasis on 7- RP, 8 – EE, 8 – F, N-Q, A-CED, A-REI, F-BF, G-MG, Modeling, and S-ID

- Tasks will draw on securely held content from previous grades and courses, include down to Grade 7, but that are at the Algebra II/Mathematics III level of rigor.
- Task prompts describe a scenario using everyday language. Mathematical language such as "function," "equation," etc. is not used.
- Tasks require the student to make simplifying assumptions autonomously in order to formulate a mathematical model. For example, the student might make a simplifying assumption autonomously that every tree in a forest has the same trunk diameter, or that water temperature is a linear function of ocean depth.
- Tasks may require the student to create a quantity of interest in the situation being described.
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References & Suggested Instructional Websites

www.internet4classrooms.com
https://www.desmos.com/
www.mathopenref.com
https://www.georgiastandards.org/Georgia-Standards/Pages/Math-9-12.aspx
www.illustrativemathematics.org/
https://www.khanacademy.org/math/algebra-home/algebra2
http://www.mathplanet.com/education/algebra-2
http://www.mathsisfun.com/algebra/index-2.html
https://parcc.pearson.com/practice-tests/math/
http://www.achieve.org/ccss-cte-classroom-tasks
http://www.nylearns.org/module/Standards/Tools/Browse?linkStandardId=0&standardId=97817
**Field Trip Ideas**

**SIX FLAGS GREAT ADVENTURE:** This educational event includes workbooks and special science and math related shows throughout the day. Your students will leave with a better understanding of real world applications of the material they have learned in the classroom. Each student will have the opportunity to experience different rides and attractions linking mathematical and scientific concepts to what they are experiencing.

[www.sixflags.com](http://www.sixflags.com)

**MUSEUM of MATHEMATICS:** Mathematics illuminates the patterns that abound in our world. The National Museum of Mathematics strives to enhance public understanding and perception of mathematics. Its dynamic exhibits and programs stimulate inquiry, spark curiosity, and reveal the wonders of mathematics. The Museum’s activities lead a broad and diverse audience to understand the evolving, creative, human, and aesthetic nature of mathematics.

[www.momath.org](http://www.momath.org)

**LIBERTY SCIENCE CENTER** - An interactive science museum and learning center located in Liberty State Park. The center, which first opened in 1993 as New Jersey's first major state science museum, has science exhibits, the largest IMAX Dome theater in the United States, numerous educational resources, and the original *Hoberman sphere*.

[http://lsc.org/plan-your-visit/](http://lsc.org/plan-your-visit/)