Algebra II: Unit 4
Making Inference, Justifying Conclusion and Conditional Probability
Course Philosophy/Description

Algebra II continues the students’ study of advanced algebraic concepts including functions, polynomials, rational expressions, systems of functions and inequalities, and matrices. Students will be expected to describe and translate among graphic, algebraic, numeric, tabular, and verbal representations of relations and use those representations to solve problems. Emphasis will be placed on practical applications and modeling. Students extend their knowledge and understanding by solving open-ended real-world problems and thinking critically through the use of high level tasks.

Students will be expected to demonstrate their knowledge in: utilizing essential algebraic concepts to perform calculations on polynomial expression; performing operations with complex numbers and graphing complex numbers; solving and graphing linear equations/inequalities and systems of linear equations/inequalities; solving, graphing, and interpreting the solutions of quadratic functions; solving, graphing, and analyzing solutions of polynomial functions, including complex solutions; manipulating rational expressions, solving rational equations, and graphing rational functions; solving logarithmic and exponential equations; and performing operations on matrices and solving matrix equations.
ESL Framework

This ESL framework was designed to be used by bilingual, dual language, ESL and general education teachers. Bilingual and dual language programs use the home language and a second language for instruction. ESL teachers and general education or bilingual teachers may use this document to collaborate on unit and lesson planning to decide who will address certain components of the SLO and language objective. ESL teachers may use the appropriate leveled language objective to build lessons for ELLs which reflects what is covered in the general education program. In this way, whether it is a pull-out or push-in model, all teachers are working on the same Student Learning Objective connected to the New Jersey Student Learning Standards. The design of language objectives are based on the alignment of the World-Class Instructional Design Assessment (WIDA) Consortium’s English Language Development (ELD) standards with the New Jersey Student Learning Standards (NJSLS). WIDA’s ELD standards advance academic language development across content areas ultimately leading to academic achievement for English learners. As English learners are progressing through the six developmental linguistic stages, this framework will assist all teachers who work with English learners to appropriately identify the language needed to meet the requirements of the content standard. At the same time, the language objectives recognize the cognitive demand required to complete educational tasks. Even though listening and reading (receptive) skills differ from speaking and writing (expressive) skills across proficiency levels the cognitive function should not be diminished. For example, an Entering Level One student only has the linguistic ability to respond in single words in English with significant support from their home language. However, they could complete a Venn diagram with single words which demonstrates that they understand how the elements compare and contrast with each other or they could respond with the support of their native language with assistance from a teacher, para-professional, peer or a technology program.

http://www.state.nj.us/education/modelcurriculum/ela/ELLOverview.pdf
<table>
<thead>
<tr>
<th>#</th>
<th>Student Learning Objective</th>
<th>NJSLS</th>
<th>Big Ideas Math Correlation</th>
<th>Instruction: 8 weeks</th>
<th>Assessment: 1 week</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Use the mean and standard deviation of a data set to fit it to a normal distribution, estimate population percentages, and recognize that there are data sets for which such a procedure is not appropriate (use calculators, spreadsheets, and tables to estimate areas under the normal curve).</td>
<td>S.ID.A.4</td>
<td>11.1</td>
<td></td>
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<tr>
<td>2</td>
<td>Identify and evaluate random sampling methods.</td>
<td>S.I.C.A.1</td>
<td>11.2, 11.3, 11.4</td>
<td></td>
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</tr>
<tr>
<td>3</td>
<td>Determine if the outcomes and properties of a specified model are consistent with results from a given data-generating process (e.g. using simulation).</td>
<td>S.I.C.A.2</td>
<td>11.5, 11.6</td>
<td></td>
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<tr>
<td>4</td>
<td>Identify the differences among and purposes of sample surveys, experiments, and observational studies, explaining how randomization relates to each.</td>
<td>S.I.C.B.3</td>
<td>11.3, 11.4</td>
<td></td>
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</tr>
<tr>
<td>5</td>
<td>Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.</td>
<td>S.I.C.B.4</td>
<td>11.5</td>
<td></td>
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<tr>
<td>6</td>
<td>Use data from a randomized experiment to compare two treatments and use simulations to decide if differences between parameters are significant; evaluate reports based on data.</td>
<td>S.I.C.B.5  S.I.C.B.6</td>
<td>11.4, 11.6</td>
<td></td>
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<tr>
<td>7</td>
<td>Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” ”not”).</td>
<td>S.C.P.A.1</td>
<td>10.1, 10.2, 10.4</td>
<td></td>
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<tr>
<td>8</td>
<td>Use two-way frequency tables to determine if events are independent and to calculate conditional probability. Use everyday language to explain independence and conditional probability in real-world situations.</td>
<td>S.C.P.A.2  S.C.P.A.3 S.C.P.A.4 S.C.P.A.5</td>
<td>10.2, 10.3, 10.4</td>
<td></td>
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<tr>
<td>9</td>
<td>Find the conditional probability of A given B as the fraction of B’s outcomes that also belong to A and apply the Addition Rule ( P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) ).</td>
<td>10.2, 10.4</td>
<td>S.CP.B.6 S.CP.B.7</td>
<td></td>
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</tbody>
</table>
Research about Teaching and Learning Mathematics

Structure teaching of mathematical concepts and skills around problems to be solved (Checkly, 1997; Wood & Sellars, 1996; Wood & Sellars, 1997)

Encourage students to work cooperatively with others (Johnson & Johnson, 1975; Davidson, 1990)

Use group problem-solving to stimulate students to apply their mathematical thinking skills (Artzt & Armour-Thomas, 1992)

Students interact in ways that support and challenge one another’s strategic thinking (Artzt, Armour-Thomas, & Curcio, 2008)

Activities structured in ways allowing students to explore, explain, extend, and evaluate their progress (National Research Council, 1999)

There are three critical components to effective mathematics instruction (Shellard & Moyer, 2002):

- Teaching for conceptual understanding
- Developing children’s procedural literacy
- Promoting strategic competence through meaningful problem-solving investigations

Teachers should be:

- Demonstrating acceptance and recognition of students’ divergent ideas.
- Challenging students to think deeply about the problems they are solving, extending thinking beyond the solutions and algorithms required to solve the problem
- Influencing learning by asking challenging and interesting questions to accelerate students’ innate inquisitiveness and foster them to examine concepts further
- Projecting a positive attitude about mathematics and about students’ ability to “do” mathematics

Students should be:

- Actively engaging in “doing” mathematics
- Solving challenging problems
- Investigating meaningful real-world problems
- Making interdisciplinary connections
- Developing an understanding of mathematical knowledge required to “do” mathematics and connect the language of mathematical ideas with numerical representations
- Sharing mathematical ideas, discussing mathematics with one another, refining and critiquing each other’s ideas and understandings
- Communicating in pairs, small group, or whole group presentations
- Using multiple representations to communicate mathematical ideas
- Using connections between pictures, oral language, written symbols, manipulative models, and real-world situations
- Using technological resources and other 21st century skills to support and enhance mathematical understanding
Mathematics is not a stagnate field of textbook problems; rather, it is a dynamic way of constructing meaning about the world around us, generating knowledge and understanding about the real world every day. Students should be metaphorically rolling up their sleeves and “doing mathematics” themselves, not watching others do mathematics for them or in front of them. (Protheroe, 2007)

Balanced Mathematics Instructional Model

Balanced math consists of three different learning opportunities: guided math, shared math, and independent math. Ensuring a balance of all three approaches will build conceptual understanding, problem solving, computational fluency, and procedural fluency. Building conceptual understanding is the focal point of developing mathematical proficiency. Students should frequently work on rigorous tasks, talk about the math, explain their thinking, justify their answer or process, build models with graphs or charts or manipulatives, and use technology.

When balanced math is used in the classroom it provides students opportunities to:

- solve problems
- make connections between math concepts and real-life situations
- communicate mathematical ideas (orally, visually and in writing)
- choose appropriate materials to solve problems
- reflect and monitor their own understanding of the math concepts
- practice strategies to build procedural and conceptual confidence

Teacher builds conceptual understanding by modeling through demonstration, explicit instruction, and think alouds, as well as guiding students as they practice math strategies and apply problem solving strategies. (Whole group or small group instruction)

Students practice math strategies independently to build procedural and computational fluency. Teacher assesses learning and reteaches as necessary. (whole group instruction, small group instruction, or centers)

Teacher and students practice mathematics processes together through interactive activities, problem solving, and discussion. (whole group or small group instruction)
<table>
<thead>
<tr>
<th>Effective Pedagogical Routines/Instructional Strategies</th>
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</thead>
<tbody>
<tr>
<td><strong>Collaborative Problem Solving</strong></td>
</tr>
<tr>
<td>Connect Previous Knowledge to New Learning</td>
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<tr>
<td>Making Thinking Visible</td>
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<tr>
<td><strong>Develop and Demonstrate Mathematical Practices</strong></td>
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<tr>
<td>Inquiry-Oriented and Exploratory Approach</td>
</tr>
<tr>
<td>Multiple Solution Paths and Strategies</td>
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<tr>
<td>Use of Multiple Representations</td>
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<tr>
<td><strong>Explain the Rationale of your Math Work</strong></td>
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<tr>
<td>Quick Writes</td>
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<tr>
<td>Pair/Trio Sharing</td>
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<tr>
<td>Turn and Talk</td>
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<tr>
<td>Charting</td>
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<td>Gallery Walks</td>
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<tr>
<td>Small Group and Whole Class Discussions</td>
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<tr>
<td>Student Modeling</td>
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</tbody>
</table>

| **Analyze Student Work**                               |
| Identify Student’s Mathematical Understanding          |
| Identify Student’s Mathematical Misunderstandings     |
| Interviews                                           |
| Role Playing                                         |
| **Anticipate Likely and Possible Student Responses** |
| Diagrams, Charts, Tables, and Graphs                  |
| Collect Different Student Approaches                  |
| **Multiple Response Strategies**                      |
| Asking Assessing and Advancing Questions              |
| Revoicing                                            |
| Marking                                              |
| Recapping                                            |
| Challenging                                          |
| **Pressing for Accuracy and Reasoning**               |
| Maintain the Cognitive Demand                        |
## Educational Technology

### Standards

**8.1.12.A.1, 8.1.12.C.1, 8.1.12.F.1, 8.2.12.E.3**

- **Technology Operations and Concepts**
  - Create a personal digital portfolio which reflects personal and academic interests, achievements, and career aspirations by using a variety of digital tools and resources.
  

- **Communication and Collaboration**
  - Develop an innovative solution to a real world problem or issue in collaboration with peers and experts, and present ideas for feedback through social media or in an online community.
  
  **Example:** Use Google Classroom for real-time communication between teachers, students, and peers to complete assignments and discuss strategies for identifying the differences among and purposes of sample surveys, experiments, and observational studies, explaining how randomization relates to each.

- **Critical Thinking, Problem Solving, and Decision Making**
  - Evaluate the strengths and limitations of emerging technologies and their impact on educational, career, personal or social needs.
  
  **Example:** Students research and use probability applications to justify conclusions about theoretical and experimental probabilities.

- **Computational Thinking: Programming**
  - Use a programming language to solve problems or accomplish a task (e.g., robotic functions, website designs, applications and games).
  
  **Example:** Students will create a set of instructions explaining how to calculate the mean and standard deviation of a data set.

**Link:** [http://www.state.nj.us/education/cccs/2014/tech/](http://www.state.nj.us/education/cccs/2014/tech/)
Career Ready Practices

Career Ready Practices describe the career-ready skills that all educators in all content areas should seek to develop in their students. They are practices that have been linked to increase college, career, and life success. Career Ready Practices should be taught and reinforced in all career exploration and preparation programs with increasingly higher levels of complexity and expectation as a student advances through a program of study.

- **CRP2. Apply appropriate academic and technical skills.**
  Career-ready individuals readily access and use the knowledge and skills acquired through experience and education to be more productive. They make connections between abstract concepts with real-world applications, and they make correct insights about when it is appropriate to apply the use of an academic skill in a workplace situation.

  **Example:** Students will apply prior knowledge when solving real world problems. Students will make sound judgements about the use of specific tools, such as number cubes, coins, random number generators, graphing calculators and technology to deepen understanding of probability.

- **CRP4. Communicate clearly and effectively and with reason.**
  Career-ready individuals communicate thoughts, ideas, and action plans with clarity, whether using written, verbal, and/or visual methods. They communicate in the workplace with clarity and purpose to make maximum use of their own and others’ time. They are excellent writers; they master conventions, word choice, and organization, and use effective tone and presentation skills to articulate ideas. They are skilled at interacting with others; they are active listeners and speak clearly and with purpose. Career-ready individuals think about the audience for their communication and prepare accordingly to ensure the desired outcome.

  **Example:** Students will communicate precisely using clear definitions and provide carefully formulated explanations when constructing arguments. Students will communicate and defend mathematical reasoning using objects, drawings, diagrams, and/or actions. Students will ask probing questions to clarify or improve arguments.
Career Ready Practices

- **CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.**
  Career-ready individuals readily recognize problems in the workplace, understand the nature of the problem, and devise effective plans to solve the problem. They are aware of problems when they occur and take action quickly to address the problem; they thoughtfully investigate the root cause of the problem prior to introducing solutions. They carefully consider the options to solve the problem. Once a solution is agreed upon, they follow through to ensure the problem is solved, whether through their own actions or the actions of others.

  **Example:** Students will understand the meaning of a problem and look for entry points to its solution. They will analyze information, make conjectures, and plan a solution pathway to solve simple rational and radical equations.

- **CRP12. Work productively in teams while using cultural global competence.**
  Career-ready individuals positively contribute to every team, whether formal or informal. They apply an awareness of cultural difference to avoid barriers to productive and positive interaction. They find ways to increase the engagement and contribution of all team members. They plan and facilitate effective team meetings.

  **Example:** Students will work collaboratively in groups to solve mathematical tasks. Students will listen to or read the arguments of others and ask probing questions to clarify or improve arguments. They will be able to explain how to find conditional probability.
WIDA Proficiency Levels

At the given level of English language proficiency, English language learners will process, understand, produce or use:

<table>
<thead>
<tr>
<th>Level</th>
<th>Description</th>
</tr>
</thead>
</table>
| 6- Reaching | Specialized or technical language reflective of the content areas at grade level  
- A variety of sentence lengths of varying linguistic complexity in extended oral or written discourse as required by the specified grade level  
- Oral or written communication in English comparable to proficient English peers |
| 5- Bridging | Specialized or technical language of the content areas  
- A variety of sentence lengths of varying linguistic complexity in extended oral or written discourse, including stories, essays or reports  
- Oral or written language approaching comparability to that of proficient English peers when presented with grade level material. |
| 4- Expanding | Specific and some technical language of the content areas  
- A variety of sentence lengths of varying linguistic complexity in oral discourse or multiple, related sentences or paragraphs  
- Oral or written language with minimal phonological, syntactic or semantic errors that may impede the communication, but retain much of its meaning, when presented with oral or written connected discourse, with sensory, graphic or interactive support |
| 3- Developing | General and some specific language of the content areas  
- Expanded sentences in oral interaction or written paragraphs  
- Oral or written language with phonological, syntactic or semantic errors that may impede the communication, but retain much of its meaning, when presented with oral or written, narrative or expository descriptions with sensory, graphic or interactive support |
| 2- Beginning | General language related to the content area  
- Phrases or short sentences  
- Oral or written language with phonological, syntactic, or semantic errors that often impede the communication when presented with one to multiple-step commands, directions, or a series of statements with sensory, graphic or interactive support |
| 1- Entering | Pictorial or graphic representation of the language of the content areas  
- Words, phrases or chunks of language when presented with one-step commands directions, WH-, choice or yes/no questions, or statements with sensory, graphic or interactive support |
# Language Development Supports for English Language Learners

## To Increase Comprehension and Communication Skills

### Environment

- Welcoming and stress-free
- Respectful of linguistic and cultural diversity
- Honors students' background knowledge
- Sets clear and high expectations
- Includes routines and norms
- Is thinking-focused vs. answer-seeking
- Offers multiple modalities to engage in content learning and to demonstrate understanding
- Includes explicit instruction of specific language targets
- Provides participation techniques to include all learners

- Integrates learning centers and games in a meaningful way
- Provides opportunities to practice and refine receptive and productive skills in English as a new language
- Integrates meaning and purposeful tasks/activities that:
  - Are accessible by all students through multiple entry points
  - Are relevant to students' lives and cultural experiences
  - Build on prior mathematical learning
  - Demonstrate high cognitive demand
  - Offer multiple strategies for solutions
  - Allow for a language learning experience in addition to content

### Sensory Supports*

- Real-life objects (realia) or concrete objects
- Physical models
- Manipulatives
- Pictures & photographs
- Visual representations or models such as diagrams or drawings
- Videos & films
- Newspapers or magazines
- Gestures
- Physical movements
- Music & songs

### Graphic Supports*

- Graphs
- Charts
- Timelines
- Number lines
- Graphic organizers
- Graphing paper

### Interactive Supports*

- In a whole group
- In a small group
- With a partner such as Turn-and-Talk
- In pairs as a group (first, two pairs work independently, then they form a group of four)
- In triads
- Cooperative learning structures such as Think-Pair-Share
- Interactive websites or software
- With a mentor or coach

### Verbal and Textual Supports

- Labeling
- Students' native language
- Modeling
- Repetitions
- Paraphrasing
- Summarizing
- Guiding questions
- Clarifying questions
- Probing questions
- Leveled questions such as What? When? Where? How? Why?
- Questioning prompts & cues
- Word Banks
- Sentence starters
- Sentence frames
- Discussion frames
- Talk moves, including Wait Time

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# Building Equity in Your Teaching Practice

How do the essential questions highlight the connection between the big ideas of the unit and equity in your teaching practice?

<table>
<thead>
<tr>
<th><strong>CONTENT INTEGRATION</strong></th>
<th><strong>KNOWLEDGE CONSTRUCTION</strong></th>
<th><strong>PREJUDICE REDUCTION</strong></th>
<th><strong>EQUITABLE PEDAGOGY</strong></th>
<th><strong>EMPOWERING SCHOOL CULTURE</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Teachers use examples and content from a variety of cultures &amp; groups.</td>
<td>Teachers help students understand how knowledge is created and influenced by cultural assumptions, perspectives &amp; biases.</td>
<td>Teachers implement lessons and activities to assert positive images of ethnic groups &amp; improve intergroup relations.</td>
<td>Teachers modify techniques and methods to facilitate the academic achievement of students from diverse backgrounds.</td>
<td>Using the other four dimensions to create a safe and healthy educational environment for all.</td>
</tr>
<tr>
<td>This unit / lesson is connected to other topics explored with students.</td>
<td>This unit / lesson provides context to the history of privilege and oppression.</td>
<td>This unit / lesson help students question and unpack biases &amp; stereotypes.</td>
<td>The instruction has been modified to meet the needs of each student.</td>
<td>There are opportunities for students to connect with the community.</td>
</tr>
<tr>
<td>There are multiple viewpoints reflected in the content of this unit / lesson.</td>
<td>This unit / lesson addresses power relationships.</td>
<td>This unit / lesson help students examine, research and question information and sources.</td>
<td>Students feel respected and their cultural identities are valued.</td>
<td>My classroom is welcoming and supportive for all students?</td>
</tr>
<tr>
<td>The materials and resources are reflective of the diverse identities and experiences of students.</td>
<td>This unit / lesson help students to develop research and critical thinking skills.</td>
<td>The curriculum encourage discussion and understanding about the groups of people being represented.</td>
<td>Additional supports have been provided for students to become successful and independent learners.</td>
<td>I am aware of and sensitive to the needs of my students and their families.</td>
</tr>
<tr>
<td>The content affirms students, as well as exposes them to experiences other than their own.</td>
<td>This curriculum creates windows and mirrors* for students.</td>
<td>This unit / lesson challenges dominant perspectives.</td>
<td>Opportunities are provided for student to reflect on their learning and provide feedback.</td>
<td>There are effective parent communication systems established. Parents can talk to me about issues as they arise in my classroom.</td>
</tr>
</tbody>
</table>

### Culturally Relevant Pedagogy Examples

- **Integrate Relevant Word Problems:** Contextualize equations using word problems that reference student interests and cultures.
  **Example:** When learning about interpreting data, problems that relate to student interests such as music, sports and art enable the students to understand and relate to the concept in a more meaningful way.

- **Everyone has a Voice:** Create a classroom environment where students know that their contributions are expected and valued.
  **Example:** Norms for sharing are established that communicate a growth mindset for mathematics. All students are capable of expressing mathematical thinking and contributing to the classroom community. Students learn new ways of looking at problem solving by working with and listening to each other.

- **Run Problem Based Learning Scenarios:** Encourage mathematical discourse among students by presenting problems that are relevant to them, the school and/or the community.
  **Example:** Using a Place Based Education (PBE) model, students explore math concepts such as systems of equations while determining ways to address problems that are pertinent to their neighborhood, school or culture.

- **Encourage Student Leadership:** Create an avenue for students to propose problem solving strategies and potential projects.
  **Example:** Students can learn to learn about and compute probability by creating problems together and deciding if the problems fit the necessary criteria. This experience will allow students to discuss and explore their current level of understanding by applying the concepts to relevant real-life experiences.

- **Present New Concepts Using Student Vocabulary:** Use student diction to capture attention and build understanding before using academic terms.
  **Example:** Teach math vocabulary in various modalities for students to remember. Use multi-modal activities, analogies, realia, visual cues, graphic representations, gestures, pictures and cognates. Directly explain and model the idea of vocabulary words having multiple meanings. Students can create the Word Wall with their definitions and examples to foster ownership.
### Differentiated Instruction

**Accommodate Based on Students Individual Needs: Strategies**

<table>
<thead>
<tr>
<th>Time/General</th>
<th>Processing</th>
<th>Comprehension</th>
<th>Recall</th>
</tr>
</thead>
<tbody>
<tr>
<td>Extra time for assigned tasks</td>
<td>Extra Response time</td>
<td>Precise processes for balanced math instructional model</td>
<td>Teacher-made checklist</td>
</tr>
<tr>
<td>Adjust length of assignment</td>
<td>Have students verbalize steps</td>
<td>Short manageable tasks</td>
<td>Use visual graphic organizers</td>
</tr>
<tr>
<td>Timeline with due dates for reports and projects</td>
<td>Repeat, clarify or reword directions</td>
<td>Brief and concrete directions</td>
<td>Reference resources to promote independence</td>
</tr>
<tr>
<td>Communication system between home and school</td>
<td>Mini-breaks between tasks</td>
<td>Provide immediate feedback</td>
<td>Visual and verbal reminders</td>
</tr>
<tr>
<td>Provide lecture notes/outline</td>
<td>Provide a warning for transitions</td>
<td>Small group instruction</td>
<td>Graphic organizers</td>
</tr>
<tr>
<td></td>
<td>Partnering</td>
<td>Emphasize multi-sensory learning</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Assistive Technology</th>
<th>Tests/Quizzes/Grading</th>
<th>Behavior/Attention</th>
<th>Organization</th>
</tr>
</thead>
<tbody>
<tr>
<td>Computer/whiteboard</td>
<td>Extended time</td>
<td>Consistent daily structured routine</td>
<td>Individual daily planner</td>
</tr>
<tr>
<td>Tape recorder</td>
<td>Study guides</td>
<td>Simple and clear classroom rules</td>
<td>Display a written agenda</td>
</tr>
<tr>
<td>Video Tape</td>
<td>Shortened tests</td>
<td>Frequent feedback</td>
<td>Note-taking assistance</td>
</tr>
<tr>
<td></td>
<td>Read directions aloud</td>
<td></td>
<td>Color code materials</td>
</tr>
</tbody>
</table>

### Assistive Technology
- Computer/whiteboard
- Tape recorder
- Video Tape

### Tests/Quizzes/Grading
- Extended time
- Study guides
- Shortened tests
- Read directions aloud

### Behavior/Attention
- Consistent daily structured routine
- Simple and clear classroom rules
- Frequent feedback

### Organization
- Individual daily planner
- Display a written agenda
- Note-taking assistance
- Color code materials
### Differentiated Instruction

**Accommodate Based on Content Specific Needs**

- Anchor charts to model strategies for finding conditional probabilities
- Review Algebra concepts to ensure students have the information needed to progress in their understanding
- Pre-teach pertinent vocabulary
- Use manipulatives to compare situations of theoretical and experimental probabilities
- Provide reference sheets that list step-by-step procedures of finding standard deviation and conditional probability
- Word wall with visual representations of mathematical terms
- Teacher modeling of thinking processes involved in making inferences, justifying conclusions and conditional probability.
- Introduce concepts embedded in real-life context to help students relate to the mathematics involved.
- Record processes and mathematical rules in reference notebooks.
- Graphing calculator to assist with computations of mean and standard deviation.
- Utilize technology through interactive sites to experiment with probability and normal distribution.

www.mathopenref.com  https://www.geogebra.org/
Interdisciplinary Connections

Model interdisciplinary thinking to expose students to other disciplines.

Social Studies Connection: Social Studies Standard 6.1.12.D.2.a

Name of Task: Margin of Error for Estimating a Population Mean

- The purpose of this task is to illustrate the development of margin of error when estimating a population mean (S.IC.4). The results from several simulations are used to develop margin of error and then a way of estimating the margin of error from a single sample is introduced. This is a challenging task, but it is well aligned with standard S.IC.4, which is one of the more complex statistics standards.

Science Connection: Science Standard MS-ESS3-2

Name of Task: Rain and Lightning

- This task uses the concept of weather prediction to explore different concepts of probability theory.
# Enrichment

## What is the Purpose of Enrichment?

- The purpose of enrichment is to provide extended learning opportunities and challenges to students who have already mastered, or can quickly master, the basic curriculum. Enrichment gives the student more time to study concepts with greater depth, breadth, and complexity.
- Enrichment also provides opportunities for students to pursue learning in their own areas of interest and strengths.
- Enrichment keeps advanced students engaged and supports their accelerated academic needs.
- Enrichment provides the most appropriate answer to the question, “What do you do when the student already knows it?”

### Enrichment is…

- Planned and purposeful
- *Different*, or differentiated, work – not just *more* work
- Responsive to students’ needs and situations
- A promotion of high-level thinking skills and making connections within content
- The ability to apply different or multiple strategies to the content
- The ability to synthesize concepts and make real world and cross-curricular connections
- Elevated contextual complexity
- Sometimes independent activities, sometimes direct instruction
- Inquiry based or open ended assignments and projects
- Using supplementary materials in addition to the normal range of resources.
- Choices for students
- Tiered/Multi-level activities with flexible groups (may change daily or weekly)

### Enrichment is not…

- Just for gifted students (some gifted students may need intervention in some areas just as some other students may need frequent enrichment)
- Worksheets that are more of the same (busywork)
- Random assignments, games, or puzzles not connected to the content areas or areas of student interest
- Extra homework
- A package that is the same for everyone
- Thinking skills taught in isolation
- Unstructured free time
Assessments

Required District/State Assessments
- Unit Assessment
- PARCC
- SGO Assessments

Suggested Formative/Summative Classroom Assessments
- Describe Learning Vertically
- Identify Key Building Blocks
- Make Connections (between and among key building blocks)
- Short/Extended Constructed Response Items
- Multiple-Choice Items (where multiple answer choices may be correct)
- Drag and Drop Items
- Use of Equation Editor
- Quizzes
- Journal Entries/Reflections/Quick-Writes
- Accountable talk
- Projects
- Portfolio
- Observation
- Graphic Organizers/Concept Mapping
- Presentations
- Role Playing
- Teacher-Student and Student-Student Conferencing
- Homework
## New Jersey Student Learning Standards

### S.ID.A.4:
Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.

### S.IC.A.1:
Understand statistics as a process for making inferences about population parameters based on a random sample from that population.

### S.IC.A.2:
Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?

### S.IC.B.3:
Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.

### S.IC.B.4:
Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.

### S.IC.B.5:
Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.

### S.IC.B.6:
Evaluate reports based on data.

### S.CP.A.1:
Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”).
<table>
<thead>
<tr>
<th>New Jersey Student Learning Standards</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>S.CP.A.2:</strong></td>
</tr>
<tr>
<td>Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.</td>
</tr>
</tbody>
</table>

| **S.CP.A.3:**                      |
| Understand the conditional probability of A given B as P(A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B. |

| **S.CP.A.4:**                      |
| Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results. |

| **S.CP.A.5:**                      |
| Recognize and explain the NEW Concepts of conditional probability and independence in everyday language and everyday situations. For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer. |

| **S.CP.B.6:**                      |
| Find the conditional probability of A given B as the fraction of B’s outcomes that also belong to A, and interpret the answer in terms of the model. |

| **S.CP.B.7:**                      |
| Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in terms of the model. |
## Mathematical Practices

1. Make sense of problems and persevere in solving them.

2. Reason abstractly and quantitatively.

3. Construct viable arguments and critique the reasoning of others.

4. Model with mathematics.

5. Use appropriate tools strategically.

6. Attend to precision.

7. Look for and make use of structure.

8. Look for and express regularity in repeated reasoning.
NJSLS:

Unit Focus:
- Summarize, represent, and interpret data on a single count or measurement variable
- Understand and evaluate random processes underlying statistical experiments
- Make inferences and justify conclusions from sample surveys, experiments and observational studies
- Understand the independence and conditional probability and use them to interpret data
- Use the rules of probability to compute probabilities of compound events in a uniform probability model

New Jersey Student Learning Standard(s):
S.ID.A.4: Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.

Student Learning Objective 1: Use the mean and standard deviation of a data set to fit it to a normal distribution, estimate population percentages, and recognize that there are data sets for which such a procedure is not appropriate (use calculators, spreadsheets, and tables to estimate areas under the normal curve).

Modified Student Learning Objectives/Standards:
M.EE.S-ID.4: Calculate the mean of a given data set (limit the number of data points to fewer than five).

<table>
<thead>
<tr>
<th>MPs</th>
<th>Evidence Statement Key/Clarifications</th>
<th>Skills, Strategies &amp; Concepts</th>
<th>Essential Understandings/Questions (Accountable Talk)</th>
<th>Tasks/Activities</th>
</tr>
</thead>
<tbody>
<tr>
<td>MP 2 MP 4</td>
<td>S.ID.4</td>
<td>Each normal distribution has a well-defined mean and standard deviation.</td>
<td>How does probability help to develop informed decisions? How are permutations and combinations helpful?</td>
<td>Type II, III: Do You Fit In This Car</td>
</tr>
<tr>
<td></td>
<td>Tasks may require finding the area associated with a z-score using technology.</td>
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<tr>
<td>Use of a z-score table will not be required.</td>
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<tr>
<td>Tasks may involve finding a value at a given percentile based on a normal distribution.</td>
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</tr>
<tr>
<td>The mean and standard deviation of a data set can be used to find the best-fit normal distribution for that data set.</td>
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<tr>
<td>Population percentages may be estimated when the data are approximately normally distributed.</td>
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<tr>
<td>Identify data sets as approximately normally distributed or not.</td>
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<tr>
<td>Explain the 68-95-99.7 rule for normal distributions (approximately 68% of the area under a normal distribution curve is within one standard deviation, approximately 95% of the area under a normal distribution curve is within two standard deviations, etc.).</td>
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</tr>
<tr>
<td>Use the mean and standard deviation of a normal distribution to estimate population percentages.</td>
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</tr>
<tr>
<td>Use calculators, spreadsheets, and tables to estimate areas under the normal curve and interpret in context.</td>
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</tr>
<tr>
<td><strong>SPED Strategies:</strong></td>
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</tr>
<tr>
<td>Pre-teach vocabulary using visual and verbal models that are connected to real life situations.</td>
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<tr>
<td>Link prior learning by reviewing measures of central tendency with students and</td>
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</tr>
</tbody>
</table>

| How can multiple events change the likelihood of an occurrence? |
| How can you use the mean and standard deviation of a set of data to fit the data to a normal curve? |
| How can you estimate the population percentages? |
| How can you estimate the area under a normal curve using a calculator, table or spreadsheet? |

| Should we send out certificate SAT Scores |
introducing new concepts of standard deviation in a real life context.

Provide students with a reference document that highlights all the necessary information with verbal and pictorial descriptions enabling students to become more proficient, independent and confident.

**ELL Strategies:**
Make a comparison chart for the terms: “Mean, Median and Mode.”

Model a real-life problem, i.e. students’ height to create a table, and guide students to find the standard deviation of the data.

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**New Jersey Student Learning Standard(s):**
S.IC.A.1: Understand statistics as a process for making inferences about population parameters based on a random sample from that population.

**Student Learning Objective 2:** Identify and evaluate random sampling methods.

**Modified Student Learning Objectives/Standards:**
M.EE.S-IC.1–2: Determine the likelihood of an event occurring when the outcomes are equally likely to occur.

<table>
<thead>
<tr>
<th>MPs</th>
<th>Evidence Statement Key / Clarifications</th>
<th>Skills, Strategies &amp; Concepts</th>
<th>Essential Understandings / Questions (Accountable Talk)</th>
<th>Tasks/Activities</th>
</tr>
</thead>
<tbody>
<tr>
<td>MP 2</td>
<td>N/A</td>
<td>Statistics is a process for making inferences about a population based on analysis of a random sample from the population.</td>
<td>How can you test theoretical probability using sample data?</td>
<td>Type II, III: Musical Preferences</td>
</tr>
<tr>
<td>MP 4</td>
<td></td>
<td></td>
<td></td>
<td>Random Walk III</td>
</tr>
</tbody>
</table>
| Identify and evaluate random sampling methods. | What are some of the considerations when undertaking a statistical study? | School Advisory Panel
Strict Parents
Why Randomize |
| Explain the importance of randomness to sampling and inference making. | How can you use an experiment to test a conjecture? |
| Explain the difference between values that describe a population and a sample, in context. | How can I obtain a reasonable estimation of a population parameter without completing a census? |
| **SPED Strategies:**
Pre-teach vocabulary related to random sampling using visual and verbal models that are connected to real life situations. |
| Model the application of random sampling techniques to real life contexts highlighting why certain methods are better suited to specific situations. |
| Create a reference document for students that highlight the different types of random sampling methods. |
| **ELL Strategies:**
Infer orally and in written form about population parameters based on a random sample from that population in the student’s native language and/or use gestures, examples and selected technical words. |
| Use turn and talk to discuss the terms: “Population and a Sample.” |
Model a random sample of a large population by asking students to conduct a survey about whether the art or sport program should be cut if their school’s budget is cut.

**New Jersey Student Learning Standard(s):**

S.IC.A.2: Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?

**Student Learning Objective 3:** Determine if the outcomes and properties of a specified model are consistent with results from a given data-generating process (e.g. using simulation).

**Modified Student Learning Objectives/Standards:**

M.EE.S-IC.1–2: Determine the likelihood of an event occurring when the outcomes are equally likely to occur.

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</tr>
</thead>
<tbody>
<tr>
<td>MP 2</td>
<td>S.IC.2</td>
<td>Random processes can be described mathematically by using a model: a list or description of possible outcomes.</td>
<td>How can you test theoretical probability using sample data?</td>
<td>Type II, III:</td>
</tr>
<tr>
<td>MP 4</td>
<td></td>
<td>Determine whether a given model is consistent with results from and experiment.</td>
<td>What are reasonable outcomes of a simulation?</td>
<td>Block Scheduling</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Know the difference between experimental and theoretical modeling.</td>
<td>What are the different ways in which data can be organized and analyzed?</td>
<td>Guess The Probability</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Know how far predictions can be projected based on sample size.</td>
<td>How can you determine if a model is consistent with the results of a simulation or experiment?</td>
<td>Last Person Standing</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Design simulations of random sampling.</td>
<td></td>
<td>Sarah, the chimpanzee</td>
</tr>
</tbody>
</table>
Experiments must be repeated to verify a model.

Large numbers of trials can be performed using computer simulations. If a model is appropriate for a given situation, the experimental probability of an event will approach the theoretical probability as the sample size increases.

**SPED Strategies:**
Pre-teach vocabulary related to theoretical and experimental probability using visual and verbal models that are connected to real life situations.

Engage students in an activity such as rolling a fair number cube after computing the theoretical probability of rolling each number on the cube. Have students work in pairs and compare their results to the theoretical. Then add all of the data from the student pairs and demonstrate how experimental approximates theoretical as the number of trials increases.

**ELL Strategies:**
Explain orally and in written form if the outcomes and properties of a specified model are consistent with results from a given data-generating process in the student’s native language and/or use gestures, examples and selected technical words.

| | | | |
Introduce the term simulation by using turn and talk to discuss “how do pollsters make predictions about an election, sometimes weeks, or months in advance of election?”

Introduce the process of conducting a statistical study. Use samples done randomly.

New Jersey Student Learning Standard(s):
S.IC.B.3: Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.

Student Learning Objective 4: Identify the differences among and purposes of sample surveys, experiments, and observational studies, explaining how randomization relates to each.

Modified Student Learning Objectives/Standards:
EE.S-ID.1–2: Given data, construct a simple graph (line, pie, bar, or picture) or table, and interpret the data.

<table>
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<tr>
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<th>Tasks/Activities</th>
</tr>
</thead>
<tbody>
<tr>
<td>MP 4</td>
<td>S.IC.3-1</td>
<td>Collecting data from a random sample of a population makes it possible to draw conclusions about the whole population. Randomly assigning individuals to different treatments allows a fair comparison of the effectiveness of those treatments. Sample surveys, experiments, and observational studies serve different statistical purposes allowing for different statistical analyses.</td>
<td>What are some of the considerations when undertaking a statistical study? How does randomization relate to sample surveys, experiments, and observational studies? How do you use data from sample surveys, observational studies, and observational studies?</td>
<td>Type II, III: High blood pressure Strict Parents Words and Music II</td>
</tr>
<tr>
<td><strong>Experiment:</strong> To estimate or compare the effects of different treatments based on randomized assignment of treatments to units for the purpose of establishing a cause and effect relationship.</td>
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<tr>
<td><strong>Observational study:</strong> To suggest patterns and/or associations among variables where treatments or conditions are inherent and not assigned to units.</td>
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</tbody>
</table>

- Distinguish between sample surveys, experiments, and observational studies.
- Explain the importance of randomization in each of these processes.
- Identify voluntary response samples and convenience samples.
- Describe simple random samples, stratified random samples, and cluster samples.
- Explain how under coverage, nonresponsive, and question wording can lead to bias in a sample survey.

**SPED Strategies:**
Provide students with descriptions and examples of sample surveys, experiments, and observational studies. Highlight the characteristics of each and the types of situations that are best studied using each method.

Discuss the importance of randomization by discussing how the results of a study can be negatively affected if random sampling/assignment principles are not adhered to.

**ELL Strategies:**
Identify orally and in written form different methods and purposes for conducting samples surveys, experiments, and observational experiments to draw inferences and conclusions?
studies and explain how randomization relates to each in the student’s native language and/or use gestures, examples and selected technical words.

Model samples survey by assigning a different type of sample to each student group. Have students conduct a survey of their sample, and compare samples types and sample results of the groups.

Make inferences about population using the result of the discussion of bias, and unbiased sample.

New Jersey Student Learning Standard(s):
S.I.C.B.4: Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.

Student Learning Objective 5: Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.

Modified Student Learning Objectives/Standards:
M.EE.S-ID.1–2: Given data, construct a simple graph (line, pie, bar, or picture) or table, and interpret the data.

<table>
<thead>
<tr>
<th>MPs</th>
<th>Evidence Statement Key/Clarifications</th>
<th>Skills, Strategies &amp; Concepts</th>
</tr>
</thead>
<tbody>
<tr>
<td>MP 1</td>
<td>N/A</td>
<td>Appropriately drawn samples of a population may be used to estimate a population mean or population proportion.</td>
</tr>
<tr>
<td>MP 2</td>
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<tr>
<td>MP 4</td>
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<tr>
<td>MP 5</td>
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<tr>
<td>MP 6</td>
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</table>

<table>
<thead>
<tr>
<th>Essential Understandings/Questions (Accountable Talk)</th>
</tr>
</thead>
<tbody>
<tr>
<td>How can you use a sample survey to infer a conclusion about a population?</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Tasks/Activities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type II, III:</td>
</tr>
<tr>
<td>The Marble Jar</td>
</tr>
<tr>
<td>Relationship between margin of error, variation with a data set, and variability in the population.</td>
</tr>
<tr>
<td>Conduct simulations of random sampling to gather samples.</td>
</tr>
<tr>
<td>Estimate population means with sample means.</td>
</tr>
<tr>
<td>Estimate population proportions with sample proportions.</td>
</tr>
<tr>
<td>Calculate margins of error for the estimates.</td>
</tr>
<tr>
<td>Explain how the results relate to variability in the population.</td>
</tr>
<tr>
<td>Students may use computer generated simulation models based upon sample surveys results to estimate population statistics and margins of error.</td>
</tr>
</tbody>
</table>

**SPED Strategies:**
Pre-teach vocabulary using visual and verbal models that are connected to real life situations.

Model how to use the data from a sample survey to estimate a population mean or proportion and develop a margin of error.
Provide students with opportunities to practice the skills learned using sample data working in small groups with peers.

**ELL Strategies:**
Describe and explain data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling in the student’s native language and/or use gestures, examples and selected technical words.

Discuss the different types of samples and ask students to write a new simpler name for each type of sample that help to define it.

Model quality control by using computer or internet to predict the number of defective products that can be expected on an assembly line.
New Jersey Student Learning Standard(s):
S.IC.B.5: Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.

S.IC.B.6: Evaluate reports based on data.

Student Learning Objective 6: Use data from a randomized experiment to compare two treatments and use simulations to decide if differences between parameters are significant; evaluate reports based on data.

Modified Student Learning Objectives/Standards:
M.EE.S-ID.1–2: Given data, construct a simple graph (line, pie, bar, or picture) or table, and interpret the data.

<table>
<thead>
<tr>
<th>MPs</th>
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<th>Tasks/Activities</th>
</tr>
</thead>
<tbody>
<tr>
<td>MP 1 MP 2 MP 4 MP 5 MP 6</td>
<td>N/A</td>
<td>A statistically significant outcome is one that is unlikely to be due to chance alone. Conduct a t-test to evaluate the effectiveness and differences in two treatments. Use simulations to generate data simulating applying two treatments. Use the results of simulations to determine if the differences are significant. Read and explain, in the context of the situation, data from outside reports – discussing experimental study design, drawing conclusions from graphical and numerical summaries, and identifying characteristics of the experimental design.</td>
<td>When is it appropriate to use a randomized experiment as opposed to a sample survey? Why do we use simulations to support inferences about data? How can you determine if a report is showing you misleading data or conclusions?</td>
<td>Type II, III: Musical Preferences</td>
</tr>
</tbody>
</table>
Reported data may be misleading due to, for example, sample size, biased survey sample, choice of interval scale, unlabeled scale, uneven scale, and outliers.

**SPED Strategies:**
Model the thinking and processes needed to compare two treatments and decide whether the results are significant.

Provide students with opportunities to practice the skills learned using sample data working in small groups with peers.

Encourage students to verbalize their thinking by asking assessment and advancing questions. Based on the information gleaned from these questions, tailor instructional strategies to meet student needs.

**ELL Strategies:**
Describe and explain data from a randomized experiment to compare two treatments and use simulations to decide if differences between parameters are significant; evaluate reports based on data in the student’s native language and/or use gestures, examples and selected technical words.

Write a prompt asking student identity methods of accurately “collecting data.”
Model a real life problem by asking students to identify the method of data collection of a research that uses technology to estimate the damage that will be done if a volcano erupts.

**New Jersey Student Learning Standard(s):**

S.CP.A.1: Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”).

**Student Learning Objective 7:** Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” ”not”).

**Modified Student Learning Objectives/Standards:**

M.EE.S-CP.1–5: Identify when events are independent or dependent.

<table>
<thead>
<tr>
<th>MPs</th>
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<th>Tasks/Activities</th>
</tr>
</thead>
<tbody>
<tr>
<td>MP 1</td>
<td>N/A</td>
<td>Events are described as subsets of a sample space. Identify a sample space, recognizing it as the set of all possible outcomes. Identify and describe subsets of a sample space as events. Describe unions, intersections and complements of events. Visualize unions, intersections and complements of events with Venn diagrams. Establish events as subsets of a sample space.</td>
<td>In what ways does one event impact the probability of another event occurring? Give an example of a sample space and describe several events based on that sample space. Explain the connection between unions, intersections, and complements of sets and the probabilities of events</td>
<td>Type II, III: The Titanic 1 Describing Events Return to Fred’s Fun Factory</td>
</tr>
<tr>
<td>MP 2</td>
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<td>MP 4</td>
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<td>MP 5</td>
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<tr>
<td>MP 6</td>
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</tbody>
</table>
### SPED Strategies:
Pre-teach vocabulary using visual and verbal models that are connected to real life situations.

Create a reference document for students that highlight the characteristics of outcomes, unions, intersections and complements with clear examples depicted verbally and pictorially.

Provide students with opportunities to practice the skills learned by working with sample problems in small groups with peers.

Encourage students to verbalize their thinking by asking assessment and advancing questions. Based on the information gleaned from these questions, tailor instructional strategies to meet student needs.

### ELL Strategies:
Describe orally and/or in written form the events as subsets of a sample space using characteristics of the outcomes, or as unions, intersections, or complements of other events in the student’s native language and/or use gestures, examples and selected technical words.

Use a graphing calculator and use “the dice rolling apps” to explore experiment and evaluate sample space.
Use a Venn diagram to represent and display outcomes data in an intersection, union or complement.

New Jersey Student Learning Standard(s):
S.CP.A.2: Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.
S.CP.A.3: Understand the conditional probability of A given B as \( P(A \text{ and } B)/P(B) \), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.
S.CP.A.4: Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.
S.CP.A.5: Recognize and explain the NEW Concepts of conditional probability and independence in everyday language and everyday situations. For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.

Student Learning Objective 8: Use two-way frequency tables to determine if events are independent and to calculate conditional probability. Use everyday language to explain independence and conditional probability in real-world situations.

Modified Student Learning Objectives/Standards:
M.EE.S-CP.1–5: Identify when events are independent or dependent.
<table>
<thead>
<tr>
<th>MP 1</th>
<th>MP 2</th>
<th>MP 4</th>
<th>MP 5</th>
<th>MP 6</th>
<th>N/A</th>
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</table>

Two events $A$ and $B$ are independent if the probability of $A$ and $B$ occurring together is the product of their probabilities.

Independence of event $A$ and event $B$ means that the conditional probability of $A$ given $B$ is the same as the probability of, and the conditional probability of $B$ given $A$ is the same as the probability of $B$.

Identify events as independent or dependent.

Interpret the conditional probability of $A$ given $B$ as answering the question ‘now that $B$ has occurred, what is the probability that event $A$ will occur?'

Determine the conditional probability of $A$ given $B$ using $P(A \text{ and } B)/P(B)$.

Represent conditional probability of $A$ given $B$ as $P(A|B)$.

Calculate conditional probabilities.

Construct two-way frequency tables for two categorical variables.

Calculate probabilities from the two-way frequency table.

Use the probabilities to assess independence of two variables.

How can you determine whether two events are independent or dependent?

How can you construct and interpret a two-way table?

How do you define and identify the independent events?

How do you calculate the probability of an event?

Type II, III:

Describing Events

Breakfast Before School

But mango is my favorite

How Do You Get to School

Rain and Lightning.

The Titanic 1

The Titanic 2

The Titanic 3

Two-Way Tables and Probability

Lucky Envelopes

Cards and Independence

The Addition Rule

Return to Fred’s Fun Factory
| Establish events as subsets of a sample space based on union, intersection, and/or complement of other events. | **SPED Strategies:**
Review two way frequency tables with students and relate this prior learning to calculating probability.

Model how to use real life examples to construct two way frequency tables and calculate probability based on those tables.

Create a reference document for students.

Provide students with opportunities to practice the skills learned by working with sample problems in small groups with peers. |
|---|---|
| **ELL Strategies:**
Explain orally and in written form how to use two-way frequency tables to determine if events are independent and to calculate conditional probability in the student’s native language and/or use gestures, examples and selected technical words.

Describe and explain independence and conditional probability in real-world situations in the student’s native language and/or use gestures, examples and selected technical words. | **But Mango is My Favorite** |
Model real life problem by conducting a survey with several questions and then ask students to create a two-way table with the result of the survey.

Practice verbal clues, and have students highlight the words in a published report that identified it as a comparative experiment, then complete a list of verbal clues to look when reports are independent or dependent.

New Jersey Student Learning Standard(s):

S.CP.B.6: Find the conditional probability of $A$ given $B$ as the fraction of $B$’s outcomes that also belong to $A$, and interpret the answer in terms of the model.

S.CP.B.7: Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, and interpret the answer in terms of the model.

Student Learning Objective #9: Find the conditional probability of $A$ given $B$ as the fraction of $B$’s outcomes that also belong to $A$ and apply the Addition Rule $[P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)]$.

Modified Student Learning Objectives/Standards:

M.EE.S-IC.1–2: Determine the likelihood of an event occurring when the outcomes are equally likely to occur.

<table>
<thead>
<tr>
<th>MPs</th>
<th>Evidence Statement Key/Clarifications</th>
<th>Skills, Strategies &amp; Concepts</th>
<th>Essential Understandings/Questions (Accountable Talk)</th>
<th>Tasks/Activities</th>
</tr>
</thead>
<tbody>
<tr>
<td>MP 1</td>
<td>N/A</td>
<td>Scatter plots of data sets can be used to identify the type of function that best represents the shape of the data (linear, quadratic or exponential).</td>
<td>Why would you want to identify trends or associations in a data set?</td>
<td>Type II, III: Coffee at Mom’s Diner</td>
</tr>
<tr>
<td>MP 2</td>
<td></td>
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<td>Rain and Lightning.</td>
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<td>MP 4</td>
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</table>
Residuals (lines of regressions) are drawn on scatter plots in order to informally assess the fit of a function to a data set. If a scatter plot has a linear association, then a line of best fit can be drawn to interpret the data set.

Mutually exclusive events exist.

Analyze event B’s outcomes to determine the proportion of B’s outcomes that also belong to event A. Interpret this proportion as conditional probability of A given B.

Identify two events as mutually exclusive (disjoint).

Calculate probabilities using the Addition rule

\[ P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) \]

**SPED Strategies:**

Pre-teach vocabulary using visual and verbal models that are connected to real life situations.

Review scatter plots and lines of regression and relate this concept to probability.

Model conditional probability scenarios and the mathematical models that correlate to each. Memorialize this learning by creating a reference document that illustrates the types of conditional probability scenarios, the mathematical model that correlates and the

identify a type of function to fit a data set?

<table>
<thead>
<tr>
<th>Venn Diagrams and the Addition Rule</th>
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<tbody>
<tr>
<td>How Do You Get to School</td>
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<tr>
<td>The Titanic 1</td>
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<tr>
<td>The Titanic 2</td>
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<tr>
<td>The Titanic 3</td>
</tr>
<tr>
<td>Describing Events</td>
</tr>
<tr>
<td>Return to Fred’s Fun Factory</td>
</tr>
</tbody>
</table>
thinking process involved in solving these problem types.

**ELL Strategies:**
Demonstrate comprehension of how to find the conditional probability of A given B as the fraction of B’s outcomes that also belong to A and by applying and interpreting the addition rule of probability in a uniform probability model in the student’s native language and/or use gestures, examples and selected technical words.

Use a graphing calculator to simulate, and then estimate conditional probabilities.

Write two examples to define dependent and conditional probability and introduce the notation P(B|A). Ask students to read aloud and describe the formula in their own words.
Integrated Evidence Statements

S-CP.Int.1: Solve multi-step contextual word problems with degree of difficulty appropriate to the course, requiring application of course-level knowledge and skills articulated in S-CP.
  • Calculating expected values of a random variable is a plus standard and not assessed; however, the word "expected" may be used informally (e.g., if you tossed a fair coin 20 times, how many heads would you expect?).

S-IC.Int.1: Solve multi-step contextual word problems with degree of difficulty appropriate to the course, requiring application of course-level knowledge and skills articulated in S-IC.
  • If the task addresses S-IC.4, the margin of error can be estimated as being 2 standard deviations of the sampling distribution of the statistic.

HS.C.17.2: Make inferences and justify conclusions from data. Content scope: S-IC.
  • For tasks that address simple random sample: A simple random sample requires that every possible group of the given sample size has an equal chance of being selected, not that every unit in the population has an equal chance of being selected.
  • For tasks that address comparing two data distributions: Comparisons of center, shape, and spread are required.

HS.C.17.3: Make inferences and justify conclusions from data. Content scope: S-IC.3
  • For tasks that address simple random sample: A simple random sample requires that every possible group of the given sample size has an equal chance of being selected, not that every unit in the population has an equal chance of being selected.

HS.C.17.4: Make inferences and justify conclusions from data. Content scope: S-IC.5
  • For tasks that address comparing two data distributions: Comparisons of center, shape, and spread are required.
  • Tasks may use the term ‘variability” and “spread”.

HS.C.17.5: Make inferences and justify conclusions from data. Content scope: S-IC.6
  • Reports should be based on content from S-IC.
  • For tasks that address simple random sample: A simple random sample requires that every possible group of the given sample size has an equal chance of being selected, not that every unit in the population has an equal chance of being selected.
  • For tasks that address comparing two data distributions: Comparisons of center, shape, and spread are required.
Integrated Evidence Statements

**HS.D.2-13:** Solve multi-step contextual word problems with degree of difficulty appropriate to the course, requiring application of course-level knowledge and skills articulated in S-ID and S-IC.

- If the content is only S-ID, the task must include Algebra 2 / Math 3 content (S-ID.4 or S-ID.6)
- Longer tasks may require some or all of the steps of the modeling cycle (CCSSM, pp. 72, 73); for example, see ITN Appendix F, "Karnataka" task (Section A "Illustrations of innovative task characteristics," subsection 7 "Modeling/Application," subsection f "Full Models"). As in the Karnataka example, algebra and function skills may be used.
- Predictions should not extrapolate far beyond the set of data provided.
- Line of best fit is always based on the equation of the least squares regression line either provided or calculated through the use of technology. Tasks may involve linear, exponential, or quadratic regressions. If the linear regression is in the task, the task must be written to allow students to choose the regression.
- To investigate associations, students may be asked to evaluate scatterplots that may be provided or created using technology. Evaluation includes shape, direction, strength, presence of outliers, and gaps.
- Analysis of residuals may include the identification of a pattern in a residual plot as an indication of a poor fit.
- Models may assess key features of the graph of the fitted model.
- Tasks that involve S-IC.2 might ask the students to look at the results of a simulation and decide how plausible the observed value is with respect to the simulation. For an example, see question 7 on the calculator section of the online practice test (http://practice.parcc.testnav.com/#).
- Tasks that involve S-ID.4, may require finding the area associated with a z-score using technology. Use of a z-score table will not be required.
- Tasks may involve finding a value at a given percentile based on a normal distribution.

**HS.D.3-5:** Decisions from data: Identify relevant data in a data source, analyze it, and draw reasonable conclusions from it. Content scope: Knowledge and skills articulated in Algebra 2.

- Tasks may result in an evaluation or recommendation.
- The purpose of tasks is not to provide a setting for the student to demonstrate breadth in data analysis skills (such as box-and-whisker plots and the like). Rather, the purpose is for the student to draw conclusions in a realistic setting using elementary techniques.
Integrated Evidence Statements

HS.D.CCR: Solve problems using modeling: Identify variables in a situation, select those that represent essential features, formulate a mathematical representation of the situation using those variables, analyze the representation and perform operations to obtain a result, interpret the result in terms of the original situation, validate the result by comparing it to the situation, and either improve the model or briefly report the conclusions. Content scope: Knowledge and skills articulated in the Standards as described in previous courses and grades, with a particular emphasis on 7- RP, 8 – EE, 8 – F, N-Q, A-CED, A-REI, F-BF, G-MG, Modeling, and S-ID

- Tasks will draw on securely held content from previous grades and courses, include down to Grade 7, but that are at the Algebra II/Mathematics III level of rigor.
- Task prompts describe a scenario using everyday language. Mathematical language such as "function," "equation," etc. is not used.
- Tasks require the student to make simplifying assumptions autonomously in order to formulate a mathematical model. For example, the student might make a simplifying assumption autonomously that every tree in a forest has the same trunk diameter, or that water temperature is a linear function of ocean depth.
- Tasks may require the student to create a quantity of interest in the situation being described.
<table>
<thead>
<tr>
<th>Number and Quantity</th>
<th>Algebra</th>
<th>Functions</th>
<th>Statistics and Probability</th>
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<tbody>
<tr>
<td>Complex number</td>
<td>Binomial Theorem</td>
<td>Absolute value function</td>
<td>2-way frequency table</td>
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<tr>
<td>Conjugate</td>
<td>Complete the square</td>
<td>Asymptote</td>
<td>Addition Rule</td>
</tr>
<tr>
<td>Determinant</td>
<td>Exponential function</td>
<td>Amplitude</td>
<td>Arithmetic sequence</td>
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<tr>
<td>Fundamental theorem of Algebra</td>
<td>Geometric series</td>
<td>Arc</td>
<td>Box plot</td>
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<tr>
<td>Identity matrix</td>
<td>Logarithmic function</td>
<td>Arithmetic sequence</td>
<td>Causation</td>
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<tr>
<td>Imaginary number</td>
<td>Function</td>
<td>Constant function</td>
<td>Combinations</td>
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<tr>
<td>Initial point</td>
<td>Maximum</td>
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<td>Minimum</td>
<td>Decreasing intervals</td>
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<td>Parallelogram rule</td>
<td>Polar form</td>
<td>Domain</td>
<td>Conditional relative frequency</td>
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<td>Rectangular form</td>
<td>End behavior</td>
<td>Correlation</td>
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<td>Polynomial</td>
<td>Scalar multiplication of Matrices</td>
<td>Exponential decay</td>
<td>Correlation coefficient</td>
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<td>Rational exponent</td>
<td>Recursive process</td>
<td>Exponential function</td>
<td>Dot plot</td>
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<td>Relative maximum</td>
<td>Exponential growth</td>
<td>Experiment</td>
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<td>Rectangular form</td>
<td>Relative minimum</td>
<td>Fibonacci sequence</td>
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<td>Scalar</td>
<td>Sine</td>
<td>Function notation</td>
<td>Frequency table</td>
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<td>multiplication of Matrices</td>
<td>Step function</td>
<td>Geometric sequence</td>
<td>Geometric sequence</td>
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<td>Terminal point</td>
<td>Symmetries</td>
<td>Increasing intervals</td>
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<td>Vector</td>
<td>Tangent</td>
<td>Intercepts</td>
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<td>Velocity</td>
<td>Invertible function</td>
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**Field Trip Ideas**

**SIX FLAGS GREAT ADVENTURE:** This educational event includes workbooks and special science and math related shows throughout the day. Your students will leave with a better understanding of real world applications of the material they have learned in the classroom. Each student will have the opportunity to experience different rides and attractions linking mathematical and scientific concepts to what they are experiencing.

[www.sixflags.com](http://www.sixflags.com)

**MUSEUM of MATHEMATICS:** Mathematics illuminates the patterns that abound in our world. The National Museum of Mathematics strives to enhance public understanding and perception of mathematics. Its dynamic exhibits and programs stimulate inquiry, spark curiosity, and reveal the wonders of mathematics. The Museum’s activities lead a broad and diverse audience to understand the evolving, creative, human, and aesthetic nature of mathematics.

[www.momath.org](http://www.momath.org)

**LIBERTY SCIENCE CENTER:** An interactive science museum and learning center located in Liberty State Park. The center, which first opened in 1993 as New Jersey's first major state science museum, has science exhibits, the largest IMAX Dome theater in the United States, numerous educational resources, and the original *Hoberman sphere*.

[http://lsc.org/plan-your-visit/](http://lsc.org/plan-your-visit/)