MATHEMATICS

Algebra I: Unit 3
Quadratic Equations, Functions & Polynomials
Course Philosophy/Description

The fundamental purpose of Algebra 1 is to formalize and extend the mathematics that students learned in the elementary and middle grades. The Standards for Mathematical Practice apply throughout each course, and, together with the New Jersey Student Learning Standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations. Conceptual knowledge behind the mathematics is emphasized. Algebra I provides a formal development of the algebraic skills and concepts necessary for students to succeed in advanced courses as well as the PARCC. The course also provides opportunities for the students to enhance the skills needed to become college and career ready.

The content shall include, but not be limited to, perform set operations, use fundamental concepts of logic including Venn diagrams, describe the concept of a function, use function notation, solve real-world problems involving relations and functions, determine the domain and range of relations and functions, simplify algebraic expressions, solve linear and literal equations, solve and graph simple and compound inequalities, solve linear equations and inequalities in real-world situations, rewrite equations of a line into slope-intercept form and standard form, graph a line given any variation of information, determine the slope, x- and y- intercepts of a line given its graph, its equation or two points on the line, write an equation of a line given any variation of information, determine a line of best fit and recognize the slope as the rate of change, factor polynomial expressions, perform operations with polynomials, simplify and solve algebraic ratios and proportions, simplify and perform operations with radical and rational expressions, simplify complex fractions, solve rational equations including situations involving mixture, distance, work and interest, solve and graph absolute value equations and inequalities, graph systems of linear equations and inequalities in two and three variables and quadratic functions, and use varied solution strategies for quadratic equations and for systems of linear equations and inequalities in two and three variables.
ESL Framework

This ESL framework was designed to be used by bilingual, dual language, ESL and general education teachers. Bilingual and dual language programs use the home language and a second language for instruction. ESL teachers and general education or bilingual teachers may use this document to collaborate on unit and lesson planning to decide who will address certain components of the SLO and language objective. ESL teachers may use the appropriate leveled language objective to build lessons for ELLs which reflects what is covered in the general education program. In this way, whether it is a pull-out or push-in model, all teachers are working on the same Student Learning Objective connected to the New Jersey Learning Student Standards. The design of language objectives are based on the alignment of the World-Class Instructional Design Assessment (WIDA) Consortium’s English Language Development (ELD) standards with the Common Core State Standards (CCSS). WIDA’s ELD standards advance academic language development across content areas ultimately leading to academic achievement for English learners. As English learners are progressing through the six developmental linguistic stages, this framework will assist all teachers who work with English learners to appropriately identify the language needed to meet the requirements of the content standard. At the same time, the language objectives recognize the cognitive demand required to complete educational tasks. Even though listening and reading (receptive) skills differ from speaking and writing (expressive) skills across proficiency levels the cognitive function should not be diminished. For example, an Entering Level One student only has the linguistic ability to respond in single words in English with significant support from their home language. However, they could complete a Venn diagram with single words which demonstrates that they understand how the elements compare and contrast with each other or they could respond with the support of their native language with assistance from a teacher, para-professional, peer or a technology program.

http://www.state.nj.us/education/modelcurriculum/ela/ELLOverview.pdf
<table>
<thead>
<tr>
<th>#</th>
<th>Student Learning Objective</th>
<th>NJSLS</th>
<th>Big Ideas Math Correlation</th>
<th>Instruction: 8 weeks</th>
<th>Assessment: 1 week</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Add, subtract, and multiply polynomials, relating these to arithmetic operations with integers. Factor to produce equivalent forms of quadratic expressions in one variable.</td>
<td>A.APR.A.1,</td>
<td>7.1, 7.2, 7.3, 7.5, 7.6, 7.7, 7.8</td>
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<td></td>
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<td>A.SSE.A.2</td>
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<tr>
<td>2</td>
<td>Derive the quadratic formula by completing the square and recognize when there are no real solutions.</td>
<td>A.REI.B.4a</td>
<td>9.4, 9.5</td>
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<td>3</td>
<td>Solve quadratic equations in one variable using a variety of methods (including inspection, taking square roots, factoring, completing the square, and the quadratic formula) and write complex solutions in a ± bi form.</td>
<td>A.REI.B.4b</td>
<td>7.4, 9.3, 9.4, 9.5</td>
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<tr>
<td>4</td>
<td>Create quadratic equations in one variable and use them to solve problems.</td>
<td>A.CED.A.1</td>
<td>9.3, 9.4, 9.5</td>
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<td>5</td>
<td>Interpret key features of quadratic functions from graphs and tables. Given a verbal description of the relationship, sketch the graph of a quadratic function, showing key features and relating the domain of the function to its graph.</td>
<td>F.IF.B.4*</td>
<td>8.4, 8.5</td>
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<td></td>
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<td>F.IF.B.5*</td>
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<tr>
<td>6</td>
<td>Use factoring and completing the square to produce equivalent forms of quadratic expressions in one variable that highlight particular properties such as the zeros or the maximum or minimum value of the function.</td>
<td>A.SSE.B.3a,3b</td>
<td>7.5, 7.6, 7.7, 7.8, 8.5, 9.4</td>
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<td>7</td>
<td>Given a context, write an explicit expression, a recursive process or steps for calculation for quadratic relationships.</td>
<td>F.BF.A.1a</td>
<td>7.5, 7.6, 7.7, 7.8, 8.5, 9.4</td>
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<td>Pacing Chart – Unit 3</td>
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<tr>
<td>8</td>
<td>Graph quadratic functions by hand in simple cases and with technology in complex cases, showing intercepts, extreme values and symmetry of the graph. Compare properties of two quadratic functions, each represented in a different way.</td>
<td>F.IF.C.7a*, F.IF.C.8a*, F.IF.C.9 *</td>
<td>7.5, 7.6, 7.7, 7.8, 8.5, 9.4</td>
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<td>9</td>
<td>Calculate and interpret the average rate of change of a quadratic function presented symbolically or as a table. Estimate and compare the rates of change from graphs of quadratic and exponential functions.</td>
<td>F.IF.B.6, F.LE.A.3</td>
<td>7.5, 7.6, 7.7, 7.8, 8.5, 9.4</td>
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<td>10</td>
<td>Identify the effects of transformations and combinations of transformations ([f(x) + k, k f(x), f(kx), \text{and } f(x + k)]) on a function; find the value of (k) given the graph.</td>
<td>F.BF.B.3</td>
<td>7.5, 7.6, 7.7, 7.8, 8.5, 9.4</td>
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<tr>
<td>11</td>
<td>Find approximate solutions of (f(x) = g(x)), where (f(x)) is a linear function and (g(x)) is a quadratic function by making a table of values, using technology to graph and finding successive approximations.</td>
<td>A.RELD.11</td>
<td>7.5, 7.6, 7.7, 7.8, 8.5, 9.4</td>
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<tr>
<td>12</td>
<td>Identify zeros of cubic functions when suitable factorizations are available and use the zeros to construct a rough graph of the function. (*cubic functions are presented as the product of a linear and a quadratic factor)</td>
<td>A.APR.B.3</td>
<td>8.5</td>
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<td>13</td>
<td>Explain and justify conclusions about sums and products of rational and irrational numbers.</td>
<td>N.RN.B.3</td>
<td>9.1</td>
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</table>
Research about Teaching and Learning Mathematics

Structure teaching of mathematical concepts and skills around problems to be solved (Checkly, 1997; Wood & Sellars, 1996; Wood & Sellars, 1997)

Encourage students to work cooperatively with others (Johnson & Johnson, 1975; Davidson, 1990)

Use group problem-solving to stimulate students to apply their mathematical thinking skills (Artzt & Armour-Thomas, 1992)

Students interact in ways that support and challenge one another’s strategic thinking (Artzt, Armour-Thomas, & Curcio, 2008)

Activities structured in ways allowing students to explore, explain, extend, and evaluate their progress (National Research Council, 1999)

There are three critical components to effective mathematics instruction (Shellard & Moyer, 2002):

- Teaching for conceptual understanding
- Developing children’s procedural literacy
- Promoting strategic competence through meaningful problem-solving investigations

Teachers should be:

- Demonstrating acceptance and recognition of students’ divergent ideas.
- Challenging students to think deeply about the problems they are solving, extending thinking beyond the solutions and algorithms required to solve the problem
- Influencing learning by asking challenging and interesting questions to accelerate students’ innate inquisitiveness and foster them to examine concepts further.
- Projecting a positive attitude about mathematics and about students’ ability to “do” mathematics

Students should be:

- Actively engaging in “doing” mathematics
- Solving challenging problems
- Investigating meaningful real-world problems
- Making interdisciplinary connections
- Developing an understanding of mathematical knowledge required to “do” mathematics and connect the language of mathematical ideas with numerical representations
- Sharing mathematical ideas, discussing mathematics with one another, refining and critiquing each other’s ideas and understandings
- Communicating in pairs, small group, or whole group presentations
- Using multiple representations to communicate mathematical ideas
- Using connections between pictures, oral language, written symbols, manipulative models, and real-world situations
- Using technological resources and other 21st century skills to support and enhance mathematical understanding
Mathematics is not a stagnate field of textbook problems; rather, it is a dynamic way of constructing meaning about the world around us, generating knowledge and understanding about the real world every day. Students should be metaphorically rolling up their sleeves and “doing mathematics” themselves, not watching others do mathematics for them or in front of them. (Protheroe, 2007)

Balanced Mathematics Instructional Model

Balanced math consists of three different learning opportunities: guided math, shared math, and independent math. Ensuring a balance of all three approaches will build conceptual understanding, problem solving, computational fluency, and procedural fluency. Building conceptual understanding is the focal point of developing mathematical proficiency. Students should frequently work on rigorous tasks, talk about the math, explain their thinking, justify their answer or process, build models with graphs or charts or manipulatives, and use technology.

When balanced math is used in the classroom it provides students opportunities to:

- solve problems
- make connections between math concepts and real-life situations
- communicate mathematical ideas (orally, visually and in writing)
- choose appropriate materials to solve problems
- reflect and monitor their own understanding of the math concepts
- practice strategies to build procedural and conceptual confidence

Teacher builds conceptual understanding by modeling through demonstration, explicit instruction, and think alouds, as well as guiding students as they practice math strategies and apply problem solving strategies. (whole group or small group instruction)

Teacher and students practice mathematics processes together through interactive activities, problem solving, and discussion. (whole group or small group instruction)

Students practice math strategies independently to build procedural and computational fluency. Teacher assesses learning and reteaches as necessary. (whole group instruction, small group instruction, or centers)
<table>
<thead>
<tr>
<th>Effective Pedagogical Routines/Instructional Strategies</th>
</tr>
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<tbody>
<tr>
<td><strong>Collaborative Problem Solving</strong></td>
</tr>
<tr>
<td>Connect Previous Knowledge to New Learning</td>
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<tr>
<td>Making Thinking Visible</td>
</tr>
<tr>
<td><strong>Develop and Demonstrate Mathematical Practices</strong></td>
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<td>Inquiry-Oriented and Exploratory Approach</td>
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<td>Multiple Solution Paths and Strategies</td>
</tr>
<tr>
<td>Use of Multiple Representations</td>
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<tr>
<td><strong>Explain the Rationale of your Math Work</strong></td>
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<td>Quick Writes</td>
</tr>
<tr>
<td>Pair/Trio Sharing</td>
</tr>
<tr>
<td>Turn and Talk</td>
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<tr>
<td>Charting</td>
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<tr>
<td>Gallery Walks</td>
</tr>
<tr>
<td><strong>Small Group and Whole Class Discussions</strong></td>
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<tr>
<td>Student Modeling</td>
</tr>
<tr>
<td><strong>Analyze Student Work</strong></td>
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<tr>
<td>Identify Student’s Mathematical Understanding</td>
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<tr>
<td>Identify Student’s Mathematical Misunderstandings</td>
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<tr>
<td>Interviews</td>
</tr>
<tr>
<td><strong>Role Playing</strong></td>
</tr>
<tr>
<td><strong>Anticipate Likely and Possible Student Responses</strong></td>
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<td><strong>Collect Different Student Approaches</strong></td>
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<tr>
<td><strong>Multiple Response Strategies</strong></td>
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<tr>
<td><strong>Asking Assessing and Advancing Questions</strong></td>
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<tr>
<td><strong>Revoicing</strong></td>
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<td><strong>Marking</strong></td>
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<tr>
<td><strong>Recapping</strong></td>
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<td><strong>Challenging</strong></td>
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<tr>
<td><strong>Pressing for Accuracy and Reasoning</strong></td>
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<td><strong>Maintain the Cognitive Demand</strong></td>
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</table>
### Educational Technology

#### Standards


<table>
<thead>
<tr>
<th>➢ Technology Operations and Concepts</th>
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<tr>
<td>• Collaborate in online courses, learning communities, social networks or virtual worlds to discuss a resolution to a problem or issue.</td>
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<tr>
<td><strong>Example:</strong> Students can work collaboratively to explore how transformations can be visualized both algebraically and graphically using graphing calculators. Students share their findings with classmates using Google Docs and/or Google Classroom. <a href="https://www.desmos.com/calculator">https://www.desmos.com/calculator</a></td>
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<tr>
<td>• Create a report from a relational database consisting of at least two tables and describe the process, and explain the report results.</td>
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<tr>
<td><strong>Example:</strong> Students can analyze the key features of quadratic functions from graphs and tables using digital tools and discuss their significance in the context of the task. <a href="http://a4a.learnport.org/page/algebra-for-all-resources">http://a4a.learnport.org/page/algebra-for-all-resources</a></td>
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<thead>
<tr>
<th>➢ Communication and Collaboration</th>
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<tr>
<td>• Develop an innovative solution to a real-world problem or issue in collaboration with peers and experts, and present ideas for feedback through social media or in an online community.</td>
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<tr>
<td><strong>Example:</strong> Students use digital tools to model functions and to solve problems involving the intersection of a linear function and a quadratic function in real life contexts. <a href="https://www.desmos.com/calculator">https://www.desmos.com/calculator</a></td>
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</table>

**Link:** [http://www.state.nj.us/education/cces/standards/8/](http://www.state.nj.us/education/cces/standards/8/)
Career Ready Practices

Career Ready Practices describe the career-ready skills that all educators in all content areas should seek to develop in their students. They are practices that have been linked to increase college, career, and life success. Career Ready Practices should be taught and reinforced in all career exploration and preparation programs with increasingly higher levels of complexity and expectation as a student advances through a program of study.

- **CRP2. Apply appropriate academic and technical skills.**
  Career-ready individuals readily access and use the knowledge and skills acquired through experience and education to be more productive. They make connections between abstract concepts with real-world applications, and they make correct insights about when it is appropriate to apply the use of an academic skill in a workplace situation.

  **Example:** Students will apply prior knowledge when solving real world problems. Students will make sound judgments about the use of specific tools, such as Graphing Calculators, to explore and deepen understanding the concepts related to quadratic equations, functions and polynomials including how to solve, write, graph, interpret and explain these relationships accurately.

- **CRP4. Communicate clearly and effectively and with reason.**
  Career-ready individuals communicate thoughts, ideas, and action plans with clarity, whether using written, verbal, and/or visual methods. They communicate in the workplace with clarity and purpose to make maximum use of their own and others’ time. They are excellent writers; they master conventions, word choice, and organization, and use effective tone and presentation skills to articulate ideas. They are skilled at interacting with others; they are active listeners and speak clearly and with purpose. Career-ready individuals think about the audience for their communication and prepare accordingly to ensure the desired outcome.

  **Example:** Students will engage in mathematical discourse on a daily basis. They will be expected to communicate the reasoning behind their solution paths, make connections to the context and the quantities involved, and use proper vocabulary. Students will be able to accurately describe the relationships depicted in quadratic equations, functions and polynomials visually, verbally and algebraically. They will be able to explain the meaning behind the solution path/representation chosen and defend their rationale. Students will also ask probing questions of others to clarify and deepen understanding.
Career Ready Practices

- **CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.**
  Career-ready individuals readily recognize problems in the workplace, understand the nature of the problem and devise effective plans to solve the problem. They are aware of problems when they occur and take action quickly to address the problem; they thoughtfully investigate the root cause of the problem prior to introducing solutions. They carefully consider the options to solve the problem. Once a solution is agreed upon, they follow through to ensure the problem is solved, whether through their own actions or the actions of others.

  **Example:** Throughout their daily lessons, students will understand the meaning of a problem by analyzing the relationships among the quantities, constraints and goals of the task. This analytic process will encourage students to find entry points that will facilitate problem solving. Students will become effective at self-monitoring, evaluating and critiquing their process. This in turn will facilitate their ability to progress as they are working and change strategy when necessary. Students will be able to use critical thinking to solve, model, interpret, create, and compare quadratic equations, functions and polynomials.

- **CRP12. Work productively in teams while using cultural global competence.**
  Career-ready individuals positively contribute to every team, whether formal or informal. They apply an awareness of cultural difference to avoid barriers to productive and positive interaction. They find ways to increase the engagement and contribution of all team members. They plan and facilitate effective team meetings.

  **Example:** Students will work in collaborative and whole group settings to develop various solutions to math tasks. They will work together to understand the terms of the problem, ask each other clarifying and challenging questions, and develop agreed upon solutions using a variety of strategies and models. Students will listen to, read and discuss arguments with respect and courtesy at all times. A willingness to assist one another will be cultivated and honored. Students will demonstrate and explain to a peer or small group how to solve, model, interpret, create, compare and explain quadratic equations, functions and polynomials.
WIDA Proficiency Levels

At the given level of English language proficiency, English language learners will process, understand, produce or use

<table>
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<tr>
<th>Level</th>
<th>Description</th>
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</table>
| 6- Reaching | Specialized or technical language reflective of the content areas at grade level  
A variety of sentence lengths of varying linguistic complexity in extended oral or written discourse as required by the specified grade level  
Oral or written communication in English comparable to proficient English peers |
| 5- Bridging | Specialized or technical language of the content areas  
A variety of sentence lengths of varying linguistic complexity in extended oral or written discourse, including stories, essays or reports  
Oral or written language approaching comparability to that of proficient English peers when presented with grade level material. |
| 4- Expanding | Specific and some technical language of the content areas  
A variety of sentence lengths of varying linguistic complexity in oral discourse or multiple, related sentences or paragraphs  
Oral or written language with minimal phonological, syntactic or semantic errors that may impede the communication, but retain much of its meaning, when presented with oral or written connected discourse, with sensory, graphic or interactive support |
| 3- Developing | General and some specific language of the content areas  
Expanded sentences in oral interaction or written paragraphs  
Oral or written language with phonological, syntactic or semantic errors that may impede the communication, but retain much of its meaning, when presented with oral or written, narrative or expository descriptions with sensory, graphic or interactive support |
| 2- Beginning | General language related to the content area  
Phrases or short sentences  
Oral or written language with phonological, syntactic, or semantic errors that often impede of the communication when presented with one to multiple-step commands, directions, or a series of statements with sensory, graphic or interactive support |
| 1- Entering | Pictorial or graphic representation of the language of the content areas  
Words, phrases or chunks of language when presented with one-step commands directions, WH-, choice or yes/no questions, or statements with sensory, graphic or interactive support |
# Language Development Supports For English Language Learners

## To Increase Comprehension and Communication Skills

<table>
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<tr>
<th>Environment</th>
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| - Welcoming and stress-free  
- Respectful of linguistic and cultural diversity  
- Honors students’ background knowledge  
- Sets clear and high expectations  
- Includes routines and norms  
- Is thinking-focused vs. answer-seeking  
- Offers multiple modalities to engage in content learning and to demonstrate understanding  
- Includes explicit instruction of specific language targets  
- Provides participation techniques to include all learners  
- Integrates learning centers and games in a meaningful way  
- Provides opportunities to practice and refine receptive and productive skills in English as a new language  
- Integrates meaning and purposeful tasks/activities that:  
  - Are accessible by all students through multiple entry points  
  - Are relevant to students’ lives and cultural experiences  
  - Build on prior mathematical learning  
  - Demonstrate high cognitive demand  
  - Offer multiple strategies for solutions  
  - Allow for a language learning experience in addition to content |

<table>
<thead>
<tr>
<th>Sensory Supports*</th>
<th>Graphic Supports*</th>
<th>Interactive Supports*</th>
<th>Verbal and Textual Supports</th>
</tr>
</thead>
</table>
| - Real-life objects (realia) or concrete objects  
- Physical models  
- Manipulatives  
- Pictures & photographs  
- Visual representations or models such as diagrams or drawings  
- Videos & films  
- Newspapers or magazines  
- Gestures  
- Physical movements  
- Music & songs | - Graphs  
- Charts  
- Timelines  
- Number lines  
- Graphic organizers  
- Graphing paper | - In a whole group  
- In a small group  
- With a partner such as Turn-and-Talk  
- In pairs as a group (first, two pairs work independently, then they form a group of four)  
- In triads  
- Cooperative learning structures such as Think-Pair-Share  
- Interactive websites or software  
- With a mentor or coach | - Labeling  
- Students’ native language  
- Modeling  
- Repetitions  
- Paraphrasing  
- Summarizing  
- Guiding questions  
- Clarifying questions  
- Probing questions  
- Leveled questions such as What? When? Where? How? Why?  
- Questioning prompts & cues  
- Word Banks  
- Sentence starters  
- Sentence frames  
- Discussion frames  
- Talk moves, including Wait Time |

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Galina (Halia) Imourko, ESOL Coach, PGCPS; 2015, Roud. 2016
BUILDING EQUITY IN YOUR TEACHING PRACTICE

How do the essential questions highlight the connection between the big ideas of the unit and equity in your teaching practice?

**CONTENT INTEGRATION**
Teachers use examples and content from a variety of cultures & groups.

This unit/lesson is connected to other topics explored with students.

There are multiple viewpoints reflected in the content of this unit/lesson.

The materials and resources are reflective of the diverse identities and experiences of students.

The content affirms students, as well as exposes them to experiences other than their own.

**KNOWLEDGE CONSTRUCTION**
Teachers help students understand how knowledge is created and influenced by cultural assumptions, perspectives & biases.

This unit/lesson provides context to the history of privilege and oppression.

This unit/lesson addresses power relationships.

This unit/lesson helps students to develop research and critical thinking skills.

This curriculum creates windows and mirrors* for students.

**PREJUDICE REDUCTION**
Teachers implement lessons and activities to assert positive images of ethnic groups & improve intergroup relations.

This unit/lesson helps students question and unpack biases & stereotypes.

This unit/lesson helps students examine, research and question information and sources.

The curriculum encourages discussion and understanding about the groups of people being represented.

This unit/lesson challenges dominant perspectives.

**EQUITABLE PEDAGOGY**
Teachers modify techniques and methods to facilitate the academic achievement of students from diverse backgrounds.

The instruction has been modified to meet the needs of each student.

Students feel respected and their cultural identities are valued.

Additional supports have been provided for students to become successful and independent learners.

Opportunities are provided for student to reflect on their learning and provide feedback.

**EMPOWERING SCHOOL CULTURE**
Using the other four dimensions to create a safe and healthy educational environment for all.

There are opportunities for students to connect with the community.

My classroom is welcoming and supportive for all students.

I am aware of and sensitive to the needs of my students and their families.

There are effective parent communication systems established. Parents can talk to me about issues as they arise in my classroom.

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## Culturally Relevant Pedagogy Examples

- **Integrate Relevant Word Problems:** Contextualize equations using word problems that reference student interests and cultures.
  
  **Example:** When learning about the key features of quadratic functions, incorporating problems that relate to student interests such as music, sports and art enable the students to understand and relate to the concept in a more meaningful way.

- **Everyone has a Voice:** Create a classroom environment where students know that their contributions are expected and valued.
  
  **Example:** Norms for sharing are established that communicate a growth mindset for mathematics. All students are capable of expressing mathematical thinking and contributing to the classroom community. Students learn new ways of looking at problem solving by working with and listening to each other.

- **Run Problem Based Learning Scenarios:** Encourage mathematical discourse among students by presenting problems that are relevant to them, the school and/or the community.
  
  **Example:** Using a Place Based Education (PBE) model, students explore math concepts while determining ways to address problems that are pertinent to their neighborhood, school or culture.

- **Encourage Student Leadership:** Create an avenue for students to propose problem solving strategies and potential projects.
  
  **Example:** Students can deepen their understanding of quadratic functions and polynomials by creating problems together and deciding if the problems fit the necessary criteria. This experience will allow students to discuss and explore their current level of understanding by applying the concepts to relevant real-life experiences.

- **Present New Concepts Using Student Vocabulary:** Use student diction to capture attention and build understanding before using academic terms.
  
  **Example:** Teach math vocabulary in various modalities for students to remember. Use multi-modal activities, analogies, realia, visual cues, graphic representations, gestures, pictures and cognates. Directly explain and model the idea of vocabulary words having multiple meanings. Students can create the Word Wall with their definitions and examples to foster ownership.
# Differentiated Instruction

**Accommodate Based on Students Individual Needs: Strategies**

<table>
<thead>
<tr>
<th>Time/General</th>
<th>Processing</th>
<th>Comprehension</th>
<th>Recall</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Extra time for assigned tasks</td>
<td>• Extra Response time</td>
<td>• Precise processes for balanced math instructional model</td>
<td>• Teacher-made checklist</td>
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<tr>
<td>• Adjust length of assignment</td>
<td>• Have students verbalize steps</td>
<td>• Short manageable tasks</td>
<td>• Use visual graphic organizers</td>
</tr>
<tr>
<td>• Timeline with due dates for reports and projects</td>
<td>• Repeat, clarify or reword directions</td>
<td>• Brief and concrete directions</td>
<td>• Reference resources to promote independence</td>
</tr>
<tr>
<td>• Communication system between home and school</td>
<td>• Mini-breaks between tasks</td>
<td>• Provide immediate feedback</td>
<td>• Visual and verbal reminders</td>
</tr>
<tr>
<td>• Provide lecture notes/outline</td>
<td>• Provide a warning for transitions</td>
<td>• Small group instruction</td>
<td>• Graphic organizers</td>
</tr>
<tr>
<td>• Partnering</td>
<td>• Partnering</td>
<td>• Emphasize multi-sensory learning</td>
<td></td>
</tr>
</tbody>
</table>

**Assistive Technology**

- Computer/whiteboard
- Tape recorder
- Video Tape

**Tests/Quizzes/Grading**

- Extended time
- Study guides
- Shortened tests
- Read directions aloud

**Behavior/Attention**

- Consistent daily structured routine
- Simple and clear classroom rules
- Frequent feedback

**Organization**

- Individual daily planner
- Display a written agenda
- Note-taking assistance
- Color code materials
## Differentiated Instruction

### Accommodate Based on Content Needs: Strategies

- Anchor charts to model strategies and process
- Reference sheets that list formulas, step-by-step procedures and model strategies
- Conceptual word wall that contains definition, translation, pictures and/or examples
- Graphic organizer to help students solve quadratic equations using different methods (such as quadratic formula, completing the square, factoring, etc.)
- Translation dictionary
- Sentence stems to provide additional language support for ELL students
- Teacher modeling
- Highlight and label solution steps for multi-step problems in different colors
- Create an interactive notebook with students with a table of contents so they can refer to previously taught material readily
- Exponent rules chart
- Graph paper
- Algebra tiles
- Graphing calculator
- Visual, verbal and algebraic models of quadratic functions
- A chart noting key features of quadratic functions based on visual, graphical and verbal presentation
- Videos to reinforce skills and thinking behind concepts
- Access to tools such as tables, graphs and charts to solve problems
## Interdisciplinary Connections

Model interdisciplinary thinking to expose students to other disciplines.

### Social Studies Connection:
**Population and Food Supply (6.2.12.B.6.a)**
- Students will analyze the population and food production growth and analyze the main challenge of food shortages.

### Financial Literacy Connection:
**Profit of a Company (9.1.8.E.1)**
- Students will analyze the profit function, the break-even point and the maximum profit point of a company.
**Stock Prices (9.1.8.D.2)**
- Students will compare the performance of different stocks to make investment decisions.

### Science Connection:
**Braking Distance & Modeling a Context from a Verbal Description (MS-PS2-2)**
- Students will learn about deceleration, speed and distance needed to stop. Safety is a factor to be considered in the class discussion.
**Flares (MS-PS1-6)**
- Students will learn about deceleration trajectory of flares and the distance traveled.

### Physical Education Connection:
**Springboard Dive (2.5.8.B.2, 2.5.12.B.2)**
- Students will analyze the time, pathway and height of a diver.
**Throwing Baseballs (2.5.8.A.2, 2.5.12.A.2)**
- Students will analyze the time, pathway and height of a baseball as the players compete.
## Enrichment

### What is the Purpose of Enrichment?

- The purpose of enrichment is to provide extended learning opportunities and challenges to students who have already mastered, or can quickly master, the basic curriculum. Enrichment gives the student more time to study concepts with greater depth, breadth, and complexity.
- Enrichment also provides opportunities for students to pursue learning in their own areas of interest and strengths.
- Enrichment keeps advanced students engaged and supports their accelerated academic needs.
- Enrichment provides the most appropriate answer to the question, “What do you do when the student already knows it?”

### Enrichment is...

- Planned and purposeful
- *Different*, or differentiated, work – not just *more* work
- Responsive to students’ needs and situations
- A promotion of high-level thinking skills and making connections within content
- The ability to apply different or multiple strategies to the content
- The ability to synthesize concepts and make real world and cross-curricular connections
- Elevated contextual complexity
- Sometimes independent activities, sometimes direct instruction
- Inquiry based or open-ended assignments and projects
- Using supplementary materials in addition to the normal range of resources
- Choices for students
- Tiered/Multi-level activities with flexible groups (may change daily or weekly)

### Enrichment is not...

- Just for gifted students (some gifted students may need intervention in some areas just as some other students may need frequent enrichment)
- Worksheets that are more of the same (busywork)
- Random assignments, games, or puzzles not connected to the content areas or areas of student interest
- Extra homework
- A package that is the same for everyone
- Thinking skills taught in isolation
- Unstructured free time
Assessments

**Required District/State Assessments**
- Unit Assessment
- PARCC
- SGO Assessments

**Suggested Formative/Summative Classroom Assessments**
- Describe Learning Vertically
- Identify Key Building Blocks
- Make Connections (between and among key building blocks)
- Short/Extended Constructed Response Items
- Multiple-Choice Items (where multiple answer choices may be correct)
- Drag and Drop Items
- Use of Equation Editor
- Quizzes
- Journal Entries/Reflections/Quick-Writes
- Accountable talk
- Projects
- Portfolio
- Observation
- Graphic Organizers/ Concept Mapping
- Presentations
- Role Playing
- Teacher-Student and Student-Student Conferencing
- Homework
# New Jersey Student Learning Standards

<table>
<thead>
<tr>
<th>Standard</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A.APR.A.1</strong></td>
<td>Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.</td>
</tr>
<tr>
<td><strong>A.SSE.A.2</strong></td>
<td>Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.</td>
</tr>
</tbody>
</table>
| **A.REI.B.4** | Solve quadratic equations in one variable.  
  
  **A.REI.B.4a**: Use the method of completing the square to transform any quadratic equation in $x$ into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.  
  
  **A.REI.B.4b**: Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers $a$ and $b$. |
| **A.CED.A.1** | Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear functions and quadratic functions, and simple rational and exponential functions. |
| **F.IF.B.4** | For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. **Key features include**: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. |
| **F.IF.B.5** | Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function $h(n)$ gives the number of person-hours it takes to assemble $n$ engines in a factory, then the positive integers would be an appropriate domain for the function. |
| **A.SSE.B.3** | Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.  
  
  **A.SSE.B.3a**: Factor a quadratic expression to reveal the zeros of the function it defines. |
**A.SSE.B.3b:** Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.

### New Jersey Student Learning Standards

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<tr>
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</table>
| **F.BF.A.1:** | Write a function that describes a relationship between two quantities.  
  
  **F.BF.A.1a:** Determine an explicit expression, a recursive process, or steps for calculation from a context. |
| **F.IF.C.7:** | Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.  
  
  **F.IF.C.7a:** Graph linear and quadratic functions and show intercepts, maxima, and minima. *[emphasize quadratic functions]* |
| **F.IF.C.8:** | Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.  
  
  **F.IF.C.8a:** Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. |
| **F.IF.C.9:** | Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).  
  
  *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum* |
| **F.IF.B.6:** | Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. |
| **F.LE.A.3:** | Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. |
| **F.BF.B.3:** | Identify the effect on the graph of replacing \( f(x) \) by \( f(x) + k \), \( k f(x) \), \( f(kx) \), and \( f(x + k) \) for specific values of \( k \) (both positive and negative); find the value of \( k \) given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. |
### A.REI.D.11
Explain why the x-coordinates of the points where the graphs of the equations \( y = f(x) \) and \( y = g(x) \) intersect are the solutions of the equation \( f(x) = g(x) \); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where \( f(x) \) and/or \( g(x) \) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

### New Jersey Student Learning Standards

| A.APR.B.3 | Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial. *[Algebra 1: limit to quadratic and cubic functions in which linear and quadratic factors are available]. |
| N.RN.B.3 | Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational. |
### Mathematical Practices

1. Make sense of problems and persevere in solving them.

2. Reason abstractly and quantitatively.

3. Construct viable arguments and critique the reasoning of others.

4. Model with mathematics.

5. Use appropriate tools strategically.

6. Attend to precision.

7. Look for and make use of structure.

8. Look for and express regularity in repeated reasoning.
| Course: Algebra 1 | Unit: 3 (Three) | Topic: Quadratic Equations, Functions & Polynomials |

**NJSLS:**

**Unit Focus:**
- Perform arithmetic operations on polynomials
- Understand the relationship between zeros and factors
- Interpret the structure of expressions
- Solve equations and inequalities in one variable
- Create equations that describe numbers or relationships
- Interpret functions that arise in applications in terms of the context
- Represent and solve equations and inequalities graphically
- Build a function that models a relationship between two quantities
- Construct & compare linear, quadratic, & exponential models
- Build new functions from existing functions
- Analyze functions using different representations
- Use properties of rational and irrational numbers
### New Jersey Student Learning Standards (s):

**A.APR.A.1**: Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

**A.SSE.A.2**: Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.

### Student Learning Objective 1:
Add, subtract, and multiply polynomials, relating these to arithmetic operations with integers. Factor to produce equivalent forms of quadratic expressions in one variable.

### Modified Student Learning Objectives/Standards: N/A

<table>
<thead>
<tr>
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<th>Evidence Statement Key/Clarifications</th>
<th>Skills, Strategies &amp; Concepts</th>
<th>Essential Understandings/Questions (Accountable Talk)</th>
<th>Tasks/Activities</th>
</tr>
</thead>
<tbody>
<tr>
<td>MP 2</td>
<td>A-APR.1-1</td>
<td>Recognize numerical expressions as a difference of squares and rewrite the expression as the product of sums/differences.</td>
<td>How might polynomials be classified?</td>
<td>Type I:</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Recognize polynomial expressions in one variable as a difference of squares and rewrite the expression as the product of sums/differences</td>
<td>How can students use polynomial operations of addition, subtraction, and multiplication in real-world situations?</td>
<td>Algebraic Expressions – The Distributive Property</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Understand the concepts of combining like terms and closure.</td>
<td>Why is the system of polynomials closed under addition, subtraction and multiplication?</td>
<td>Multiplying Polynomials</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Add, subtract, and multiply polynomials and understand how closure applies under these operations.</td>
<td>How is the system of polynomials similar to and different from the system of integers?</td>
<td>Type II – III:</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Polynomials form a system analogous to the integers.</td>
<td></td>
<td>Bean Bag Toss</td>
</tr>
<tr>
<td>MP 7</td>
<td>HS.C.8.1</td>
<td>A-SSE.2-1</td>
<td>How might polynomials be classified?</td>
<td>Type I:</td>
</tr>
<tr>
<td></td>
<td>Construct autonomously, chains of reasoning that will justify or refute algebraic propositions or conjectures. Content scope: A-APR.1</td>
<td>Recognize numerical expressions as a difference of squares and rewrite the expression as the product of sums/differences.</td>
<td>How can students use polynomial operations of addition, subtraction, and multiplication in real-world situations?</td>
<td>Algebraic Expressions – The Distributive Property</td>
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<td></td>
<td>A-SSE.2-1</td>
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<td>Why is the system of polynomials closed under addition, subtraction and multiplication?</td>
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<td>Understand the concepts of combining like terms and closure.</td>
<td>How is the system of polynomials similar to and different from the system of integers?</td>
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</tr>
<tr>
<td></td>
<td></td>
<td>Polynomials form a system analogous to the integers.</td>
<td></td>
<td>Quadratic</td>
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<td>Released Items / Suggested Tasks:</td>
</tr>
<tr>
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<td>EOY 10</td>
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<td>PBA 5</td>
</tr>
</tbody>
</table>
- Examples: Recognize $53^2 - 47^2$ as a difference of squares and see an opportunity to rewrite it in the easier-to-evaluate form $(53 + 47)(53 - 47)$.
- Limit to problems intended to be solved with one step.
- Tasks do not have a context.

**A-SSE.2-4**
- Example: Factor completely: $x^2 - 1 + (x-1)$. (A first iteration might give $(x+1)(x-1) + (x-1)^2$, which could be rewritten as $(x-1)(x+1+x-1)$ on the way to factorizing completely as $2x(x-1)$. Or the student might first expand, as $-1 + x^2 - 2x + 1$, rewriting as $2x^2 - 2x$, then factorizing as $2x(x-1)$.)
- Tasks do not have a real-world context.

**SPED Strategies:**
Provide students with a review of adding, subtracting and multiplying integers and then make the connections to adding, subtracting and multiplying polynomials.

Model the thinking process involved in difference of squares and how an equivalent expression can be expressed as the product of sums/differences.

**ELL Support:**
Create an anchor chart with students that review integer addition, subtraction, multiplication and division using diagrams, linguistically simplified terms and native language as needed.

Model adding, subtracting, multiplying and dividing polynomials by relating it to simpler examples.

Encourage students to work with peers and discuss the thinking and processes involved so as to increase mathematical and academic language proficiency.

How does the distributive property show that you can combine like terms?

Explain how the distributive property is used to multiply any size polynomials.

**Powers of 11**

**A Cubic Identity**

**Adding and Subtracting Polynomials**

**Algebraic Expression, the Commutitive Property**

**Animal Populations**

**Computation with Complex Numbers**

**Equivalent Expressions**

**Non-negative Polynomials**

**Seeing Dots**

**Solar Panels**

**Sum of Even and Odd**
New Jersey Student Learning Standard(s):
A.REI.B.4: Solve quadratic equations in one variable.

A.REI.B.4a: Use the method of completing the square to transform any quadratic equation in $x$ into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.

A.REI.B.4b: Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers $a$ and $b$.

**Student Learning Objective 2:** Derive the quadratic formula by completing the square and recognize when there are no real solutions.

**Modified Student Learning Objectives/Standards:** N/A

<table>
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<th>Essential Understandings/Questions (Accountable Talk)</th>
<th>Tasks/Activities</th>
</tr>
</thead>
</table>
| MP 1 MP 3 MP 5 MP 7 | **HS.C.5.5**  
- Given an equation or system of equations, reason about the number or nature of the solutions. Content scope: A-REI.4a, A-REI.4b, limited to real solutions only.  
**HS.C.16.2**  
- Given an equation or system of equations, present the solution steps as a logical argument that concludes with the set | There are multiple methods for solving quadratic equations. Use the method of completing the square to transform a quadratic equation in $x$ into an equation of the form $(x - p)^2 = q$. Derive the quadratic formula from $(x - p)^2 = q$. Solve quadratic equations in one variable by inspection. Solve quadratic equations in one variable by taking square roots. Solve quadratic equations in one variable by completing the square. | Transforming a quadratic equation into the form $(x - p)^2 = q$ yields an equation having the same solutions. How do you complete the square? How can we identify a difference of squares? When might it be a good idea to not factor and use a different method for solving quadratic equations? | IFL Set of Related Tasks: “Developing an Understanding of Quadratics”  
PBAs: Fencing for Josephine’s Backyard  
**Type I:** Two Squares are Equal  
**Quadratic Equations I**  
**Type II – III:** |
Tasks are limited to quadratic equations. Content scope: A-REI.1, A-REI.4a, A-REI.4b, limited to real solutions only.

**A-REI.4A-1**
- The derivation part of the standard is not assessed here; it is assessed under Sub-Claim C.

- **Analyze the quadratic formula, recognizing the conditions leading to complex solutions (discriminant).**

**SPED Strategies:**
Use real-life examples with visual models to demonstrate how to solve quadratics using all methods listed i.e.: inspection, square roots, completing the square, quadratic formula and factoring.

Provide graphic organizers/notes/Google Docs/Anchor Charts that include helpful hints and strategies to employ when solving quadratics including the quadratic formula. Students should be able to use this as a resource to increase confidence, proficiency and understanding.

**ELL Support:**
If students are receiving mathematics instruction in Spanish, use the Khan Academy video from Khan Academy en Español to provide students with a native language explanation of the quadratic formula, how it was derived and how it is used to solve polynomials.


<table>
<thead>
<tr>
<th>Braking Distance</th>
<th>Hitting a Baseball</th>
<th>Completing the Square</th>
</tr>
</thead>
<tbody>
<tr>
<td>Visualizing</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Released Items</td>
<td></td>
<td></td>
</tr>
<tr>
<td>EOY18, EOY 33</td>
<td></td>
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</tr>
<tr>
<td>PBA3</td>
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<td></td>
</tr>
</tbody>
</table>
Model the derivation of the quadratic formula by illustrating the steps involved using appropriate language and pictorial displays.

New Jersey Student Learning Standard(s):
A.REI.B.4: Solve quadratic equations in one variable.

A.REI.B.4a: Use the method of completing the square to transform any quadratic equation in x into an equation of the form \((x - p)^2 = q\) that has the same solutions. Derive the quadratic formula from this form.

A.REI.B.4b: Solve quadratic equations by inspection (e.g., for \(x^2 = 49\)), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as \(a \pm bi\) for real numbers \(a\) and \(b\).

Student Learning Objective 3: Solve quadratic equations in one variable using a variety of methods (including inspection, taking square roots, factoring, completing the square,

Modified Student Learning Objectives/Standards: N/A

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<tbody>
<tr>
<td>MP 1</td>
<td>MP 3 MP 5 MP 7</td>
<td>HS.C.5.5</td>
<td>There are multiple methods for solving quadratic equations.</td>
<td>IFL Set of Related Tasks: “Developing an Understanding of Quadratics”</td>
</tr>
<tr>
<td></td>
<td>- Given an equation or system of equations, reason about the number or nature of the solutions Content scope: A-REI.4a, A-REI.4b, limited to real solutions only.</td>
<td>Use the method of completing the square to transform a quadratic equation in x into an equation of the form ((x - p)^2 = q).</td>
<td>Transforming a quadratic equation into the form ((x - p)^2 = q) yields an equation having the same solutions. How do students complete the square?</td>
<td></td>
</tr>
<tr>
<td></td>
<td>HS.C.16.2</td>
<td>Derive the quadratic formula from ((x - p)^2 = q).</td>
<td>How can students identify a difference of squares?</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Solve quadratic equations in one variable by inspection.</td>
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<td></td>
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<td>PBAs: Fencing for Josephine’s Backyard</td>
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<td></td>
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<td>Type I:</td>
<td></td>
</tr>
</tbody>
</table>
- Given an equation or system of equations, present the solution steps as a logical argument that concludes with the set of solutions (if any). Tasks are limited to quadratic equations. Content scope: A-REI.1, A-REI.4a, A-REI.4b, limited to real solutions only.

A-REI.4A-1
- The derivation part of the standard is not assessed here; it is assessed under Sub-Claim C.

A-REI.4B-1
- Tasks should exhibit variety in initial forms. Examples of quadratic equations with real solutions: $t^2= 49$, $3a^2 = 4$, $7 = x^2$, $r^2 = 0$, $(1/2)^2 = 1/5$, $y^2 - 8y + 15 = 0$, $2x^2 - 16x + 30 = 0$, $2p = p^2 + 1$, $t^2 = 4t$, $7x^2 + 5x - 3 = 0$, $(3/4)c(c - 1) = c$, $(3c-2)^2 = 6x-4$.

<table>
<thead>
<tr>
<th>Solve quadratic equations in one variable by taking square roots.</th>
<th>Write complex solutions of the quadratic formula in $a \pm bi$ form.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solve quadratic equations in one variable by completing the square.</td>
<td>Analyze the quadratic formula, recognizing the conditions leading to complex solutions (discriminant).</td>
</tr>
<tr>
<td>Solve quadratic equations in one variable using the quadratic formula.</td>
<td>SPED Strategies: Provide students with ample opportunity to work with quadratics in context and determine the appropriate approach to solving the problem.</td>
</tr>
<tr>
<td>Solve quadratic equations in one variable by factoring.</td>
<td>Explain the concept of complex numbers thoroughly and connect it to real life situations such as electrical circuits.</td>
</tr>
</tbody>
</table>

When might it be a good idea to not factor and use a different method for solving quadratic equations?

Why would you want to transform a quadratic equation to the form $(x-p)^2 = q$?

How do you determine which method is best for solving a quadratic equation?

Why do some quadratic equations have extraneous and/or complex solutions?

Two Squares are Equal

Quadratic Equations

Type II – III:
- Springboard Dive
- Braking Distance
- Hitting a Baseball

Released Items / Suggested Tasks
- EOY18, EOY 33
- PBA3

Visualizing Completing the Square
- Methods are not explicitly assessed; strategy is assessed indirectly by presenting students with a variety of initial forms.

- For rational solutions, exact values are required. For irrational solutions, exact or decimal approximations may be required. Simplifying or rewriting radicals is not required; however, students will not be penalized if they simplify the radicals correctly.

- Prompts integrate mathematical practices by not indicating that the equation is quadratic. (e.g., "Find all real solutions of the equation \( t^2 = 4t \) ... not, "Solve the quadratic equation \( t^2 = 4t \).")

### A-REI.4B-2
- Writing solutions in the form \( a \pm bi \) is not

| Develop a resource document that explains the differences between solving a quadratic with real solutions as compared to complex solutions. Model how to use this document to facilitate problem solving. |

**ELL Support:**
Create anchor chart/notes with students that document the different ways of solving quadratic equations: completing the square, inspection, square roots, quadratic formula, etc.

Use think-aloud as a support that allows students to discuss how they are processing information with peers and the teacher.

Encourage discourse by asking assessing and advancing questions and referring students to resources that encourage self-sufficiency.
assessed here. (Assessed under N-CN.7.)

**New Jersey Student Learning Standard(s):**

**A.CED.A.1:** Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear functions and quadratic functions, and simple rational and exponential functions.

**Student Learning Objective 4:** Create quadratic equations in one variable and use them to solve problems.

**Modified Student Learning Objectives/Standards:**

**M.EE.A-CED.1.** Create an equation involving one operation with one variable, and use it to solve areal-world problem.

<table>
<thead>
<tr>
<th>MPs</th>
<th>Evidence Statement Key/Clarifications</th>
<th>Skills, Strategies &amp; Concepts</th>
<th>Essential Understandings/Questions (Accountable Talk)</th>
<th>Tasks/Activities</th>
</tr>
</thead>
<tbody>
<tr>
<td>MP 2</td>
<td></td>
<td></td>
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<tr>
<td>MP 6</td>
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<tr>
<td>MP 7</td>
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<tr>
<td>HS.D.2-5</td>
<td></td>
<td>Create quadratic equations and inequalities in one variable and use them in a contextual situation to solve problems.</td>
<td>How can students determine the relationships between numbers?</td>
<td>Type I Solving Quads</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Solve multi-variable formulas or literal equations, for a specific variable.</td>
<td>How can students use a pattern to identify multiplicative comparisons?</td>
<td>More and More Quads</td>
</tr>
<tr>
<td></td>
<td></td>
<td>SPED Strategies: Create visual and verbal models for writing quadratic equations to solve problems with students.</td>
<td>One of the factors in multiplication indicates the number of objects in a group and the other factor indicates the number of groups.</td>
<td>Type II-III Quadratic in Action</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Model how to determine patterns and relationships between numbers and decipher meaning from them.</td>
<td>In the multiplicative expression A x B, A can be defined as a scaling factor. (NCTM, Essential Understanding, 2011).</td>
<td>Swirling Glass</td>
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<tr>
<td></td>
<td></td>
<td>Ask students assessing and advancing questions to elicit their level of understanding and propel thinking forward.</td>
<td></td>
<td>Released Items / Suggested Tasks</td>
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<td>Bernardo and Sylvia</td>
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<td>Play a Game</td>
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<td>Buying a Car</td>
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<td>Planes and Wheat</td>
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<tr>
<td>ELL Support: Create a document with students that clearly depict visual and verbal models for writing quadratic equations to solve problems using appropriate language level or native language. Use relevant contextual examples to reinforce and extend understanding of writing quadratic equations to solve problems. Ensure that language supports are provided such as access to word-to-word dictionary, notes, anchor charts, linguistically simpler explanations and visual models.</td>
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<tr>
<td>A situation that can be represented by multiplication has an element that represents the scalar and an element that represents the quantity to which the scalar applies. (NCTM, Essential Understanding, 2011). A multiplicative comparison involves a constant increase that x is more times or x times less; whereas an additive comparison only involves determining how many more than or how many less than another set. How do you translate real-world situations into mathematical equations and inequalities? How do you determine if a situation is best represented by an equation, an inequality, a system of equations or a system of inequalities? Why would you want to create an equation or inequality to represent a real-world problem?</td>
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<tr>
<td>Sums of Angles in a Polygons</td>
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</tbody>
</table>
### New Jersey Student Learning Standard(s):

**F.IF.B.4:** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include:* intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

**F.IF.B.5:** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*

### Student Learning Objective 5:
Interpret key features of quadratic functions from graphs and tables. Given a verbal description of the relationship, sketch the graph of a quadratic function, showing key features and relating the domain of the function to its graph.

### Modified Student Learning Objectives/Standards:

**M.EE.F-IF.4–6:** Construct graphs that represent linear functions with different rates of change and interpret which is faster/slower, higher/lower, etc.

<table>
<thead>
<tr>
<th>MPs</th>
<th>Evidence Statement Key/Clarifications</th>
<th>Skills, Strategies &amp; Concepts</th>
<th>Essential Understandings/Questions (Accountable Talk)</th>
<th>Tasks/Activities</th>
</tr>
</thead>
<tbody>
<tr>
<td>MP 4</td>
<td>F-IF.4-1</td>
<td>Interpret maximum/minimum and intercepts of quadratic functions from graphs and tables in the context of the problem.</td>
<td>Where does the function increase or decrease?</td>
<td>IFL Set of Related Tasks: “Developing</td>
</tr>
<tr>
<td>MP 6</td>
<td>F-IF.4-1</td>
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<tr>
<td>See illustrations for F-IF.4 at the following sites:</td>
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<td>- <a href="http://illustrativemathematics.org">http://illustrativemathematics.org</a> e.g.</td>
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<td>- <a href="http://illustrativemathematics.org/illustrations/649">http://illustrativemathematics.org/illustrations/649</a></td>
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<td>- <a href="http://illustrativemathematics.org/illustrations/637">http://illustrativemathematics.org/illustrations/637</a></td>
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<td><strong>F-IF.5-1, 5-2</strong></td>
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<tr>
<td>- Tasks have a real-world context.</td>
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</tbody>
</table>

| Sketch graphs of quadratic functions given a verbal description of the relationship between the quantities. |
| Identify intercepts and intervals where function is increasing/decreasing |
| Determine the practical domain of a function. |

**SPED Strategies:**
Fortify students’ understanding of quadratic functions by reviewing and creating a document that highlights the key features of a quadratic function and how to notice them in a table or graph.

| Provide opportunities to practice sketching graphs based on verbal models of quadratic functions. |
| Model the thinking that is needed when creating sketches from verbal models. |

**ELL Support:**
Create a document that describes the key features of a quadratic and provides helpful hints on how to notice them in a table or graph using visual/pictorial representations, simplified language or native language as appropriate.

| What are the relative maximums and minimums of the function? What does that tell us? |
| Is the function symmetric? If yes, where and what does that mean? |
| What happens to the ends of the graph of the function? |
| Is there a period in the function? Where? |
| How is a graph related to its algebraic function? |
| How could you use function notation to represent a specific output of a function. |
| With the exception of linear functions, all polynomial functions have variable rates of change, because the value of the average rate of change is not constant over every interval of the function. |
| The domain and range of a relationship are constrained by the context of the situation because certain values may not make sense in a given context. |

**an Understanding of Quadratics”**

**PBAs:**
- Bottle Rocket
- Buddy Bags

**Type I: Key Features in Graphs**

**Type II-II:**
- Average Cost Containers
- The Canoe Trip

**Released Items / Suggested Tasks:**
- EOY 22 , EOY 5
- The Restaurant
- Words-Tables-Graphs
understanding of quadratics and simultaneously build language proficiency. This can be done by planning effective questions, filling in the background knowledge gaps and providing language supports.

Key features of the graph of a quadratic function—vertex, average rate of change, and symmetry—can be analyzed and interpreted within a problem context and can be used to solve problems.

**New Jersey Student Learning Standard(s):**
A.SSE.B.3: Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

A.SSE.B.3a: Factor a quadratic expression to reveal the zeros of the function it defines.

A.SSE.B.3b: Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.

**Student Learning Objective 6:** Use factoring and completing the square to produce equivalent forms of quadratic expressions in one variable that highlights particular properties such as the zeros or the maximum or minimum value of the function.

**Modified Student Learning Objectives/Standards:**
M.EE.A-SSE.3: Solve simple algebraic equations with one variable using multiplication and division.

<table>
<thead>
<tr>
<th>MPs</th>
<th>Evidence Statement Key/Clarifications</th>
<th>Skills, Strategies &amp; Concepts</th>
<th>Essential Understandings/Questions (Accountable Talk)</th>
<th>Tasks/Activities</th>
</tr>
</thead>
<tbody>
<tr>
<td>MP 1</td>
<td>A-SSE.3a, A-SSE.3b</td>
<td>Factor a quadratic expression for the purpose of revealing the zeros of a function. Complete the square for the purpose of revealing the maximum or minimum of a function.</td>
<td>Alternate, equivalent forms of a quadratic expression may reveal specific attributes of the function that it defines Can students simplify expressions and recognize equivalent forms of the same expression?</td>
<td>Type I: Graphs of Quadratic Functions Profit of a company assessment variation</td>
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<td>MP 2</td>
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<td>MP 4</td>
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<td>MP 7</td>
<td>- The equivalent form must reveal the zeros of the function. Tasks require students to make the connection</td>
<td>Write expressions in equivalent forms by factoring to find the zeros of a quadratic function and explain the meaning of the zeros.</td>
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<td>Type II- III A Cubic Identity</td>
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</tbody>
</table>
| between the equivalent forms of the expression. | Given a quadratic function explain the meaning of the zeros of the function. That is if 
\[ f(x) = (x - c)(x - a) \] then \( f(a) = 0 \) and \( f(c) = 0 \).

Given a quadratic expression, explain the meaning of the zeros graphically. That is for an expression \((x - a)(x - c)\), \(a\) and \(c\) correspond to the \(x\)-intercepts (if \(a\) and \(c\) are real).

Write expressions in equivalent forms by completing the square to convey the vertex form, to find the maximum or minimum value of a quadratic function, and to explain the meaning of the vertex.

Use properties of exponents (such as power of a power, product of powers, power of a product, and rational exponents, etc.) to write an equivalent form of an exponential function to reveal and explain specific information about its approximate rate of growth or decay.

**SPED Strategies:**
Illustrate to students how equivalent forms of a quadratic expression make it easier to see the key features of the function.

Create notes/Google Doc/Anchor Chart with students to illustrate explicitly what the different forms reveal. Ex: factoring and zeros, vertex form and the coordinates of the vertex (maxima and minima).

**ELL Support:**

| Rewriting a quadratic expression | Profit of a company
Camels | Released Items / Suggested Tasks: |
<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>EOY 19, PBA 7</td>
<td>Rewriting a Quadratic Equation</td>
<td></td>
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<tr>
<td>Ice Cream</td>
<td>Increasing or Decreasing Variation 2</td>
<td></td>
</tr>
</tbody>
</table>
Demonstrate how factoring and completing the square produce equivalent forms of quadratic expressions using language that meets students’ learning needs.

Illustrate how the different forms of the quadratic highlight key features differently and discuss when you might use them. Document this learning on an anchor chart or in notes so that students can refer to it when working independently.

New Jersey Student Learning Standard(s):
F.BF.A.1: Write a function that describes a relationship between two quantities.

F.BF.A.1a: Determine an explicit expression, a recursive process, or steps for calculation from a context.

Student Learning Objective 7: Given a context, write an explicit expression, a recursive process or steps for calculation for quadratic relationships.

Modified Student Learning Objectives/Standards: N/A

<table>
<thead>
<tr>
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<th>Evidence Statement Key/Clarifications</th>
<th>Skills, Strategies &amp; Concepts</th>
<th>Essential Understandings/Questions (Accountable Talk)</th>
<th>Tasks/Activities</th>
</tr>
</thead>
<tbody>
<tr>
<td>MP 2 MP 4</td>
<td>HS.D.2-8 HS.D.2-9</td>
<td>Given a context, write explicit expressions, a recursive process or steps for calculation for quadratic relationships. From context, either write an explicit expression, define a recursive process, or describe the calculations needed to model a function between two quantities</td>
<td>A function is a relationship between two quantities. The function representing a given situation may be a combination of more than one standard function.</td>
<td>Type I Building an Explicit Quadratic Function Type II – III Compounding with 5% interest rate</td>
</tr>
</tbody>
</table>

SPED Strategies:
and skills articulated in F-BF.1a, F-BF.3, ACED.1, A-SSE.3, F-IF.B, F-IF.7, limited to linear functions and exponential functions with domains in the integers.

- F-BF.1a is the primary content; other listed content elements may be involved in tasks as well.

**Model the thinking and provide students with multiple examples of how to write an explicit expression, a recursive process or steps for calculation for quadratic relationships in context.**

Create a mnemonic device with students to help them remember how to write an explicit expression, a recursive process or steps for calculation for quadratic relationships in context.

**ELL Support:**
Refer students back to the work done on recursive processes and explicit expressions in Unit 2 SLO 6. Extend their learning by relating the process from the linear and exponential contexts covered in Unit 2 to the quadratic relationships in Unit 3.

Use contexts of high interest to students to explore the writing of explicit expressions and recursive processes.

Ensure that students are provided with appropriate language supports such as native language instruction, access to notes, dictionaries and effective questioning so that they can further their conceptual understanding.

| Can students develop a recursive rule using an equation? |
| What data would you need to write a linear, basic quadratic, or basic exponential function? |
| How do you translate a description of the relationship between two quantities into an algebraic equation or inequality? |

| Compounding with 100% interest rate |
| Susita’s Account |
| Graphing Quadratic Functions from the Standard Form |
| Modeling a Context From a Verbal Description |
| Skeleton Tower |
| Released Items / Suggested Tasks: |
| Bears |
| Coffee |
| Kimi and Jordan Lake Algae |
| Summer Intern |
New Jersey Student Learning
Standard(s):
F.IF.C.7: Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

  
  F.IF.C.7a: Graph linear and quadratic functions and show intercepts, maxima, and minima. *[emphasize quadratic functions]*

F.IF.C.8: Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

  
  F.IF.C.8a: Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

F.IF.C.9: Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

Student Learning Objective 8: Graph quadratic functions by hand in simple cases and with technology in complex cases, showing intercepts, extreme values and symmetry of the graph. Compare properties of two quadratic functions, each represented in a different way.

Modified Student Learning Objectives/Standards:
M.EE.F-IF.1–3: Use the concept of function to solve problems.

<table>
<thead>
<tr>
<th>MPs</th>
<th>Evidence Statement Key/Clarifications</th>
<th>Skills, Strategies &amp; Concepts</th>
<th>Essential Understandings/Questions (Accountable Talk)</th>
<th>Tasks/Activities</th>
</tr>
</thead>
<tbody>
<tr>
<td>MP 1</td>
<td>F-IF.7a-1</td>
<td>Graph quadratic functions expressed symbolically.</td>
<td>Can students find key parts of a quadratic function from a table or graph?</td>
<td>Type I:</td>
</tr>
<tr>
<td>MP 3</td>
<td>F-IF.7a-2</td>
<td></td>
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<td>Graphs of Quadratic Functions Which Function?</td>
</tr>
<tr>
<td>MP 5</td>
<td>F-IF.8a</td>
<td>Graph more complicated cases of quadratic functions using technology.</td>
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<tr>
<td>MP 6</td>
<td>F-IF.8a</td>
<td></td>
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<tr>
<td>MP 8</td>
<td>• Tasks have a real-world context.</td>
<td>Identify and describe key features of the graphs of quadratic functions.</td>
<td>Can we compare two functions that are represented differently (algebraic, graphical, tabular, verbal)?</td>
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<tr>
<td>F-IF.9-1</td>
<td>Tasks may have a real-world context.</td>
<td>Given two quadratic functions, each represented in a different way, compare the properties of the functions.</td>
<td>How do the parts of a graph of a function relate to its real-world context?</td>
<td></td>
</tr>
<tr>
<td>HS.C.12.1</td>
<td>Construct, autonomously, chains of reasoning that will justify or refute propositions or conjectures about functions Content scope: F-IF.8a</td>
<td><strong>SPED Strategies:</strong> Work with students to help them synthesize the skills, strategies and concepts they have learned about quadratic functions in this unit regarding key features, representation and comparing functions.</td>
<td>Can quadratic polynomials be written in different forms to find the zeros, vertex and axis/lines of symmetry?</td>
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<td>--------------------------------</td>
<td>• Tasks involve using algebra to prove properties of given functions. For example, prove algebraically that the function h(t) = t(t-1) has minimum value 1/4; prove algebraically that the graph of g(x) = x^2 - x + 1/4 is symmetric about the line x = 1/2; prove that x^2 + 1 is never less than -2x.</td>
<td><strong>ELL Support:</strong> Model the process and thinking behind graphing quadratic functions by hand in simple cases and with technology in complex cases using appropriate language and visual depictions.</td>
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<tr>
<td>--------------------------------</td>
<td>• Scaffolding is provided to ensure tasks have appropriate level of</td>
<td>Artifacts (notes and anchor charts) of this learning that shows intercepts, extreme values and symmetry of the graph should be available to students when they work in small groups or independently.</td>
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<td></td>
<td>Engage students in tasks that require them to compare properties of two quadratic functions, each represented in a different way and discuss their findings with peers.</td>
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</tbody>
</table>

**Type II – III:**

Springboard Dive
- Flares

Released Items / Suggested Tasks:
- EOY 8
  - Throwing Baseball
  - Flying T-Shirt
  - Value of an Antique
difficulty. (For example, the prompt could show the graphs of $x^2 + 1$ and $-2x$ on the same set of axes, and say, "From the graph, it looks as if $x^2 + 1$ is never less than $-2x$. In this task, you will use algebra to prove it." And so on, perhaps with additional hints or scaffolding.)

- Tasks may have a mathematical or real-world context.

**New Jersey Student Learning Standard(s):**

**F.IF.B.6.** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

**F.LE.A.3.** Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

**Student Learning Objective 9:** Calculate and interpret the average rate of change of a quadratic function presented symbolically or as a table. Estimate and compare the rates of change from graphs of quadratic and exponential functions.

**Modified Student Learning Objectives/Standards:**

**M.EE.F-IF.4–6:** Construct graphs that represent linear functions with different rates of change and interpret which is faster/slower, higher/lower, etc.

**M.EE.F-LE.1–3:** Model a simple linear function such as $y = mx$ to show that these functions increase by equal amounts over equal intervals.

<table>
<thead>
<tr>
<th>MPs</th>
<th>Evidence Statement Key/Clarifications</th>
<th>Skills, Strategies &amp; Concepts</th>
<th>Essential Understandings/Questions (Accountable Talk)</th>
<th>Tasks/Activities</th>
</tr>
</thead>
</table>

44 | Page
<table>
<thead>
<tr>
<th>MP 1</th>
<th>MP 4</th>
<th>MP 5</th>
<th>MP 7</th>
</tr>
</thead>
<tbody>
<tr>
<td>F-IF.6-1a</td>
<td>F-IF.6-1b</td>
<td>F-IF.6-6a</td>
<td>F-IF.6-6b</td>
</tr>
</tbody>
</table>

- Tasks have a real-world context.
- Tasks must include the interpret part of the evidence statement.

### SPED Strategies:
Using contextualized problems, model how to find the rate of change of a quadratic function from a graph or a symbolic representation. Highlight the “look fors” that students should notice and why they are important.

Engage students in the task of analyzing graphs and tables of exponential and quadratic function to compare rates of change. Have students work in groups to write down what they notice and what it means.

### ELL Support:
Refer students back to the work done on rate of change Unit 2 SLO 10. Extend their learning by relating the vocabulary and the process from the linear and exponential contexts covered in Unit 2 to the quadratic relationships in Unit 3.

### Type I: Company's Profit
- **Type II – III:**
  - **Stock Prices**
  - 1000 is half of 2000
  - Exponential growth versus linear growth I
  - Exponential growth versus linear growth II
  - Exponential growth versus polynomial growth
  - Laptop Battery charge 2
  - Temperature Change
  - The High School Gym
  - Population and Food Supply
  - Released Items / Suggested Tasks:
    - EOY 9
    - Marathon Relay

### A quantity increasing exponentially eventually exceeds a quantity increasing quadratically.
Given a function that describes a real-world situation, what can the average rate of change of the function tell you?

Can I interpret graphs and tables that are increasing exponentially?

What are the strategies and methods used to construct and compare linear, quadratic and exponential models and solve problems?
Encourage students to work on contextual problems to illustrate rate of change in quadratics in a meaningful and memorable way.

Use assessing and advancing questions to determine what additional pedagogical and linguistic supports students need.

**New Jersey Student Learning Standards (s):**

**F.BF.B.3:** Identify the effect on the graph of replacing \( f(x) \) by \( f(x) + k \), \( k f(x) \), \( f(kx) \), and \( f(x + k) \) for specific values of \( k \) (both positive and negative); find the value of \( k \) given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

**Student Learning Objective 10:** Identify the effects of transformations and combinations of transformations \([f(x) + k, k f(x), f(kx),\text{ and } f(x + k)]\) on a function; find the value of \( k \) given the graph.

**Modified Student Learning Objectives/Standards:** N/A

<table>
<thead>
<tr>
<th>MPs</th>
<th>Evidence Statement Key/Clarifications</th>
<th>Skills, Strategies &amp; Concepts</th>
<th>Essential Understandings/Questions (Accountable Talk)</th>
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</tr>
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<tbody>
<tr>
<td>MP 3</td>
<td>HS.C.9.1</td>
<td>Perform transformations on graphs of linear and quadratic functions. Identify the effect on the graph of replacing ( f(x) ) by ( f(x) + k ); ( k f(x) ); ( f(kx) ); and ( f(x + k) ) for specific values of ( k ) (both positive and negative). Identify the effect on the graph of combinations of transformations. Given the graph, find the value of ( k ).</td>
<td>What are the characteristics of even and odd functions in graphs and algebraic expressions? How do functions change based on transformations? Can students represent vertical and horizontal shifts? How does a graphed function changed when values are</td>
<td>Type I Building a quadratic function from ( f(x)=x^2 ) Identifying Quadratic Functions (Standard Form)</td>
</tr>
<tr>
<td>MP 5</td>
<td>F-BF.3-1</td>
<td></td>
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<tr>
<td>MP 7</td>
<td></td>
<td></td>
<td></td>
<td>Type II, III Building a General Quadratic Function Medieval Archer</td>
</tr>
</tbody>
</table>
- Tasks do not involve recognizing even and odd functions.
- Experimenting with cases and illustrating an explanation are not assessed here. They are assessed under Sub-claim C.
- Tasks may involve more than one transformation.

<table>
<thead>
<tr>
<th>Illustrate an explanation of the effects on linear and quadratic graphs using technology. Recognize even and odd functions from their graphs and from algebraic expressions for them. Using technology, identify effects of single transformations on graphs of functions.</th>
</tr>
</thead>
</table>

**SPED Strategies:**
When introducing the topic of transformations of functions, have students graph the original function and the transformation on the same graph in different colors to enable students to see changes more clearly.

Make connections between the graph and the equation and highlight what is known about the function by looking at the graph and the equation.

Memorialize the learning of students by creating a document (Google Doc, Anchor chart, Notes) that highlights what they know about a function based on the graph or the equation so that students can use it as a reference.

**ELL Support:**
Use Desmos and/or graphing calculators to provide students with a visual model of what replaced (both positive and negative)?

<table>
<thead>
<tr>
<th>Released Items / suggested Tasks</th>
</tr>
</thead>
<tbody>
<tr>
<td>EOY 27 – EOY 24</td>
</tr>
<tr>
<td>Identifying Even and Odd Functions</td>
</tr>
<tr>
<td>Transforming the Graph of a Function</td>
</tr>
<tr>
<td>Laboratory Rats</td>
</tr>
</tbody>
</table>
happens when linear and quadratic functions are transformed.

Create an anchor chart that links the transformation algebraically to the changes graphically using appropriate language to meet student needs.

**New Jersey Student Learning Standard(s):**

A.REI.D.11: Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

**Student Learning Objective 11:** Find approximate solutions of f(x) = g(x), where f(x) is a linear function and g(x) is a quadratic function by making a table of values, using technology to graph and finding successive approximations

**Modified Student Learning Objectives/Standards:***

M.EE.A-REI.10–12: Interpret the meaning of a point on the graph of a line. For example, on a graph of pizza purchases, trace the graph to a point and tell the number of pizzas purchased and the total cost of the pizzas.

<table>
<thead>
<tr>
<th>MPs</th>
<th>Evidence Statement Key/Clarifications</th>
<th>Skills, Strategies &amp; Concepts</th>
<th>Essential Understandings/Questions (Accountable Talk)</th>
<th>Tasks/Activities</th>
</tr>
</thead>
<tbody>
<tr>
<td>MP 1 MP 5</td>
<td>HS.C.5.10-1</td>
<td>Find approximate solutions of f(x) = g(x), where f(x) is a linear function and g(x) is a quadratic function by making a table of values, using technology to graph and finding successive approximations.</td>
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<tr>
<td></td>
<td>• Given an equation or system of equations, reason about the number or nature of the solutions. Content scope: A-REI.11,</td>
<td></td>
<td>Can I use various models to solve a system of two or more equations in two variables?</td>
<td>Type I: Non-Linear Systems</td>
</tr>
<tr>
<td></td>
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<td></td>
<td>Why are the x-coordinates of the points where the graphs of the equations y = f(x) and y =</td>
<td>Type II, III: What a Beautiful Arch Error Analysis -1</td>
</tr>
</tbody>
</table>

**SPED Strategies:**
limited to equations of the form \( f(x) = g(x) \) where \( f \) and \( g \) are linear or quadratic.

**A-REI.11-1**
- The "explain" part of standard A-REI.11 is not assessed here. For this aspect of the standard, see Sub-Claim C.

Introduce the topic in a contextual way so that students can visualize how the graphing of a system of a linear and a quadratic function would apply to real life.

Have students graph the functions in different colors to enable students to see the intersection more clearly.

Use graphing calculators to enable students to practice the skills needed, to see the intersections clearly and to check their work.

**ELL Support:**
Use a contextual example of a linear and a quadratic equation intersecting to ground student understanding of this new concept.

Ensure that new vocabulary is explained thoroughly using visual cues, native language and by modifying the linguistic complexity of explanations.

\( g(x) \) intersect equal to the solutions of the equations \( f(x) = g(x) \)?

What does graphing or using a table give approximate solutions?

In what situations would you want an exact solution rather than an approximate solution or vice versa?

<table>
<thead>
<tr>
<th>Population and food supply</th>
</tr>
</thead>
<tbody>
<tr>
<td>Student task 618</td>
</tr>
<tr>
<td>Released Items / suggested Tasks</td>
</tr>
<tr>
<td>Hot Air Balloons</td>
</tr>
<tr>
<td>Ideal Gas Law</td>
</tr>
<tr>
<td>Intro to Polynomials</td>
</tr>
<tr>
<td>College Fund</td>
</tr>
</tbody>
</table>

**New Jersey Student Learning Standards:**

**A.APR.B.3.** Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial. *Algebra 1: limit to quadratic and cubic functions in which linear and quadratic factors are available*

**Student Learning Objective 12:** Identify zeros of cubic functions when suitable factorizations are available and use the zeros to construct a rough graph of the function. (*cubic functions are presented as the product of a linear and a quadratic factor*)

**Modified Student Learning Objectives/Standards:** N/A
A-APR.3-1

- For example, find the zeros of \((x-2)(x^2-9)\).
- Sketching graphs is limited to quadratics.
- For cubic polynomials, at least one linear factor must be provided or one of the linear factors must be a GCF.

Find the zeros of a polynomial (quadratic and cubic).

Test domain intervals to determine where \(f(x)\) is greater than or less than zero.

Use zeros of a function to sketch a graph.

**SPED Strategies:**
Introduce the topic in a contextual way so that students can visualize the meaning of the zeros in a quadratic and a cubic function.

Review the process involved in finding zeros of quadratic and cubic functions.

Make explicit connections between the graph and the equation so that students can sketch the graph of a function just by looking at the equation.

Encourage students to use graphing calculators to check their work.

**ELL Support:**
Illustrate and annotate the process of finding zeroes using visual displays and carefully chosen vocabulary. The artifacts of this process should be available to students as notes or anchor charts that they can reference when working independently.

**Accountable Talk**

- How can students determine the general shape(s) and end behavior of cubic functions?

- How do you determine how many zeros a polynomial function will have?

- What information do you need to sketch a rough graph of a polynomial function?

- How are the zeros of a polynomial related to its graph?

**Type I:**
Solving a Simple cubic Equation
Matching Function

**Type II, III:**
Graphing from Factors II

**Released Items / suggested Tasks:**
EOY 1
Graphing from Factors I
Exponential Growth vs Linear Growth I
Exponential Growth vs Linear Growth II
Graphing from Factors III
Foster the partner work strategy to help students support their partners’ language and conceptual development while learning how to sketch graphs when given the zeros.

The teacher should ask assessing and advancing questions that encourage mathematical discourse and deepen understanding of concepts and related language.

**New Jersey Student Learning Standards (s):**

N.RN.B.3: Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

**Student Learning Objective 13:** Explain and justify conclusions about sums and products of rational and irrational numbers.

**Modified Student Learning Objectives/Standards:** N/A

<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>MP 3</td>
<td>HS.C.2-1</td>
<td>The sum or product of two rational numbers is rational.</td>
<td>How do mathematical operations affect the nature of the relation between rational and irrational numbers?</td>
<td>Type I: Real Challenges</td>
</tr>
<tr>
<td>MP 6</td>
<td></td>
<td>The sum of a rational number and an irrational number is irrational.</td>
<td>Why is the sum or product of rational numbers rational?</td>
<td>Type II, III: Sums of rational and irrational numbers</td>
</tr>
<tr>
<td></td>
<td>• Base explanations / reasoning on the properties of rational and irrational numbers. Content scope: N-RN.3</td>
<td>The product of a nonzero rational number and an irrational number is irrational.</td>
<td>SPED Strategies:</td>
<td></td>
</tr>
</tbody>
</table>

**SPED Strategies:**
• For rational solutions, exact values are required. For irrational solutions, exact or decimal approximations may be required. Simplifying or rewriting radicals is not required; however, students will not be penalized if they simplify the radicals correctly.

Review the concept of rational and irrational numbers which was introduced in Grade 8 create a graphic organizer that can help students remember the concept.

Have students add and multiply rational and irrational numbers to discover the rules and then create a document with the class that formalizes what they discovered.

**ELL Support:**
If students are receiving mathematics instruction in Spanish, use the Khan Academy video from Khan Academy en Español to provide students with a native language explanation of rational and irrational numbers.


Create a graphic organizer with students that illustrate the differences between rational and irrational numbers and the rules regarding adding and multiplying using specific examples.

Why is the sum of a rational number and irrational number irrational?

Why is the product of a nonzero rational number and an irrational number irrational?

**Released Items / suggested Tasks:**

PBA 13
Operations with Rational and Irrational Numbers
Integrated Evidence Statements

F-IF.A.Int.1: Understand the concept of a function and use function notation.

- Tasks require students to use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a real-world context.
- About a quarter of tasks involve functions defined recursively on a domain in the integers.

F-INT.1-1: Given a verbal description of a linear or quadratic functional dependence, write an expression for the function and demonstrate various knowledge and skills articulated in the Functions category in relation to this function.

- Given a verbal description of a functional dependence, the student would be required to write an expression for the function and then, e.g., identify a natural domain for the function given the situation; use a graphing tool to graph several input-output pairs; select applicable features of the function, such as linear, increasing, decreasing, quadratic, nonlinear; and find an input value leading to a given output value.
- E.g., a functional dependence might be described as follows: "The area of a square is a function of the length of its diagonal." The student would be asked to create an expression such as \( f(x) = \frac{1}{2} x^2 \) for this function. The natural domain for the function would be the positive real numbers. The function is increasing and nonlinear. And so on.
- E.g., a functional dependence might be described as follows: "The slope of the line passing through the points (1, 3) and (7, y) is a function of y." The student would be asked to create an expression such as \( s(y) = \frac{3-y}{1-7} \) for this function. The natural domain for this function would be the real numbers. The function is increasing and linear. And so on.
Integrated Evidence Statements

HS-INT.1: Solve multi-step contextual problems with degree of difficulty appropriate to the course by constructing quadratic function models and/or writing and solving quadratic equations

- A scenario might be described and illustrated with graphics (or even with animations in some cases).
- Solutions may be given in the form of decimal approximations. For rational solutions, exact values are required. For irrational solutions, exact or decimal approximations may be required. Simplifying or rewriting radicals is not required; however, students will not be penalized if they simplify the radicals correctly.
- Some examples: - A company sells steel rods that are painted gold. The steel rods are cylindrical in shape and 6 cm long. Gold paint costs $0.15 per square inch. Find the maximum diameter of a steel rod if the cost of painting a single steel rod must be $0.20 or less. You may answer in units of centimeters or inches. Give an answer accurate to the nearest hundredth of a unit. - As an employee at the Gizmo Company, you must decide how much to charge for a gizmo. Assume that if the price of a single gizmo is set at P dollars, then the company will sell 1000 - 0.2P gizmos per year. Write an expression for the amount of money the company will take in each year if the price of a single gizmo is set at P dollars. What price should the company set in order to take in as much money as possible each year? How much money will the company make per year in this case? How many gizmos will the company sell per year? (Students might use graphical and/or algebraic methods to solve the problem.) - At t=0, a car driving on a straight road at a constant speed passes a telephone pole. From then on, the car's distance from the telephone pole is given by C(t) = 30t, where t is in seconds and C is in meters. Also at t=0, a motorcycle pulls out onto the road, driving in the same direction, initially 90 m ahead of the car. From then on, the motorcycle's distance from the telephone pole is given by M(t) = 90 + 2.5 t², where t is in seconds and M is in meters. At what time t does the car catch up to the motorcycle? Find the answer by setting C and M equal. How far are the car and the motorcycle from the telephone pole when this happens? (Students might use graphical and/or algebraic methods to solve the problem.)

HS-INT.2: Solve multi-step mathematical problems with degree of difficulty appropriate to the course that requires analyzing quadratic functions and/or writing and solving quadratic equations.

- Tasks do not have a real-world context.
- Exact answers may be required or decimal approximations may be given. Students might choose to take advantage of the graphing utility to find approximate answers or clarify the situation at hand. For rational solutions, exact values are required. For irrational solutions, exact or decimal approximations may be required. Simplifying or rewriting radicals is not required. Some examples:
  - Given the function \( f(x) = x^2 + x \), find all values of \( k \) such that \( f(3 - k) = f(3) \). (Exact answers are required.)
  - Find a value of \( c \) so that the equation \( 2x^2 - cx + 1 = 0 \) has a double root. Give an answer accurate to the tenths place.
### Integrated Evidence Statements

**HS-INT.3-1:** Solve multi-step contextual word problems with degree of difficulty appropriate to the course, requiring application of course-level knowledge and skills articulated in F-LE, A-CED.1, A-SSE.3, F-IF.B, F-IF.7, limited to linear functions and exponential functions with domains in the integers.★
- F-L.E.A, Construct and compare linear, quadratic, and exponential models and solve problems, is the primary content and at least one of the other listed content elements will be involved in tasks as well.

**HS-INT.3-2:** Solve multi-step contextual word problems with degree of difficulty appropriate to the course, requiring application of course-level knowledge and skills articulated in F-LE, A-CED.1, A-SSE.3, F-IF.B, F-IF.7, limited to linear, quadratic, and exponential functions.★
- F-L.E.A, Construct and compare linear, quadratic, and exponential models and solve problems, is the primary content and at least one of the other listed content elements will be involved in tasks as well. For rational solutions, exact values are required. For irrational solutions, exact or decimal approximations may be required. Simplifying or rewriting radicals is not required; however, students will not be penalized if they simplify the radicals correctly.

### Unit 3 Vocabulary

<table>
<thead>
<tr>
<th>• Average rate of change</th>
<th>• Minima</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Axis of symmetry</td>
<td>• Odd function</td>
</tr>
<tr>
<td>• Completing the square</td>
<td>• Parabola</td>
</tr>
<tr>
<td>• Complex solutions</td>
<td>• Quadratic formula</td>
</tr>
<tr>
<td>• Conjugates</td>
<td>• Quadratic function</td>
</tr>
<tr>
<td>• Counterexample</td>
<td>• Radical expression</td>
</tr>
<tr>
<td>• Cubic function</td>
<td>• Rational numbers</td>
</tr>
<tr>
<td>• Discriminant</td>
<td>• Rationalizing the denominator</td>
</tr>
<tr>
<td>• End behavior</td>
<td>• Recursive process</td>
</tr>
<tr>
<td>• Equivalent expressions</td>
<td>• Simplest form</td>
</tr>
</tbody>
</table>
- Even function
- Explicit expression
- Exponential function
- Intercept form
- Irrational Numbers
- Like radicals
- Maxima
- Solving by inspection
- System of nonlinear equations
- Vertex form of a quadratic function
- Vertex of a parabola
- Zero of a function
- Transformations

**References & Suggested Instructional Websites**


Kahn Academy [https://www.khanacademy.org](https://www.khanacademy.org)

Achieve the Core [http://achievethecore.org](http://achievethecore.org)

Illustrative Mathematics [https://www.illustrativemathematics.org/](https://www.illustrativemathematics.org/)

Inside Mathematics [www.insidemathematics.org](http://www.insidemathematics.org)

Learn Zillion [https://learnzillion.com](https://learnzillion.com)


Big Ideas Math [https://www.bigideasmath.com/](https://www.bigideasmath.com/)
# References & Suggested Instructional Websites

<table>
<thead>
<tr>
<th>Website</th>
<th>URL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Youcubed</td>
<td><a href="https://www.youcubed.org/week-of-inspirational-math/">https://www.youcubed.org/week-of-inspirational-math/</a></td>
</tr>
<tr>
<td>NCTM Illuminations</td>
<td><a href="https://illuminations.nctm.org/Search.aspx?view=search&amp;type=ls&amp;gr=9-12">https://illuminations.nctm.org/Search.aspx?view=search&amp;type=ls&amp;gr=9-12</a></td>
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<tr>
<td>Howard County Public School System</td>
<td><a href="https://hcpss.instructure.com/courses/99">https://hcpss.instructure.com/courses/99</a></td>
</tr>
<tr>
<td>Desmos</td>
<td><a href="https://www.desmos.com/">https://www.desmos.com/</a></td>
</tr>
<tr>
<td>Geogebra</td>
<td><a href="http://www.geogebra.org/">http://www.geogebra.org/</a></td>
</tr>
<tr>
<td>CPALMS</td>
<td><a href="http://www.cpalms.org/Public/ToolkitGradeLevelGroup/Toolkit?id=14">http://www.cpalms.org/Public/ToolkitGradeLevelGroup/Toolkit?id=14</a></td>
</tr>
<tr>
<td>Partnership for Assessment of Readiness for College and Careers</td>
<td><a href="https://parcc.pearson.com/#">https://parcc.pearson.com/#</a></td>
</tr>
</tbody>
</table>
Field Trip Ideas

**SIX FLAGS GREAT ADVENTURE:** This educational event includes workbooks and special science and math related shows throughout the day. Your students will leave with a better understanding of real world applications of the material they have learned in the classroom. Each student will have the opportunity to experience different rides and attractions linking mathematical and scientific concepts to what they are experiencing.

[www.sixflags.com](http://www.sixflags.com)

**MUSEUM of MATHEMATICS:** Mathematics illuminates the patterns that abound in our world. The National Museum of Mathematics strives to enhance public understanding and perception of mathematics. Its dynamic exhibits and programs stimulate inquiry, spark curiosity, and reveal the wonders of mathematics. The Museum’s activities lead a broad and diverse audience to understand the evolving, creative, human, and aesthetic nature of mathematics.

[www.momath.org](http://www.momath.org)

**LIBERTY SCIENCE CENTER:** An interactive science museum and learning center located in Liberty State Park. The center, which first opened in 1993 as New Jersey's first major state science museum, has science exhibits, the largest IMAX Dome theater in the United States, numerous educational resources, and the original *Hoberman sphere.*

[http://lsc.org/plan-your-visit/](http://lsc.org/plan-your-visit/)