Algebra I: Unit 2 Modeling with Linear Functions, Linear Systems, & Exponential Functions
Course Philosophy/Description

The fundamental purpose of Algebra 1 is to formalize and extend the mathematics that students learned in the elementary and middle grades. The Standards for Mathematical Practice apply throughout each course, and, together with the New Jersey Student Learning Standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations. Conceptual knowledge behind the mathematics is emphasized. Algebra I provides a formal development of the algebraic skills and concepts necessary for students to succeed in advanced courses as well as the PARCC. The course also provides opportunities for the students to enhance the skills needed to become college and career ready.

The content shall include, but not be limited to, perform set operations, use fundamental concepts of logic including Venn diagrams, describe the concept of a function, use function notation, solve real-world problems involving relations and functions, determine the domain and range of relations and functions, simplify algebraic expressions, solve linear and literal equations, solve and graph simple and compound inequalities, solve linear equations and inequalities in real-world situations, rewrite equations of a line into slope-intercept form and standard form, graph a line given any variation of information, determine the slope, x- and y-intercepts of a line given its graph, its equation or two points on the line, write an equation of a line given any variation of information, determine a line of best fit and recognize the slope as the rate of change, factor polynomial expressions, perform operations with polynomials, simplify and solve algebraic ratios and proportions, simplify and perform operations with radical and rational expressions, simplify complex fractions, solve rational equations including situations involving mixture, distance, work and interest, solve and graph absolute value equations and inequalities, graph systems of linear equations and inequalities in two and three variables and quadratic functions, and use varied solution strategies for quadratic equations and for systems of linear equations and inequalities in two and three variables.
This ESL framework was designed to be used by bilingual, dual language, ESL and general education teachers. Bilingual and dual language programs use the home language and a second language for instruction. ESL teachers and general education or bilingual teachers may use this document to collaborate on unit and lesson planning to decide who will address certain components of the SLO and language objective. ESL teachers may use the appropriate leveled language objective to build lessons for ELLs which reflects what is covered in the general education program. In this way, whether it is a pull-out or push-in model, all teachers are working on the same Student Learning Objective connected to the New Jersey Student Learning Standards. The design of language objectives are based on the alignment of the World-Class Instructional Design Assessment (WIDA) Consortium’s English Language Development (ELD) standards with the Common Core State Standards (CCSS). WIDA’s ELD standards advance academic language development across content areas ultimately leading to academic achievement for English learners. As English learners are progressing through the six developmental linguistic stages, this framework will assist all teachers who work with English learners to appropriately identify the language needed to meet the requirements of the content standard. At the same time, the language objectives recognize the cognitive demand required to complete educational tasks. Even though listening and reading (receptive) skills differ from speaking and writing (expressive) skills across proficiency levels the cognitive function should not be diminished. For example, an Entering Level One student only has the linguistic ability to respond in single words in English with significant support from their home language. However, they could complete a Venn diagram with single words which demonstrates that they understand how the elements compare and contrast with each other or they could respond with the support of their native language with assistance from a teacher, para-professional, peer or a technology program.

http://www.state.nj.us/education/modelcurriculum/ela/ELLOverview.pdf
<table>
<thead>
<tr>
<th>#</th>
<th>Student Learning Objective</th>
<th>NJSLS</th>
<th>Big Ideas Math Correlation</th>
<th>Instruction: 8 weeks</th>
<th>Assessment: 1 week</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Solve multistep contextual problems by identifying variables, writing equations, and solving systems of linear equations in two variables algebraically and graphically.</td>
<td>A.REI.C.6, A.CED.A.3, A.REI.C.5</td>
<td>5.1, 5.2, 5.3, 5.4, 5.5, 5.6, 5.7</td>
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<tr>
<td>2</td>
<td>Graph linear inequalities and systems of linear inequalities in two variables and explain that the solution to the system.</td>
<td>A.REI.D.12, A.CED.A.3</td>
<td>5.1, 5.2, 5.3, 5.4, 5.5, 5.6, 5.7</td>
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<td>3</td>
<td>Explain the definition of a function, including the relationship between the domain and range. Use function notation, evaluate functions and interpret statements in context.</td>
<td>F.IF.A.1, F.IF.A.2</td>
<td>3.1, 3.3</td>
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<td>4</td>
<td>Distinguish between and explain situations modeled with linear functions and with exponential functions.</td>
<td>F.LE.A.1, F.LE.A.1a, F.LE.A.1b, F.LE.A.1c</td>
<td>3.2, 4.1, 4.2, 6.3, 6.4</td>
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<tr>
<td>5</td>
<td>Write linear and exponential functions given a graph, table of values, or written description; construct arithmetic and geometric sequences.</td>
<td>F.LE.A.2, F.IF.A.3</td>
<td>4.1, 4.2, 4.3, 4.6, 6.3, 6.4, 6.6, 6.7</td>
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<td>6</td>
<td>Write explicit expressions, recursive processes and steps for calculation from a context that describes a linear or exponential relationship between two quantities.</td>
<td>F.BF.A.1, A.SSE.A.1, A.SSE.A.1a, A.SSE.A.1b</td>
<td>4.1, 4.2, 4.6, 6.3, 6.4, 6.7</td>
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<tr>
<td>7</td>
<td>Use properties of exponents to produce equivalent forms of exponential expressions in one variable.</td>
<td>A.SSE.B.3, A.SSE.B.3c</td>
<td>6.4</td>
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<tr>
<td>8</td>
<td>Sketch graphs of linear and exponential functions expressed symbolically or from a verbal description. Show key features and interpret parameters in context.</td>
<td>F.IF.B.4, F.LE.B.5, F.IF.B.5</td>
<td>3.2, 3.5, 4.4, 4.5, 6.3</td>
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</tbody>
</table>
|   | Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). | F.IF.C.9  
F.IF.B.6 | 3.3, 6.3, 6.4 |
|---|---|---|---|
| 10 | Calculate and interpret the average rate of change of a function presented symbolically or as a table; estimate the rate of change from a graph. | F.IF.C.9  
F.IF.B.6 | 3.3, 6.3, 6.4 |
| 11 | Graph linear, square root, cube root, and piecewise-defined functions (including step and absolute value functions) expressed symbolically. Graph by hand in simple cases and using technology in more complex cases, showing key features of the graph. | F.IF.C.7  
F.IF.C.7a  
F.IF.C.7b | 3.2, 3.3, 3.4, 3.5, 3.6, 3.7, 4.7, 10.1, 10.2 |
Research about Teaching and Learning Mathematics

Structure teaching of mathematical concepts and skills around problems to be solved (Checkly, 1997; Wood & Sellars, 1996; Wood & Sellars, 1997)

Encourage students to work cooperatively with others (Johnson & Johnson, 1975; Davidson, 1990)

Use group problem-solving to stimulate students to apply their mathematical thinking skills (Artzt & Armour-Thomas, 1992)

Students interact in ways that support and challenge one another’s strategic thinking (Artzt, Armour-Thomas, & Curcio, 2008)

Activities structured in ways allowing students to explore, explain, extend, and evaluate their progress (National Research Council, 1999)

There are three critical components to effective mathematics instruction (Shellard & Moyer, 2002):

- Teaching for conceptual understanding within the Balanced Math approach
- Developing children’s procedural literacy
- Promoting strategic competence through meaningful problem-solving investigations

Teachers should be:

- Demonstrating acceptance and recognition of students’ divergent ideas.
- Challenging students to think deeply about the problems they are solving, extending thinking beyond the solutions and algorithms required to solve the problem
- Influencing learning by asking challenging and interesting questions to accelerate students’ innate inquisitiveness and foster them to examine concepts further.
- Projecting a positive attitude about mathematics and about students’ ability to “do” mathematics

Students should be:

- Actively engaging in “doing” mathematics
- Solving challenging problems
- Investigating meaningful real-world problems
- Making interdisciplinary connections
- Developing an understanding of mathematical knowledge required to “do” mathematics and connect the language of mathematical ideas with numerical representations
- Sharing mathematical ideas, discussing mathematics with one another, refining and critiquing each other’s ideas and understandings
- Communicating in pairs, small group, or whole group presentations
- Using multiple representations to communicate mathematical ideas
- Using connections between pictures, oral language, written symbols, manipulative models, and real-world situations
- Using technological resources and other 21st century skills to support and enhance mathematical understanding
Mathematics is not a stagnant field of textbook problems; rather, it is a dynamic way of constructing meaning about the world around us, generating knowledge and understanding about the real world every day. Students should be metaphorically rolling up their sleeves and “doing mathematics” themselves, not watching others do mathematics for them or in front of them. (Protheroe, 2007)

**Balanced Mathematics Instructional Model**

Balanced math consists of three different learning opportunities; guided math, shared math, and independent math. Ensuring a balance of all three approaches will build conceptual understanding, problem solving, computational fluency, and procedural fluency. Building conceptual understanding is the focal point of developing mathematical proficiency. Students should frequently work on rigorous tasks, talk about the math, explain their thinking, justify their answer or process, build models with graphs or charts or manipulatives, and use technology.

When balanced math is used in the classroom it provides students opportunities to:

- solve problems
- make connections between math concepts and real-life situations
- communicate mathematical ideas (orally, visually and in writing)
- choose appropriate materials to solve problems
- reflect and monitor their own understanding of the math concepts
- practice strategies to build procedural and conceptual confidence

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Teacher builds conceptual understanding by modeling through demonstration, explicit instruction, and think-alouds, as well as guiding students as they practice math strategies and apply problem solving strategies. (whole group or small group instruction)

Teacher and students practice mathematics processes together through interactive activities, problem solving, and discussion. (whole group or small group instruction)

Students practice math strategies independently to build procedural and computational fluency. Teacher assesses learning and re-teaches as necessary. (whole group instruction, small group instruction, or centers)
<table>
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<td>Collaborative Problem Solving</td>
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<td>Connect Previous Knowledge to New Learning</td>
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<td>Making Thinking Visible</td>
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<td>Develop and Demonstrate Mathematical Practices</td>
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<td>Inquiry-Oriented and Exploratory Approach</td>
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<td>Multiple Solution Paths and Strategies</td>
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<td>Use of Multiple Representations</td>
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<td>Explain the Rationale of your Math Work</td>
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<td>Quick Writes</td>
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<td>Gallery Walks</td>
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<td>Small Group and Whole Class Discussions</td>
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<td>Analyze Student Work</td>
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<td>Identify Student’s Mathematical Understanding</td>
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<td>Identify Student’s Mathematical Misunderstandings</td>
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<td>Interviews</td>
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<td>Role Playing</td>
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<td>Diagrams, Charts, Tables, and Graphs</td>
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<td>Anticipate Likely and Possible Student Responses</td>
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<td>Collect Different Student Approaches</td>
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<td>Asking Assessing and Advancing Questions</td>
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<td>Re-voicing</td>
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<td>Challenging</td>
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<td>Pressing for Accuracy and Reasoning</td>
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<td>Maintain the Cognitive Demand</td>
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## Educational Technology

### Standards

| 8.1.12.A.3, 8.1.12.E.1, 8.2.12.E.1 |

- **Technology Operations and Concepts**
  - Collaborate in online courses, learning communities, social networks or virtual worlds to discuss a resolution to a problem or issue.

  **Example:** Students can use Google Docs and digital tools as a means of discussing and collaboratively solving the Introduction to Polynomials - College Fund task.

- **Research and Information Fluency**
  - Produce a position statement about a real world problem by developing a systematic plan of investigation with peers and experts synthesizing information from multiple sources.

  **Example:** Use digital tools to graph linear systems of inequalities that model a solution to a real world situation. Students can then write a position statement to justify their solution, mathematical thinking and modeling with peers.

  [http://www.mathsisfun.com/data/graphs-index.html](http://www.mathsisfun.com/data/graphs-index.html)

- **Computational Thinking: Programming**
  - Demonstrate an understanding of the problem solving capacity of computers in our world. 8.2.12.E.1

  **Example:** Students can use and explain the advantages of using graphing calculators, GeoGebra or Desmos to graph systems of inequalities to solve contextual problems.


  **Link:** [https://www.state.nj.us/education/cccs/2014/tech/](https://www.state.nj.us/education/cccs/2014/tech/)
Career Ready Practices

Career Ready Practices describe the career-ready skills that all educators in all content areas should seek to develop in their students. They are practices that have been linked to increase college, career, and life success. Career Ready Practices should be taught and reinforced in all career exploration and preparation programs with increasingly higher levels of complexity and expectation as a student advances through a program of study.

- **CRP2. Apply appropriate academic and technical skills.**
  Career-ready individuals readily access and use the knowledge and skills acquired through experience and education to be more productive. They make connections between abstract concepts with real-world applications, and they make correct insights about when it is appropriate to apply the use of an academic skill in a workplace situation.

  **Example:** Students will apply prior knowledge when solving real world problems. Students will make sound judgments about the use of specific tools, such as Graphing Calculators, to explore and deepen understanding the concepts related to linear functions, exponential functions and linear systems of equations including how to solve, write, graph and explain these relationships accurately.

- **CRP4. Communicate clearly and effectively and with reason.**
  Career-ready individuals communicate thoughts, ideas, and action plans with clarity, whether using written, verbal, and/or visual methods. They communicate in the workplace with clarity and purpose to make maximum use of their own and others’ time. They are excellent writers; they master conventions, word choice, and organization, and use effective tone and presentation skills to articulate ideas. They are skilled at interacting with others; they are active listeners and speak clearly and with purpose. Career-ready individuals think about the audience for their communication and prepare accordingly to ensure the desired outcome.

  **Example:** Students will engage in mathematical discourse on a daily basis. They will be expected to communicate the reasoning behind their solution paths, make connections to the context and the quantities involved and use proper vocabulary. Students will be able to accurately describe the relationships depicted in linear functions, exponential functions and linear systems of equations visually, verbally and algebraically. They will be able to explain the meaning behind the solution path/representation and defend their rationale. Students will also ask probing questions of others to clarify and deepen understanding.
Career Ready Practices

- **CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.**
  Career-ready individuals readily recognize problems in the workplace, understand the nature of the problem and devise effective plans to solve the problem. They are aware of problems when they occur and take action quickly to address the problem; they thoughtfully investigate the root cause of the problem prior to introducing solutions. They carefully consider the options to solve the problem. Once a solution is agreed upon, they follow through to ensure the problem is solved, whether through their own actions or the actions of others.
  
  **Example:** Throughout their daily lessons, students will understand the meaning of a problem by analyzing the relationships among the quantities, constraints and goals of the task. This analytic process will encourage students to find entry points that will facilitate problem solving. Students will become effective at self-monitoring, evaluating and critiquing their process. This in turn will facilitate their ability to progress as they are working and change strategy when necessary. Students will be able to use critical thinking to solve, model, interpret, create, compare and explain linear functions, linear systems and exponential functions.

- **CRP12. Work productively in teams while using cultural global competence.**
  Career-ready individuals positively contribute to every team, whether formal or informal. They apply an awareness of cultural difference to avoid barriers to productive and positive interaction. They find ways to increase the engagement and contribution of all team members. They plan and facilitate effective team meetings.
  
  **Example:** Students will work in collaborative and whole group settings to develop various solutions to math tasks. They will work together to understand the terms of the problem, ask each other clarifying and challenging questions, and develop agreed upon solutions using a variety of strategies and models. Students will listen to, read and discuss arguments with respect and courtesy at all times. A willingness to assist one another will be cultivated and honored. Students will demonstrate and explain to a peer or small group how to solve, model, interpret, create, compare and explain linear functions, linear systems and exponential functions.
### WIDA Proficiency Levels

At the given level of English language proficiency, English language learners will process, understand, produce or use

<table>
<thead>
<tr>
<th>Level</th>
<th>Language and Communication Skills</th>
</tr>
</thead>
</table>
| **6- Reaching** | - Specialized or technical language reflective of the content areas at grade level  
- A variety of sentence lengths of varying linguistic complexity in extended oral or written discourse as required by the specified grade level  
- Oral or written communication in English comparable to proficient English peers |
| **5- Bridging** | - Specialized or technical language of the content areas  
- A variety of sentence lengths of varying linguistic complexity in extended oral or written discourse, including stories, essays or reports  
- Oral or written language approaching comparability to that of proficient English peers when presented with grade level material. |
| **4- Expanding** | - Specific and some technical language of the content areas  
- A variety of sentence lengths of varying linguistic complexity in oral discourse or multiple, related sentences or paragraphs  
- Oral or written language with minimal phonological, syntactic or semantic errors that may impede the communication, but retain much of its meaning, when presented with oral or written connected discourse, with sensory, graphic or interactive support |
| **3- Developing** | - General and some specific language of the content areas  
- Expanded sentences in oral interaction or written paragraphs  
- Oral or written language with phonological, syntactic or semantic errors that may impede the communication, but retain much of its meaning, when presented with oral or written, narrative or expository descriptions with sensory, graphic or interactive support |
| **2- Beginning** | - General language related to the content area  
- Phrases or short sentences  
- Oral or written language with phonological, syntactic, or semantic errors that often impede the communication when presented with one to multiple-step commands, directions, or a series of statements with sensory, graphic or interactive support |
| **1- Entering** | - Pictorial or graphic representation of the language of the content areas  
- Words, phrases or chunks of language when presented with one-step commands directions, WH-, choice or yes/no questions, or statements with sensory, graphic or interactive support |
# Language Development Supports For English Language Learners

## To Increase Comprehension and Communication Skills

### Environment

- Welcoming and stress-free
- Respectful of linguistic and cultural diversity
- Honors students’ background knowledge
- Sets clear and high expectations
- Includes routines and norms
- Is thinking-focused vs. answer-seeking
- Offers multiple modalities to engage in content learning and to demonstrate understanding
- Includes explicit instruction of specific language targets
- Provides participation techniques to include all learners
- Integrates learning centers and games in a meaningful way
- Provides opportunities to practice and refine receptive and productive skills in English as a new language
- Integrates meaning and purposeful tasks/activities that:
  - Are accessible by all students through multiple entry points
  - Are relevant to students’ lives and cultural experiences
  - Build on prior mathematical learning
  - Demonstrate high cognitive demand
  - Offer multiple strategies for solutions
  - Allow for a language learning experience in addition to content

### Sensory Supports*

- Real-life objects (realia) or concrete objects
- Physical models
- Manipulatives
- Pictures & photographs
- Visual representations or models such as diagrams or drawings
- Videos & films
- Newspapers or magazines
- Gestures
- Physical movements
- Music & songs

### Graphic Supports*

- Graphs
- Charts
- Timelines
- Number lines
- Graphic organizers
- Graphing paper

### Interactive Supports*

- In a whole group
- In a small group
- With a partner such as Turn-and-Talk
- In pairs as a group (first, two pairs work independently, then they form a group of four)
- In triads
- Cooperative learning structures such as Think-Pair-Share
- Interactive websites or software
- With a mentor or coach

### Verbal and Textual Supports

- Labeling
- Students’ native language
- Modeling
- Repetitions
- Paraphrasing
- Summarizing
- Guiding questions
- Clarifying questions
- Probing questions
- Leveled questions such as What? When? Where? How? Why?
- Questioning prompts & cues
- Word Banks
- Sentence starters
- Sentence frames
- Discussion frames
- Talk moves, including Wait Time

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BUILDING EQUITY IN YOUR TEACHING PRACTICE

How do the essential questions highlight the connection between the big ideas of the unit and equity in your teaching practice?

**CONTENT INTEGRATION**
Teachers use examples and content from a variety of cultures & groups.

This unit / lesson is connected to other topics explored with students.

There are multiple viewpoints reflected in the content of this unit / lesson.

The materials and resources are reflective of the diverse identities and experiences of students.

The content affirms students, as well as exposes them to experiences other than their own.

**KNOWLEDGE CONSTRUCTION**
Teachers help students understand how knowledge is created and influenced by cultural assumptions, perspectives & biases.

This unit / lesson provides context to the history of privilege and oppression.

This unit / lesson addresses power relationships.

This unit / lesson helps students to develop research and critical thinking skills.

This curriculum creates windows and mirrors* for students.


**PREJUDICE REDUCTION**
Teachers implement lessons and activities to assert positive images of ethnic groups & improve intergroup relations.

This unit / lesson helps students question and unpack biases & stereotypes.

This unit / lesson helps students examine, research and question information and sources.

The curriculum encourages discussion and understanding about the groups of people being represented.

This unit / lesson challenges dominant perspectives.

**EQUITABLE PEDAGOGY**
Teachers modify techniques and methods to facilitate the academic achievement of students from diverse backgrounds.

The instruction has been modified to meet the needs of each student.

Students feel respected and their cultural identities are valued.

Additional supports have been provided for students to become successful and independent learners.

Opportunities are provided for students to reflect on their learning and provide feedback.

**EMPOWERING SCHOOL CULTURE**
Using the other four dimensions to create a safe and healthy educational environment for all.

There are opportunities for students to connect with the community.

My classroom is welcoming and supportive for all students?

I am aware of and sensitive to the needs of my students and their families.

There are effective parent communication systems established. Parents can talk to me about issues as they arise in my classroom.
Culturally Relevant Pedagogy Examples

- **Everyone has a Voice:** Create a classroom environment where students know that their contributions are expected and valued.
  
  **Example:** Norms for sharing are established that communicate a growth mindset for mathematics. All students are capable of expressing mathematical thinking and contributing to the classroom community. Students learn new ways of looking at problem solving by working with and listening to each other.

- **Present New Concepts Using Student Vocabulary:** Use student diction to capture attention and build understanding before using academic terms.
  
  **Example:** Teach math vocabulary in various modalities for students to remember. Use multi-modal activities, analogies, visual cues, graphic representations, gestures, pictures, practice and cognates. Model to students that some vocabulary has multiple meanings. Have students create the Word Wall with their definitions and examples to foster ownership.

- **Establish Inclusion:** Highlight how the topic may relate or apply to students.
  
  **Example:** After a brief explanation of slope, have students come up with examples of slope at home, in their neighborhood and outside of their neighborhood. After having a volunteer list a few in each category, use the examples in class with the students. Establishing inclusion also involves regularly grouping students with different classmates to share unique perspectives.

- **Integrate Relevant Word Problems:** Contextualize equations using word problems that reference student interests and cultures.
  
  **Example:** When learning different types of functions, problems that relate to student interests such as music, sports and art enable the students to understand and relate to the concept in a more meaningful way.

- **Encourage Student Leadership:** Create an avenue for students to propose problem solving strategies and potential projects.
  
  **Example:** Students can learn about different function types by creating problems together and deciding if the problems fit the necessary criteria. This experience will allow students to discuss and explore their current level of understanding by applying the concepts to relevant real-life experiences.
### Differentiated Instruction

Accommodate Based on Students Individual Needs: Strategies

<table>
<thead>
<tr>
<th>Time/General</th>
<th>Processing</th>
<th>Comprehension</th>
<th>Recall</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Extra time for assigned tasks</td>
<td>• Extra Response time</td>
<td>• Precise processes for balanced math instructional model</td>
<td>• Teacher-made checklist</td>
</tr>
<tr>
<td>• Adjust length of assignment</td>
<td>• Have students verbalize steps</td>
<td>• Short manageable tasks</td>
<td>• Use visual graphic organizers</td>
</tr>
<tr>
<td>• Timeline with due dates for reports and projects</td>
<td>• Repeat, clarify or reword directions</td>
<td>• Brief and concrete directions</td>
<td>• Reference resources to promote independence</td>
</tr>
<tr>
<td>• Communication system between home and school</td>
<td>• Mini-breaks between tasks</td>
<td>• Provide immediate feedback</td>
<td>• Visual and verbal reminders</td>
</tr>
<tr>
<td>• Provide lecture notes/outline</td>
<td>• Provide a warning for transitions</td>
<td>• Small group instruction</td>
<td>• Graphic organizers</td>
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<td></td>
<td>• Partnering</td>
<td>• Emphasize multi-sensory learning</td>
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<thead>
<tr>
<th>Assistive Technology</th>
<th>Tests/Quizzes/Grading</th>
<th>Behavior/Attention</th>
<th>Organization</th>
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<tbody>
<tr>
<td>• Computer/whiteboard</td>
<td>• Extended time</td>
<td>• Consistent daily structured routine</td>
<td>• Individual daily planner</td>
</tr>
<tr>
<td>• Tape recorder</td>
<td>• Study guides</td>
<td>• Simple and clear classroom rules</td>
<td>• Display a written agenda</td>
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<tr>
<td>• Video Tape</td>
<td>• Shortened tests</td>
<td>• Frequent feedback</td>
<td>• Note-taking assistance</td>
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<td></td>
<td>• Read directions aloud</td>
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<td>• Color code materials</td>
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</tbody>
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- **Behavior/Attention**
  - Consistent daily structured routine
  - Simple and clear classroom rules
  - Frequent feedback

- **Organization**
  - Individual daily planner
  - Display a written agenda
  - Note-taking assistance
  - Color code materials
## Differentiated Instruction

### Accommodate Based on Content Needs: Strategies

- Anchor charts to model strategies and process
- Reference sheets that list formulas, step-by-step procedures and model strategies
- Conceptual word wall that contain definitions, translations, pictures and/or examples
- Graphic organizer to help students solve quadratic equations using different methods (such as quadratic formula, completing the square, factoring, etc.)
- Translation dictionary
- Sentence stems to provide additional language support for ELL students
- Teacher modeling
- Highlight and label solution steps for multi-step problems in different colors
- Create an interactive notebook with students with a table of contents so they can refer to previously taught material readily
- Targeted assistance for students when summarizing and interpreting two-way frequency tables by using real world examples
- Graph paper
- Step by step directions on how to use a graphing calculator to fit functions to data and plot residuals
- Visual, verbal and algebraic models of quadratic functions
- A chart noting key features of functions from graphs and tables
- Videos to reinforce skills and thinking behind concepts
- Use real world data sets to facilitate students’ ability to compare center and spread of two sets of data
Interdisciplinary Connections

Model interdisciplinary thinking to expose students to other disciplines.

Social Studies Connection:
Sieves of Eratosthenes (6.2.8.D.3.c)
- Students will learn to identify prime numbers using the Sieve of Eratosthenes. They will also study how the sieve was discovered and about Eratosthenes, the Greek mathematician who was responsible for the sieve. See more about the incredible things that Eratosthenes did at: [http://www.socialstudiesforkids.com/www/world/eratosthenesdef.htm](http://www.socialstudiesforkids.com/www/world/eratosthenesdef.htm) and [http://encyclopedia.kids.net.au/page/er/Eratosthenes](http://encyclopedia.kids.net.au/page/er/Eratosthenes)

Science Connection:
Cicadas Brood X (MS-LS1-4)
- Students will review and learn about cicadas found in North America that emerge from the ground every 17 years. These cicadas are called Magicicada Septendecim. They will discuss the life cycle of an insect and the predators that an insect has. Learn more information about cicadas at: [http://bugfacts.net/cicada.php](http://bugfacts.net/cicada.php)

Earth Day Project (MS-ESS3-3)
- Students will learn about recycling and Earth Day. They will discuss different ways to recycle and activities that can be done for Earth Day. Learn more information about recycling and Earth Day at: [http://www.earthday.org/](http://www.earthday.org/) or choose a video to watch at [http://www.bing.com/videos/search?q=earth+day&qpvt=earth+day&FORM=VDRE](http://www.bing.com/videos/search?q=earth+day&qpvt=earth+day&FORM=VDRE)
# Enrichment

## What is the Purpose of Enrichment?

- The purpose of enrichment is to provide extended learning opportunities and challenges to students who have already mastered, or can quickly master, the basic curriculum. Enrichment gives the student more time to study concepts with greater depth, breadth, and complexity.
- Enrichment also provides opportunities for students to pursue learning in their own areas of interest and strengths.
- Enrichment keeps advanced students engaged and supports their accelerated academic needs.
- Enrichment provides the most appropriate answer to the question, “What do you do when the student already knows it?”

## Enrichment is…

- Planned and purposeful
- *Different*, or differentiated, work – not just *more* work
- Responsive to students’ needs and situations
- A promotion of high-level thinking skills and making connections within content
- The ability to apply different or multiple strategies to the content
- The ability to synthesize concepts and make real world and cross-curricular connections
- Elevated contextual complexity
- Sometimes independent activities, sometimes direct instruction
- Inquiry based or open-ended assignments and projects
- Using supplementary materials in addition to the normal range of resources
- Choices for students
- Tiered/Multi-level activities with flexible groups (may change daily or weekly)

## Enrichment is not…

- Just for gifted students (some gifted students may need intervention in some areas just as some other students may need frequent enrichment)
- Worksheets that are more of the same (busywork)
- Random assignments, games, or puzzles not connected to the content areas or areas of student interest
- Extra homework
- A package that is the same for everyone
- Thinking skills taught in isolation
- Unstructured free time
# Assessments

## Required District/State Assessments
- Unit Assessment
- PARCC
- SGO Assessments

## Suggested Formative/Summative Classroom Assessments
- Describe Learning Vertically
- Identify Key Building Blocks
- Make Connections (between and among key building blocks)
- Short/Extended Constructed Response Items
- Multiple-Choice Items (where multiple answer choices may be correct)
- Drag and Drop Items
- Use of Equation Editor
- Quizzes
- Journal Entries/Reflections/Quick-Writes
- Accountable talk
  - Projects
  - Portfolio
  - Observation
- Graphic Organizers/Concept Mapping
  - Presentations
  - Role Playing
- Teacher-Student and Student-Student Conferencing
- Homework
New Jersey Student Learning Standards

A.REI.C.6: Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

A.CED.A.3: Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*

A.REI.C.5: Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

A.REI.D.12: Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

A.CED.A.3: Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*

A.CED.A.2: Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

F.IF.A.1: Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).

F.IF.A.2: Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

F.LE.A.1: Distinguish between situations that can be modeled with linear functions and with exponential functions.

F.LE.A.1a: Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
F.LE.A.1b: Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

F.LE.A.1c: Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

F.LE.A.2: Construct linear and exponential functions - including arithmetic and geometric sequences - given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
* [Algebra 1 limitation: exponential expressions with integer exponents]

F.IF.A.3: Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$ for $n \geq 1$.

F.BF.A.1: Write a function that describes a relationship between two quantities.

F.BF.A.1a. Determine an explicit expression, a recursive process, or steps for calculation from a context

A.SSE.A.1: Interpret expressions that represent a quantity in terms of its context.

A.SSE.A.1a: Interpret parts of an expression, such as terms, factors, and coefficients.

A.SSE.A.1b: Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret $P(1+r)n$ as the product of $P$ and a factor not depending on $P$.
* [Algebra 1 limitation: exponential expressions with integer exponents]

F.IF.B.4: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. * [Focus on exponential functions]

F.LE.B.5: Interpret the parameters in a linear or exponential function in terms of a context.
**New Jersey Student Learning Standards**

**F.IF.B.5:** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function $h(n)$ gives the number of person-hours it takes to assemble $n$ engines in a factory, then the positive integers would be an appropriate domain for the function.

**F.IF.B.6:** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

**F.IF.C.7:** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

**F.IF.C.7a:** Graph linear and quadratic functions and show intercepts, maxima, and minima.

**F.IF.C.7b:** Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
1. Make sense of problems and persevere in solving them.

2. Reason abstractly and quantitatively.

3. Construct viable arguments and critique the reasoning of others.

4. Model with mathematics.

5. Use appropriate tools strategically.

6. Attend to precision.

7. Look for and make use of structure.

8. Look for and express regularity in repeated reasoning.
<table>
<thead>
<tr>
<th>Course: Algebra I</th>
<th>Unit: 2 (Two)</th>
<th>Topic: Modeling with Linear Functions, Linear Systems, &amp; Exponential Functions</th>
</tr>
</thead>
</table>

**NJSLS:**

**Unit Focus:**
- Solve linear systems of equations
- Create equations that describe numbers or relationships
- Interpret the structure of expressions
- Represent and solve equations and inequalities graphically
- Construct & compare linear & exponential models
- Interpret expressions for functions in terms of the situation
- Build a function that models a relationship between two quantities
- Understand the concept of a function and use function notation
- Interpret functions that arise in applications in terms of the context
- Analyze functions using different representations

**New Jersey Student Learning Standard(s):**
A.REI.C.6: Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

A.CED.A.3: Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*

A.REI.C.5: Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

**Student Learning Objective 1:** Solve multistep contextual problems by identifying variables, writing equations, and solving systems of linear equations in two variables algebraically and graphically.
## Modified Student Learning Objectives/Standards:

**M.EE.A-REI.10–12:** Interpret the meaning of a point on the graph of a line. *For example, on a graph of pizza purchases, trace the graph to a point and tell the number of pizzas purchased and the total cost of the pizzas.*

**M.EE.A-CED.2–4:** Solve one-step inequalities.

<table>
<thead>
<tr>
<th>MPs</th>
<th>Evidence Statement Key/Clarifications</th>
<th>Skills, Strategies &amp; Concepts</th>
<th>Essential Understandings/Questions (Accountable Talk)</th>
<th>Tasks/Activities</th>
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</thead>
<tbody>
<tr>
<td>MP 1 MP 2 MP 3 MP 4</td>
<td>A-CED.3-1</td>
<td>Identify and define variables representing essential features of the model.</td>
<td>What are the characteristics of a problem that determine which method of solving is most efficient?</td>
<td>Type II, III:</td>
</tr>
<tr>
<td></td>
<td>• Solve multi-step contextual problems that require writing and analyzing systems of linear inequalities in two variables to find viable solutions. <a href="https://dese.mo.gov/sites/default/files/asmt-dlm-essential-elements-math.pdf">https://dese.mo.gov/sites/default/files/asmt-dlm-essential-elements-math.pdf</a></td>
<td>Model real world situations by creating a system of linear equations.</td>
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<td>Cash box</td>
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<td>Solve systems of linear equations using the elimination or substitution method.</td>
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<td>Quinoa Pasta 2</td>
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<td>Solve systems of linear equations by graphing.</td>
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<td>Dimes and Quarters</td>
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<td>Interpret the solution(s) in context.</td>
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<td>Additional Tasks:</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Systems of equations can be solved exactly (algebraically) and approximately (graphically).</td>
<td></td>
<td>Accurately Weighing Pennies I</td>
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<td><strong>SPED Strategies:</strong></td>
<td></td>
<td>Accurately Weighing Pennies II</td>
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<td>Create a Google Doc/Anchor Chart/Notes that illustrates the similarities and differences between equations and inequalities using real life examples.</td>
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<td>Coffee</td>
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<td>Use assessing and advancing questions to help students verbalize and move their level of thinking and understanding to a higher level.</td>
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<td>Find A System</td>
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<td>Fishing Adventures 3</td>
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<td>Graphs</td>
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<td>Growing Coffee</td>
</tr>
</tbody>
</table>
Use real life examples of inequalities to reinforce understanding

**Examples:**
- Height restrictions at amusement parks.
- Number of tickets that are needed to be sold to reach a financial goal.

**ELL Support:**
Teacher provides the necessary support (linguistic and conceptual) so that students can work independently on problems involving systems of equations.

Strategies include reframing questions, filling in background knowledge gaps and use of native language.

Give students notes that include expectations, common misconceptions and vocabulary (English and native language) relevant to systems of equations. This will facilitate independence and increased proficiency.
New Jersey Student Learning Standard(s):
A.REI.D.12: Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

A.CED.A.3: Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*

**Student Learning Objective 2:** Graph linear inequalities and systems of linear inequalities in two variables and explain that the solution to the system.

**Modified Student Learning Objectives/Standards:** N/A

<table>
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<tr>
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<th>Tasks/Activities</th>
</tr>
</thead>
<tbody>
<tr>
<td>MP 1</td>
<td>A.REI.12</td>
<td>Model real world situations by creating a system of linear inequalities represent the context.</td>
<td>Graphing provides a model for depicting the relationship of variables in a system of inequalities.</td>
<td>Type I: Dimes and Quarters</td>
</tr>
<tr>
<td>MP 2</td>
<td>• Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.</td>
<td>Interpret the solution(s) in context.</td>
<td>How can the graphical model of a system of inequalities be used to reason and draw conclusions?</td>
<td>Type II, III: Fishing Adventures 3</td>
</tr>
<tr>
<td>MP 4</td>
<td>A.CED.3-1</td>
<td>Use technology to graph the equations and understand that all solutions to an equation in two variables are contained within the graph of that equation.</td>
<td></td>
<td>Writing constraints</td>
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<tr>
<td>MP 5</td>
<td>• Solve multi-step contextual problems that require writing and analyzing systems of inequalities</td>
<td><strong>SPED Strategies:</strong> Encourage students to use the resources related to systems of inequalities.</td>
<td></td>
<td>Growing coffee</td>
</tr>
<tr>
<td>MP 6</td>
<td></td>
<td></td>
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<td>Cash box</td>
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<tr>
<td>linear inequalities in two variables to find viable solutions. <a href="https://dese.mo.gov/sites//default/files/asmt-dlm-essential-elements-math.pdf">https://dese.mo.gov/sites//default/files/asmt-dlm-essential-elements-math.pdf</a></td>
<td>inequalities such as the Google Docs/Anchor Chart/Notes created for SLO 1.</td>
<td>Model the thinking process involved in solving real life systems of inequalities by relating to previous learning. Choose topics of high interest for students to explore real life application of systems of inequalities so that time on task is maximized. <strong>ELL Support:</strong> Illustrate the relationship between terms related to inequalities and systems of inequalities in English and the students’ native language. Use cognates, words or word parts that sound similar and have similar meanings in a native language and a second language. Create heterogeneous groups where students with more developed mathematical understanding work with peers to provide support. Use real life situations such as financial goals for a fundraiser to ground student understanding of inequalities.</td>
<td>How Much Folate? Additional Tasks: Bernardo and Sylvia Play a Game Calculators Potato Farmer Solution Sets</td>
<td></td>
</tr>
</tbody>
</table>
New Jersey Student Learning Standard(s):
F.IF.A.1: Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If \( f \) is a function and \( x \) is an element of its domain, then \( f(x) \) denotes the output of \( f \) corresponding to the input \( x \). The graph of \( f \) is the graph of the equation \( y = f(x) \).

F.IF.A.2: Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

Student Learning Objective 3: Explain the definition of a function, including the relationship between the domain and range. Use function notation, evaluate functions and interpret statements in context.

Modified Student Learning Objectives/Standards:
M.EE.F-IF.1–3: Use the concept of function to solve problems.

<table>
<thead>
<tr>
<th>MPs</th>
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<th>Tasks/Activities</th>
</tr>
</thead>
<tbody>
<tr>
<td>MP 2</td>
<td>F.IF.A.1</td>
<td>Use the definition of a function to determine whether a relationship is a function given a table, graph or words.</td>
<td>Functions are relations that pair the input with exactly one output.</td>
<td>Type I:</td>
</tr>
<tr>
<td>MP 6</td>
<td>- Understand the concept of a function and use function notation. <a href="https://dese.mo.gov/sites//default/files/asmt-dlm-essential-elements-math.pdf">https://dese.mo.gov/sites//default/files/asmt-dlm-essential-elements-math.pdf</a></td>
<td>Given the function ( f(x) ), identify ( x ) as an element of the domain, the input, and ( f(x) ) is an element in the range, the output.</td>
<td>How can functions be used to find solutions to real-world problems and predict outcomes?</td>
<td>Cell phones</td>
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<tr>
<td>MP 7</td>
<td></td>
<td>Know that the graph of the function, ( f ), is the graph of the equation ( y=f(x) ).</td>
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<td>Domains</td>
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<td>When a relation is determined to be a function, use ( f(x) ) notation.</td>
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<td>Type II, III:</td>
</tr>
<tr>
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<td></td>
<td>Evaluate functions for inputs in their domain. Interpret statements that use function notation in terms of the context in which they are used.</td>
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<td>The Parking Lot</td>
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<td>Yam in the Oven</td>
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<td>Do two points always determine a linear function</td>
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<td>Finding the domain</td>
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<tr>
<td><strong>SPED Strategies:</strong></td>
<td><strong>Parabolas and Inverse Functions</strong></td>
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<td>Create examples of tables, graphs and verbal models that are functions and those that are not so that students have a reference to use when determining if a set of ordered pairs, a graph or a verbal model is a function.</td>
<td>Points on a graph</td>
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<td>Develop questions that encourage students to think through their determination of function vs. not a function.</td>
<td>Random Walk II</td>
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<td><strong>ELL Support:</strong></td>
<td>The customers</td>
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<tr>
<td>Provide students with notes that illustrate the essential characteristics of a function in various forms: tables, graphs, verbal models.</td>
<td>The random walk</td>
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<tr>
<td>Simplify linguistic complexity, use native language and provide real life examples to increase likelihood of conceptual understanding.</td>
<td>Using Function Notation I</td>
<td></td>
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<tr>
<td>Provide students with examples and non-examples of functions in tables, graphs and verbal models that are grounded in a context so that they can detect if a relationship is a function.</td>
<td>Using Function Notation II</td>
<td></td>
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</tr>
<tr>
<td>Provide necessary language support so that students can gain a greater level of conceptual understanding, i.e. native language explanations, word-to-word dictionary, simplified linguistic complexity, assessing and advancing questions.</td>
<td>Your Father</td>
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<td>Additional Tasks:</td>
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<td>Pets</td>
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<td>Printing Tickets</td>
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<td>Temperature</td>
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</tbody>
</table>
New Jersey Student Learning Standard(s):
F.LE.A.1: Distinguish between situations that can be modeled with linear functions and with exponential functions.

- **F.LE.A.1a**: Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
- **F.LE.A.1b**: Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
- **F.LE.A.1c**: Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

**Student Learning Objective 4**: Distinguish between and explain situations modeled with linear functions and with exponential functions.

**Modified Student Learning Objectives/Standards**:
M.EE.F-LE.1–3. Model a simple linear function such as $y = mx$ to show that these functions increase by equal amounts over equal intervals.

<table>
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<tr>
<th>MPs</th>
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<th>Essential Understandings/ Questions (Accountable Talk)</th>
<th>Tasks/Activities</th>
</tr>
</thead>
</table>
| MP 3 MP 6 | **F-LE.A.1**  
- Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).  
- **F-LE.2-2**  
  - Solve multi-step contextual problems with degree of difficulty | Identify and describe situations in which one quantity changes at a constant rate.  
Identify and describe situations in which a quantity grows or decays by a constant percent.  
Show that linear functions grow by equal differences over equal intervals.  
Show that exponential functions grow by equal factors over equal intervals.  
Use real life situations to illustrate both linear and exponential functions.  
**SPED Strategies:**  
Create Google Doc/Anchor Chart/Notes that verbally and pictorially describe the characteristics of linear and exponential functions. | What are the differentiating characteristics of linear vs. exponential functions?  
Linear and exponential functions can be used to draw conclusions, make predictions and support reasoning.  
Linear and exponential functions can be modeled in many ways including: verbally, graphically and as a table. | IFL “Solving Problems Using Linear and Exponential Models.”  
*Also addressed in IFL unit is F.LE.B.5. (F.IF.A.3 is not addressed in IFL Unit) |
| Type II, III:  
Algae Blooms  
Basketball Rebounds  
Equal Differences over Equal Intervals 1  
Equal Differences over Equal Intervals 2 |
appropriate to the course by constructing linear and/or exponential function models, where exponentials are limited to integer exponents.

Use high interest real life examples of linear and exponential functions to increase time on task.

Relate new concepts to prior learning to build knowledge and confidence.

**ELL Support:**
Use graphs of linear functions and exponential functions in context to illustrate the differences.

Facilitate student understanding by coupling the graphs and contexts with appropriate language support.

Create an anchor chart using appropriate language supports that provides a quick reference for students to utilize when determining whether a function is linear or exponential.

<table>
<thead>
<tr>
<th>Equal Factors over Equal Intervals</th>
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</thead>
<tbody>
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<td>Identifying Functions</td>
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<td>Illegal Fish</td>
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<td>In the Billions and Exponential Modeling</td>
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<td>In the Billions and Linear Modeling</td>
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<tr>
<td>Solving Problems with Linear and Exponential Models</td>
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<td>U.S. Population 1790-1860</td>
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<tr>
<td>Finding Linear and Exponential Models</td>
</tr>
<tr>
<td>PBA’s: Bottle Rocket</td>
</tr>
</tbody>
</table>
New Jersey Student Learning Standard(s):  
F.LE.A.2: Construct linear and exponential functions - including arithmetic and geometric sequences - given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).  
* [Algebra 1 limitation: exponential expressions with integer exponents]  

F.IF.A.3: Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by \( f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1) \) for \( n \geq 1 \).

**Student Learning Objective 5:** Write linear and exponential functions given a graph, table of values, or written description; construct arithmetic and geometric sequences

**Modified Student Learning Objectives/Standards:**  
M.EE.F-LE.1–3. Model a simple linear function such as \( y = mx \) to show that these functions increase by equal amounts over equal intervals.

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<th>Tasks/Activities</th>
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<tbody>
<tr>
<td>MP 2</td>
<td>F.LE.2-2</td>
<td>Create arithmetic and geometric sequences from verbal descriptions.</td>
<td>What information can be determined about the relationship of the values once they are represented in the appropriate function model?</td>
<td></td>
</tr>
<tr>
<td>MP 4</td>
<td></td>
<td>Create arithmetic sequences from linear functions.</td>
<td>How can equations be modeled to determine if a function is recursive?</td>
<td></td>
</tr>
<tr>
<td>MP 1</td>
<td></td>
<td>Create geometric sequences from exponential functions.</td>
<td>IFL “Solving Problems Using Linear and Exponential Models.” *Also addressed in IFL unit is F.LE.B.5. (F.IF.A.3 is not addressed in IFL Unit)</td>
<td></td>
</tr>
<tr>
<td>MP 5</td>
<td></td>
<td>Identify recursively defined sequences as functions.</td>
<td>Type II, III:</td>
<td></td>
</tr>
<tr>
<td>MP 6</td>
<td></td>
<td>Create linear and exponential functions given: a graph; a description of a relationship; or a table of values.</td>
<td>A valuable quarter</td>
<td></td>
</tr>
<tr>
<td>MP 7</td>
<td></td>
<td>Interpret statements that use function notation in terms of the context in which they are used.</td>
<td>Algae Blooms</td>
<td></td>
</tr>
</tbody>
</table>

Recognize that sequences, sometimes defined recursively, are functions whose domain is a subset of the set of integers.

**SPED Strategies:**
Review the notes and resources given regarding linear and exponential functions.

Link the task of writing the function given the verbal, graphical or pictorial depiction of the information to prior learning and review.

Create Google Doc/Anchor Chart/Notes that verbally and pictorially describe the characteristics of arithmetic and geometric sequences using concrete examples.

Explicitly describe a recursive sequence and provide examples that progress from concrete to abstract.

**ELL Support:**
Create an anchor chart/notes/graphic organizer that verbally and pictorially describes the characteristics of arithmetic and geometric sequences using concrete examples.

Ensure that students are provided with the necessary language support to access these concepts.

Provide students with the necessary support to write simpler functions based on a context and

<table>
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<th>Boiling Water</th>
<th>Boom Town</th>
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<td>Carbon 14 dating in practice II</td>
<td>Choosing and appropriate growth model</td>
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<tr>
<td>Do two points always determine a linear function</td>
<td>Exponential Parameters</td>
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<td>Moore’s Law and computers</td>
<td>Snake on a plane</td>
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<tr>
<td>Additional Tasks:</td>
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<td>Branches</td>
<td>Glaciers</td>
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<td>Parasitic Wasps</td>
<td>Population and Food Supply</td>
</tr>
<tr>
<td>Taxi!</td>
<td></td>
</tr>
</tbody>
</table>
relate the required skills to the task of writing arithmetic and geometric sequences.

New Jersey Student Learning Standard(s):
F.BF.A.1: Write a function that describes a relationship between two quantities.

   F.BF.A.1a: Determine an explicit expression, a recursive process, or steps for calculation from a context.

A.SSE.A.1: Interpret expressions that represent a quantity in terms of its context.

   A.SSE.A.1a: Interpret parts of an expression, such as terms, factors, and coefficients.

   A.SSE.A.1b: Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P. *[Algebra 1 limitation: exponential expressions with integer exponents]*

Student Learning Objective 6: Write explicit expressions, recursive processes and steps for calculation from a context that describes a linear or exponential relationship between two quantities.

Modified Student Learning Objectives/Standards:
M.EE.F-BF.1: Select the appropriate graphical representation (first quadrant) given a situation involving constant rate of change.
M.EE.A-SSE.1: Identify an algebraic expression involving one arithmetic operation to represent a real-world problem.

<table>
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<tr>
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</thead>
<tbody>
<tr>
<td>MP 2</td>
<td></td>
<td>Given a context, write explicit expressions, a recursive process or steps for calculation for linear and exponential relationships.</td>
<td>Functions represent relationships between variables and can be written to model real life scenarios in multiple forms.</td>
<td>Type II, III:</td>
</tr>
<tr>
<td>MP 4</td>
<td></td>
<td>Interpret parts of linear and exponential functions in context.</td>
<td>What do the different parts of the expressions</td>
<td></td>
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<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Skeleton Tower</td>
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<tr>
<td></td>
<td>- A-SSE.1-1</td>
<td></td>
<td></td>
<td>Mixing Candies</td>
</tr>
<tr>
<td></td>
<td>- Interpret exponential expressions, including related numerical expressions that represent a quantity in terms of its context.</td>
<td></td>
<td></td>
<td>Seeing Dots</td>
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<tr>
<td></td>
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<td></td>
<td>Animal populations</td>
</tr>
</tbody>
</table>
• Interpret parts of an expression, such as terms, factors, and coefficients.

• Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret \( P(1+r)n \) as the product of \( P \) and a factor not depending on \( P \).

Identify the different parts of the expression and explain their meaning within the context of a problem.

Decompose expressions and make sense of the multiple factors and terms by explaining the meaning of the individual parts.

Rewrite algebraic expressions in different equivalent forms such as factoring or combining like terms.

**SPED Strategies:**
Review the notes and resources given regarding linear functions, exponential functions and recursive process with students to prepare them.

Review the meaning of the parts of an expression by linking it to a real-life context of high interest.

Provide support to students as they work on writing functions by cuing them to the available resources, asking assessing and advancing questions and encouraging them to discuss their work with peers.

**ELL Support:**
Create an anchor chart with students that illustrate the essential characteristics of explicit expressions and recursive processes based on real life situations.

Use think-alouds as a support that allows students to discuss how they are processing information.

**Delivery Trucks**

**Exponential Parameters**

**Increasing or Decreasing Variation**

1

**Kimi and Jordan**

**Lake Algae**

**Mixing Fertilizer**

**Radius of a Cylinder**

**Summer Intern**

**The bank account**

**Additional Tasks:**

**Bears**

**Coffee**

**Conference Tables**

**Modeling a Context from a Verbal Description**
### New Jersey Student Learning Standard(s):**

**A.SSE.B.3:** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

**A.SSE.B.3c:** Use the properties of exponents to transform expressions for exponential functions. *For example the expression $1.15^t$ can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%. *[Algebra 1: limit to exponential expressions with integer exponents]*

### Student Learning Objective 7:*

Use properties of exponents to produce equivalent forms of exponential expressions in one variable.

### Modified Student Learning Objectives/Standards: N/A

**M.EE.A-SSE.3.** Solve simple algebraic equations with one variable using multiplication and division.

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<tr>
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</thead>
<tbody>
<tr>
<td>MP 1</td>
<td><strong>A-SSE.3c-1</strong></td>
<td>Use the properties of exponents to simplify or expand exponential expressions, recognizing these are equivalent forms.</td>
<td>The properties of exponents can be used to develop equivalent forms of the same expression.</td>
<td>Type II, III:</td>
</tr>
<tr>
<td>MP 2</td>
<td>Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression, where exponentials are limited to integer exponents.</td>
<td>Use properties of exponents (such as power of a power, product of powers, power of a product, and rational exponents, etc.) to write an equivalent form of an exponential function to reveal and explain specific information about its approximate rate of growth or decay.</td>
<td><strong>Profit of a company Ice Cream</strong></td>
<td></td>
</tr>
<tr>
<td>MP 4</td>
<td>Use the properties of exponents to transform</td>
<td><strong>Additional Tasks: Camels</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MP 7</td>
<td><strong>SPED Strategies:</strong> Review the properties of exponents and provide clear examples to remind students of this prior learning.</td>
<td><strong>Forms of Exponential Expressions</strong></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

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Students should be encouraged to work collaboratively to write explicit expresses and recursive processes related to meaningful contexts.
| Expressions for exponential functions. | Develop mnemonic devices with students to help them remember the properties of exponents and the circumstances under which the rules apply. Create a Google Doc/Anchor Chart/Notes recapping the properties of exponents with students for their reference when problem solving.  
**ELL Support:**  
Create an anchor chart/graphic organizer that lists the properties of exponents with a clear example for each using native language or simpler English terminology to engage all students. Model the thinking and processes involved in using the properties of exponents to create equivalent expressions. | Graphs of Quadratic Functions  
Profit of a company, assessment variation |
### New Jersey Student Learning Standards (s):

**F.IF.B.4:** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.* *(Focus on exponential functions)*

**F.LE.B.5:** Interpret the parameters in a linear or exponential function in terms of a context.

**F.IF.B.5:** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*

### Student Learning Objective 8:
Sketch graphs of linear and exponential functions expressed symbolically or from a verbal description. Show key features and interpret parameters in context.

### Modified Student Learning Objectives/Standards: N/A

**M.EE.F-IF.4-6:** Construct graphs that represent linear functions with different rates of change and interpret which is faster/slower, higher/lower, etc.

**EE.F-IF.1-3:** Use the concept of function to solve problems.

**EE.F-IF.4-6:** Construct graphs that represent linear functions with different rates of change and interpret which is faster/slower, higher/lower, etc.

### Skills, Strategies & Concepts

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<th>Evidence Statement Key/Clarifications</th>
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<th>F.LE.B.5</th>
<th>F.IF.B.5</th>
<th>Modified Standards</th>
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<tbody>
<tr>
<td>Solve multi-step contextual problems with degree of difficulty appropriate to the course by constructing linear and/or exponential function models, where exponentials are limited to integer exponents.</td>
<td>Given a verbal description of a relationship, sketch linear and exponential functions.</td>
<td>Identify intercepts and intervals where the function is positive/negative.</td>
<td>Interpreting parameters in context.</td>
<td>Determine the practical domain of a function.</td>
</tr>
<tr>
<td>How can the parameters of a function be determined when given a verbal or symbolic model?</td>
<td>Parts of the function reveal qualities of the relationship between the two variables without needing to graph.</td>
<td>IFL “Creating and Interpreting Functions.” <em>(F.LE.B.5 is not addressed in IFL Unit)</em></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Type II, III:</td>
<td>Warming and Cooling</td>
<td></td>
<td></td>
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<tr>
<td>Determine if the function is symmetric.</td>
<td>The following questions can be answered:</td>
<td>Words - Tables – Graphs</td>
<td></td>
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<tr>
<td><strong>SPED Strategies:</strong> Provide a recap of the parts of a function and what they mean to the behavior of the function.</td>
<td>• What are the X and Y intercepts?</td>
<td>Influenza Epidemic</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Create a Google Doc/Anchor Chart/Notes with the students detailing the relationship between the parts of a function and what can be determined without graphing.</td>
<td>• Where does the function increase or decrease?</td>
<td>PBA’s: Buddy Bags</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Encourage students to verbalize their thinking and approach to problem solving by asking assessing and advancing questions.</td>
<td>• What are the relative maximums and minimums of the function?</td>
<td>Additional Tasks: Golf</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>ELL Support:</strong> Illustrate the relationship between the parts of a function and how it is graphed using appropriate language and conceptual support.</td>
<td>• Is the function symmetric? If yes, where?</td>
<td>Graphs(2004)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Link the work to prior experiences and knowledge to increase confidence and proficiency.</td>
<td>• What happens to the ends of the graph of the function?</td>
<td>Modeling London’s Population</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Use Desmos or a graphing calculator to relate the parts of a function to the graph that is produced.</td>
<td>• Is there a period in the function? Where?</td>
<td>Printing Tickets</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ask students assessing and advancing questions as they explore the relationships between the parts of functions and graphs and document them in English and native language.</td>
<td></td>
<td>Squirrel Population</td>
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<td>Taxi! Average Cost</td>
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<tr>
<td></td>
<td></td>
<td>Newton’s Law of Cooling</td>
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</tbody>
</table>
**New Jersey Student Learning Standard(s):**

F.IF.C.9: Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

*Limit to linear and exponential*

**Student Learning Objective 9:** Compare properties of two functions each represented in a different way

**Modified Student Learning Objectives/Standards:** N/A

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<tbody>
<tr>
<td>MP 1</td>
<td>F-IF.9-1</td>
<td>Compare key features of two linear functions represented in different ways.</td>
<td>What information can be determined about the properties of two functions by comparing them algebraically, graphically, verbally or numerically (in tables)?</td>
<td>Type II, III:</td>
</tr>
<tr>
<td>MP 3</td>
<td></td>
<td>Compare key features of two exponential functions represented in different ways.</td>
<td></td>
<td>Throwing Baseballs</td>
</tr>
<tr>
<td>MP 5</td>
<td></td>
<td>Calculate the rate of change from a table of values or from a function presented symbolically.</td>
<td></td>
<td>Comparing Quadratic, Square Root, and Cube Root Functions Represented in Different Ways</td>
</tr>
<tr>
<td>MP 6</td>
<td></td>
<td>Estimate the rate of change from a graph.</td>
<td></td>
<td>Additional Tasks:</td>
</tr>
<tr>
<td>MP 8</td>
<td></td>
<td><strong>SPED Strategies:</strong> Model the thinking process behind comparing the properties of two functions represented in different ways.</td>
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<td>Flares</td>
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<tr>
<td></td>
<td></td>
<td><strong>ELL Support:</strong> Document the thinking process with students by note taking in the form of an Anchor Chart or Google Doc.</td>
<td></td>
<td>Printing Tickets</td>
</tr>
<tr>
<td>functions), and exponential functions with domains in the integers.</td>
<td>Model the thinking process behind comparing the key features of two functions represented in different ways.</td>
<td>Chronicle this in the form of notes/graphic organizer/anchor chart that includes linguistic support appropriate to the language proficiency level of the students. Use real life scenarios to illustrate the concept of comparing functions and calculating rate of change when the information is depicted in different ways such as graphically, symbolically, verbally or pictorially.</td>
<td></td>
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</tbody>
</table>

<https://dese.mo.gov/sites/default/files/asmt-dlm-essential-elements-math.pdf> New Jersey Student Learning Standard(s): F.IF.B.6: Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. Student Learning Objective 10: Calculate and interpret the average rate of change of a function presented symbolically or as a table; estimate the rate of change from a graph. Modified Student Learning Objectives/Standards: N/A M.EE.F-IF.4–6. Construct graphs that represent linear functions with different rates of change and interpret which is faster/slower, higher/lower, etc. |
<table>
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<th>MP 5</th>
<th>MP 6</th>
<th>MP 8</th>
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<tbody>
<tr>
<td><strong>F-IF.6-6a</strong></td>
<td><strong>Estimate the rate of change from a graph of functions.</strong></td>
<td>calculate the average rate of change over a specified interval of a function presented symbolically or in a table.</td>
<td><strong>Estimate the average rate of change over a specified interval of a function from the function’s graph.</strong></td>
<td><strong>Interpret, in context, the average rate of change of a function over a specified interval.</strong></td>
</tr>
<tr>
<td></td>
<td><strong>SPED Strategies:</strong></td>
<td>Model the strategy for students using contextualized examples to make retention of the concept more likely.</td>
<td></td>
<td><strong>ELL Support:</strong></td>
</tr>
<tr>
<td></td>
<td>Circulate while students are working on this concept to monitor understanding and provide instructional reinforcement.</td>
<td></td>
<td></td>
<td>Use think-aloud as a support that allows students to discuss how they are calculating average rate of change with peers.</td>
</tr>
<tr>
<td></td>
<td>Teacher encourages mathematical discourse and understanding by asking assessing and advancing</td>
<td>The average rate of change of a function over a specific time interval can be calculated and interpreted through the symbolic representation of the function or from the table of values.</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>IFL “Creating and Interpreting Functions.”</strong> (F.IF.B.5 is not addressed in IFL Unit)</td>
<td>Type II, III: Temperature Change</td>
<td>1,000 is half of 2,000</td>
<td><strong>Mathemafish Population</strong></td>
</tr>
<tr>
<td></td>
<td>The High School Gym</td>
<td>Additional Tasks:</td>
<td><strong>Modeling a Context from a Verbal Description</strong></td>
<td>Stock Prices</td>
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<tr>
<td></td>
<td>Laptop Battery charge 2</td>
<td></td>
<td></td>
<td>Laptop Battery charge 2</td>
</tr>
</tbody>
</table>
questions as groups are working collaboratively.

### New Jersey Student Learning Standard(s):**

**F.IF.C.7:** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

- **F.IF.C.7a.** Graph linear and quadratic functions and show intercepts, maxima, and minima.
- **F.IF.C.7b.** Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

**Student Learning Objective 11:** Graph linear, square root, cube root, and piecewise-defined functions (including step and absolute value functions) expressed symbolically. Graph by hand in simple cases and using technology in more complex cases, showing key features of the graph.

**Modified Student Learning Objectives/Standards:**

**M.EE.F-IF.1–3.** Use the concept of function to solve problems.

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</thead>
</table>
| MP 1 MP 5 MP 6 | **F-IF.7b**
- Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. | Graph linear, square root, cube root, and piecewise-defined functions. Graph more complicated cases of functions using technology. Identify and describe key features of the graphs of square root, cube root, and piecewise-defined functions. | What are the essential features of linear, square root, cube root, absolute value and piecewise functions? | Type II, III:
Bank Account Balance
Graphs of Quadratic Functions
Additional Tasks: |
- Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.


**SPED Strategies:**
Provide students with a cloze note taking sheet about graphing different types of functions to make the process of identifying important information more engaging. This will also be a reference sheet for students going forward.

Use examples for each type of graph that are interesting to the students to promote on task behavior.

Encourage students to work with a peer as they are working on graphing more complex functions using a graphing calculator.

**ELL Support:**
Create a reference sheet with students that illustrate how to graph linear, square root, cube root and piecewise functions using relevant examples and appropriate language. Information about key features of each type of graph should also be highlighted in this document. Model the thinking process involved in graphing different types of functions with students by using native language or decreasing linguistic complexity.

**Functions**

- Modeling London’s Population
- Velocity of a Free Falling Object
# Integrated Evidence Statements

**F-IF.A.Int.1: Understand the concept of a function and use function notation.**
- Tasks require students to use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a real-world context
- About a quarter of tasks involve functions defined recursively on a domain in the integers.

**F-Int.1-1: Given a verbal description of a linear or quadratic functional dependence, write an expression for the function and demonstrate various knowledge and skills articulated in the Functions category in relation to this function.**
- Given a verbal description of a functional dependence, the student would be required to write an expression for the function and then, e.g., identify a natural domain for the function given the situation; use a graphing tool to graph several input-output pairs; select applicable features of the function, such as linear, increasing, decreasing, quadratic, nonlinear; and find an input value leading to a given output value.
  - e.g., a functional dependence might be described as follows: "The area of a square is a function of the length of its diagonal." The student would be asked to create an expression such as \( f(x) = (1/2) x^2 \) for this function. The natural domain for the function would be the positive real numbers. The function is increasing and nonlinear. And so on.
  - e.g., a functional dependence might be described as follows: "The slope of the line passing through the points (1, 3) and (7, y) is a function of y." The student would be asked to create an expression such as \( s(y) = (3 - y)/(1 - 7) \) for this function. The natural domain for this function would be the real numbers. The function is increasing and linear. And so on.

**HS-Int.2: Solve multi-step mathematical problems with degree of difficulty appropriate to the course that requires analyzing quadratic functions and/or writing and solving quadratic equations.**
- Tasks do not have a real-world context.
- Exact answers may be required or decimal approximations may be given. Students might choose to take advantage of the graphing utility to find approximate answers or clarify the situation at hand. For rational solutions, exact values are required. For irrational solutions, exact or decimal approximations may be required. Simplifying or rewriting radicals is not required.
  - Some examples: Given the function \( f(x) = x^2 + x \), find all values of \( k \) such that \( f(3 - k) = f(3) \). (Exact answers are required.) Find a value of \( c \) so that the equation \( 2x^2 - cx + 1 = 0 \) has a double root. Give an answer accurate to the tenths place.

**HS.C.12.1: Construct, autonomously, chains of reasoning that will justify or refute propositions or conjectures about functions. Content scope: F-IF.8a**
- Tasks involve using algebra to prove properties of given functions. For example, prove algebraically that the function \( h(t) = t(t - 1) \) has minimum value 14; prove algebraically that the graph of \( g(x) = x^2 - x + 14 \) is symmetric about the line \( x = 12 \); prove that \( x^2 + 1 \) is never less than -2x.
Integrated Evidence Statements

- Scaffolding is provided to ensure tasks have appropriate level of difficulty. (For example, the prompt could show the graphs of $x^2+1$ and $-2x$ on the same set of axes, and say, "From the graph, it looks as if $x^2+1$ is never less than $-2x$. In this task, you will use algebra to prove it." And so on, perhaps with additional hints or scaffolding.)
- Tasks may have a mathematical or real-world context.

HS.D.2-6: Solve multi-step contextual word problems with degree of difficulty appropriate to the course, requiring application of course-level knowledge and skills articulated in A-CED, N-Q.2, A-SSE.3, A-REI.6, A-REI.12, A-REI.11-1, limited to linear and quadratic equations
  - A-CED is the primary content; other listed content elements may be involved in tasks as well.

HS.D.2-8: Solve multi-step contextual word problems with degree of difficulty appropriate to the course, requiring application of course-level knowledge and skills articulated in F-BF.1a, F-BF.3, A-CED.1, A-SSE.3, F-IF.B, F-IF.7, limited to linear functions and exponential functions with domains in the integers.
  - F-BF.1a is the primary content; other listed content elements may be involved in tasks as well.

HS.D.2-9: Solve multi-step contextual word problems with degree of difficulty appropriate to the course, requiring application of course-level knowledge and skills articulated in F-BF.1a, F-BF.3, A-CED.1, A-SSE.3, F-IF.B, F-IF.7, limited to linear and quadratic functions.
  - F-BF.1a is the primary content; other listed content elements may be involved in tasks as well.
# Unit 2 Vocabulary

- Algorithm
- Associative Property of Multiplication
- Area Model
- Array
- Benchmark Fractions
- Common Denominator
- Common Factor
- Common Multiple
- Commutative Property of Multiplication
- Compare
- Compatible Numbers
- Composite Number
- Decompose
- Denominator
- Distributive Property
- Divisibility Rules
- Divisor
- Dividend
- Equation
- Equivalent Fractions
- Estimate
- Expanded Form
- Expression
- Fact Family
- Factors
- Factor Pairs
- Fraction
- Greatest Common Factor
- Identity Property of Multiplication
- Inverse Operations
- Interpret
- Matrix Model
- Mental Math/ Mental Calculation
- Mixed Number
- Model/ Visual Model
- Multiples
- Multiplicative Identity Property of 1
- Numerator
- Partial Product
- Partial Quotient
- Pattern
- Place Value
- Prime Number
- Product
- Quotient
- Reasonableness
- Related Facts
- Remainder
- Rule
- Sequence
- Simplest Form
- Simplify
- Term
- Unit Fraction
- Unlike Denominators
- Unlike Numerators
- Variable
- Zero Property of Multiplication
## References & Suggested Instructional Websites

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Field Trip Ideas

**SIX FLAGS GREAT ADVENTURE:** This educational event includes workbooks and special science and math related shows throughout the day. Your students will leave with a better understanding of real world applications of the material they have learned in the classroom. Each student will have the opportunity to experience different rides and attractions linking mathematical and scientific concepts to what they are experiencing.

[www.sixflags.com](http://www.sixflags.com)

**MUSEUM of MATHEMATICS:** Mathematics illuminates the patterns that abound in our world. The National Museum of Mathematics strives to enhance public understanding and perception of mathematics. Its dynamic exhibits and programs stimulate inquiry, spark curiosity, and reveal the wonders of mathematics. The Museum’s activities lead a broad and diverse audience to understand the evolving, creative, human, and aesthetic nature of mathematics.

[www.momath.org](http://www.momath.org)

**LIBERTY SCIENCE CENTER:** An interactive science museum and learning center located in Liberty State Park. The center, which first opened in 1993 as New Jersey's first major state science museum, has science exhibits, the largest IMAX Dome theater in the United States, numerous educational resources, and the original *Hoberman sphere*.

[http://lsc.org/plan-your-visit/](http://lsc.org/plan-your-visit/)