Pre-Calculus Honors: Unit 3

Analytic Trigonometry and Additional Topics in Trigonometry
Course Philosophy/Description

Pre-Calculus Honors course is for accelerated students. This course is designed for students who exhibit high interest and knowledge in math and science. In this course, students will extend topics introduced in Algebra II and learn to manipulate and apply more advanced functions and algorithms. Students extend their knowledge and understanding by solving open-ended real-world problems and thinking critically through the use of high level tasks and long-term projects. This course provides a mathematically sound foundation for students who intend to study Calculus.

The course covers mathematical topics ranging from Basics of Functions to Limits of Functions. It provides opportunities to the students to expand their knowledge base and understanding of mathematics in general. The overarching goal of the course is to build a solid foundation for the students who choose Mathematics, Engineering, Sciences, or Business as their college major and/or career options. The major topics in the course such as, Polynomials, Exponents, Trigonometry, Logarithms, Complex numbers, Series/sequences, and Limits help generate students’ inquiries about the mathematical nature, complexities, and applications of these topics in real-life situations. Students not only acquire new knowledge, but also deepen their topical and overall understanding of the content for future transfer to new situations or other disciplines.

Assessment results from this course may be used for the purpose of placements into Calculus/AP Calculus, Statistics, Physics/AP Physics, or other higher level courses.
This ESL framework was designed to be used by bilingual, dual language, ESL and general education teachers. Bilingual and dual language programs use the home language and a second language for instruction. ESL teachers and general education or bilingual teachers may use this document to collaborate on unit and lesson planning to decide who will address certain components of the SLO and language objective. ESL teachers may use the appropriate leveled language objective to build lessons for ELLs which reflects what is covered in the general education program. In this way, whether it is a pull-out or push-in model, all teachers are working on the same Student Learning Objective connected to the Common Core standard. The design of language objectives are based on the alignment of the World-Class Instructional Design Assessment (WIDA) Consortium’s English Language Development (ELD) standards with the Common Core State Standards (CCSS). WIDA’s ELD standards advance academic language development across content areas ultimately leading to academic achievement for English learners. As English learners are progressing through the six developmental linguistic stages, this framework will assist all teachers who work with English learners to appropriately identify the language needed to meet the requirements of the content standard. At the same time, the language objectives recognize the cognitive demand required to complete educational tasks. Even though listening and reading (receptive) skills differ from speaking and writing (expressive) skills across proficiency levels the cognitive function should not be diminished. For example, an Entering Level One student only has the linguistic ability to respond in single words in English with significant support from their home language. However, they could complete a Venn diagram with single words which demonstrates that they understand how the elements compare and contrast with each other or they could respond with the support of their home language (L1) with assistance from a teacher, para-professional, peer or a technology program.

http://www.state.nj.us/education/modelcurriculum/ela/ELLOverview.pdf
## Pacing Chart – Unit 3

<table>
<thead>
<tr>
<th>#</th>
<th>Student Learning Objective</th>
<th>NJSLS</th>
<th>Pre-Calculus with Limits Correlation</th>
</tr>
</thead>
</table>
| 1  | • Use sum and difference formula to evaluate trigonometric identities, and solve trigonometric equations.  
     • Use multiple – angle formulas to rewrite and evaluate trigonometric functions.  
     • Use power reducing formulas to rewrite and evaluate trigonometric functions.  
     • Use product to sum and sum to product formulas to rewrite and evaluate trigonometric functions. | F.TF.B.7  
     F.TF.C.9  
     G.SRT.D.9 | 5.3, 5.4, 5.5 |
| 2  | • Use the law of Sine to solve oblique triangles (ASA or ASA and SSA), find their areas while modeling and solving real life problems.  
     • Use the law of Cosine to solve oblique triangles (SSS or SAS), find their areas while modeling and solving real life problems. | G.SRT.D.10  
     G.SRT.D.11 | 6.1, 6.2 |
| 3  | • Represent vectors as directed line segments.  
     • Perform basic vector operations and represent vectors graphically.  
     • Find the direction angles of vectors.  
     • Write vectors as the sum of two vector components.  
     • Write trigonometric forms of complex numbers.  
     • Multiply and divide complex numbers written in trigonometric form.  
     • Use DeMoivre’s Theorem to find powers of complex numbers.  
     • Use vectors to model and solve real-life problems. | N.VM.A.1  
     N.VM.A.2  
     N.VM.A.3  
     N.VM.B.4  
     N.VM.B.5 | 6.3, 6.4, 6.5, 6.6 |
Research about Teaching and Learning Mathematics

Structure teaching of mathematical concepts and skills around problems to be solved (Checkly, 1997; Wood & Sellars, 1996; Wood & Sellars, 1997)
Encourage students to work cooperatively with others (Johnson & Johnson, 1975; Davidson, 1990)
Use group problem-solving to stimulate students to apply their mathematical thinking skills (Artzt & Armour-Thomas, 1992)
Students interact in ways that support and challenge one another’s strategic thinking (Artzt, Armour-Thomas, & Curcio, 2008)
Activities structured in ways allowing students to explore, explain, extend, and evaluate their progress (National Research Council, 1999)
There are three critical components to effective mathematics instruction (Shellard & Moyer, 2002):

- Teaching for conceptual understanding
- Developing children’s procedural literacy
- Promoting strategic competence through meaningful problem-solving investigations

Teachers should be:
- Demonstrating acceptance and recognition of students’ divergent ideas.
- Challenging students to think deeply about the problems they are solving, extending thinking beyond the solutions and algorithms required to solve the problem
- Influencing learning by asking challenging and interesting questions to accelerate students’ innate inquisitiveness and foster them to examine concepts further.
- Projecting a positive attitude about mathematics and about students’ ability to “do” mathematics

Students should be:
- Actively engaging in “doing” mathematics
- Solving challenging problems
- Investigating meaningful real-world problems
- Making interdisciplinary connections
- Developing an understanding of mathematical knowledge required to “do” mathematics and connect the language of mathematical ideas with numerical representations
- Sharing mathematical ideas, discussing mathematics with one another, refining and critiquing each other’s ideas and understandings
- Communicating in pairs, small group, or whole group presentations
- Using multiple representations to communicate mathematical ideas
- Using connections between pictures, oral language, written symbols, manipulative models, and real-world situations
- Using technological resources and other 21st century skills to support and enhance mathematical understanding
Mathematics is not a stagnate field of textbook problems; rather, it is a dynamic way of constructing meaning about the world around us, generating knowledge and understanding about the real world every day. Students should be metaphorically rolling up their sleeves and “doing mathematics” themselves, not watching others do mathematics for them or in front of them. (Protheroe, 2007)

**Balanced Mathematics Instructional Model**

Balanced math consists of three different learning opportunities; guided math, shared math, and independent math. Ensuring a balance of all three approaches will build conceptual understanding, problem solving, computational fluency, and procedural fluency. Building a balanced mathematical understanding is the focal point of developing mathematical proficiency. Students should frequently work on rigorous tasks, talk about the math, explain their thinking, justify their answer or process, build models with graphs or charts or manipulatives, and use technology.

When balanced math is used in the classroom it provides students opportunities to:

- solve problems
- make connections between math concepts and real-life situations
- communicate mathematical ideas (orally, visually and in writing)
- choose appropriate materials to solve problems
- reflect and monitor their own understanding of the math concepts
- practice strategies to build procedural and conceptual confidence

Teacher builds conceptual understanding by modeling through demonstration, explicit instruction, and think alouds, as well as guiding students as they practice math strategies and apply problem solving strategies. (whole group or small group instruction)

Students practice math strategies independently to build procedural and computational fluency. Teacher assesses learning and reteaches as necessary. (whole group instruction, small group instruction, or centers)

Teacher and students practice mathematics processes together through interactive activities, problem solving, and discussion. (whole group or small group instruction)
<table>
<thead>
<tr>
<th>Effective Pedagogical Routines/Instructional Strategies</th>
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</thead>
<tbody>
<tr>
<td>Collaborative Problem Solving</td>
</tr>
<tr>
<td>Connect Previous Knowledge to New Learning</td>
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<tr>
<td>Making Thinking Visible</td>
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<tr>
<td>Develop and Demonstrate Mathematical Practices</td>
</tr>
<tr>
<td>Inquiry-Oriented and Exploratory Approach</td>
</tr>
<tr>
<td>Multiple Solution Paths and Strategies</td>
</tr>
<tr>
<td>Use of Multiple Representations</td>
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<tr>
<td>Explain the Rationale of your Math Work</td>
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<tr>
<td>Quick Writes</td>
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<tr>
<td>Pair/Trio Sharing</td>
</tr>
<tr>
<td>Turn and Talk</td>
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<tr>
<td>Charting</td>
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<tr>
<td>Gallery Walks</td>
</tr>
<tr>
<td>Small Group and Whole Class Discussions</td>
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<tr>
<td>Student Modeling</td>
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<tr>
<td>Analyze Student Work</td>
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<tr>
<td>Identify Student’s Mathematical Understanding</td>
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<tr>
<td>Identify Student’s Mathematical Misunderstandings</td>
</tr>
<tr>
<td>Interviews</td>
</tr>
<tr>
<td>Role Playing</td>
</tr>
<tr>
<td>Diagrams, Charts, Tables, and Graphs</td>
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<tr>
<td>Anticipate Likely and Possible Student Responses</td>
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<tr>
<td>Collect Different Student Approaches</td>
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<tr>
<td>Multiple Response Strategies</td>
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<tr>
<td>Asking Assessing and Advancing Questions</td>
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<tr>
<td>Revoicing</td>
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<tr>
<td>Marking</td>
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<tr>
<td>Recapping</td>
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<tr>
<td>Challenging</td>
</tr>
<tr>
<td>Pressing for Accuracy and Reasoning</td>
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<tr>
<td>Maintain the Cognitive Demand</td>
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</tbody>
</table>
## Educational Technology

### Standards

|-----------------------------------------------|

#### Technology Operations and Concepts
- Construct a spreadsheet workbook with multiple worksheets, rename tabs to reflect the data on the worksheet, and use mathematical or logical functions, charts and data from all worksheets to convey the results.

**Example:** Using digital tools such as math software, TI-83/84/89, students are able to easily draw any trigonometric graphs and explore them by tracing or using automatic procedures to explore key points, noting periodicity, amplitude, and so on.

#### Digital Citizenship
- Analyze the capabilities and limitations of current and emerging technology resources and assess their potential to address personal, social, lifelong learning and career needs.

**Example:** Students will be able to use critical thinking skills to plan and conduct, research, manage projects, solve problems, and make informed decisions using appropriate digital tools and resource.

#### Research and Information Literacy
- Produce a position statement about a real world problem by developing a systematic plan of investigation with peers and experts synthesizing information from multiple sources.

**Example:** Students will be able to produce a position statement about the real world problem by developing systematic plan of investigation with peers and experts synthesizing information from multiple source.

#### Critical Thinking, Problem Solving, Decision Making
- Evaluate the strengths and limitations of emerging technologies and their impact on educational, career, personal and or social needs.

**Example:** A problem-solving approach will allow students to construct their own ideas about mathematics and to take responsibility for their own learning.
# Career Ready Practices

Career Ready Practices describe the career-ready skills that all educators in all content areas should seek to develop in their students. They are practices that have been linked to increase college, career, and life success. Career Ready Practices should be taught and reinforced in all career exploration and preparation programs with increasingly higher levels of complexity and expectation as a student advances through a program of study.

- **CRP2. Apply appropriate academic and technical skills.**
  Career-ready individuals readily access and use the knowledge and skills acquired through experience and education to be more productive. They make connections between abstract concepts with real-world applications, and they make correct insights about when it is appropriate to apply the use of an academic skill in a workplace situation.

  **Example:** Students will use technology to acquire, manipulate, analyze and report data, display and communicate STEM information and apply science and mathematical concepts to the development of plans, processes and projects that address real world problems.

- **CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.**
  Career-ready individuals readily recognize problems in the workplace, understand the nature of the problem, and devise effective plans to solve the problem. They are aware of problems when they occur and take action quickly to address the problem; they thoughtfully investigate the root cause of the problem prior to introducing solutions. They carefully consider the options to solve the problem. Once a solution is agreed upon, they follow through to ensure the problem is solved, whether through their own actions or the actions of others.

  **Example:** Throughout this unit, students work to improve their critical-thinking skills and problem-solving techniques.

- **CRP11. Use technology to enhance productivity.**
  Career-ready individuals find and maximize the productive value of existing and new technology to accomplish workplace tasks and solve workplace problems. They are flexible and adaptive in acquiring new technology. They are proficient with ubiquitous technology applications. They understand the inherent risks-personal and organizational-of technology applications, and they take actions to prevent or mitigate these risks.

  **Example** Students will use digital tools such as TI-83/84/89, Interactive Whiteboard, etc. to access, manage, evaluate, and synthesize information in order to solve problems individually and collaboratively to create and communicate knowledge.
## WIDA Proficiency Levels

At the given level of English language proficiency, English language learners will process, understand, produce or use

<table>
<thead>
<tr>
<th>Level</th>
<th>Description</th>
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</table>
| 6- Reaching | - Specialized or technical language reflective of the content areas at grade level  
- A variety of sentence lengths of varying linguistic complexity in extended oral or written discourse as required by the specified grade level  
- Oral or written communication in English comparable to proficient English peers |
| 5- Bridging | - Specialized or technical language of the content areas  
- A variety of sentence lengths of varying linguistic complexity in extended oral or written discourse, including stories, essays or reports  
- Oral or written language approaching comparability to that of proficient English peers when presented with grade level material. |
| 4- Expanding | - Specific and some technical language of the content areas  
- A variety of sentence lengths of varying linguistic complexity in oral discourse or multiple, related sentences or paragraphs  
- Oral or written language with minimal phonological, syntactic or semantic errors that may impede the communication, but retain much of its meaning, when presented with oral or written connected discourse, with sensory, graphic or interactive support |
| 3- Developing | - General and some specific language of the content areas  
- Expanded sentences in oral interaction or written paragraphs  
- Oral or written language with phonological, syntactic or semantic errors that may impede the communication, but retain much of its meaning, when presented with oral or written, narrative or expository descriptions with sensory, graphic or interactive support |
| 2- Beginning | - General language related to the content area  
- Phrases or short sentences  
- Oral or written language with phonological, syntactic, or semantic errors that often impede the communication when presented with one to multiple-step commands, directions, or a series of statements with sensory, graphic or interactive support |
| 1- Entering | - Pictorial or graphic representation of the language of the content areas  
- Words, phrases or chunks of language when presented with one-step commands directions, WH-, choice or yes/no questions, or statements with sensory, graphic or interactive support |
# Language Development Supports for English Language Learners

To Increase Comprehension and Communication Skills

<table>
<thead>
<tr>
<th>Environment</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Welcoming and stress-free</td>
<td>Integrates learning centers and games in a meaningful way</td>
</tr>
<tr>
<td>Respectful of linguistic and cultural diversity</td>
<td>Provides opportunities to practice and refine receptive and productive skills in English as a new language</td>
</tr>
<tr>
<td>Honors students’ background knowledge</td>
<td>Integrates meaning and purposeful tasks/activities that:</td>
</tr>
<tr>
<td>Sets clear and high expectations</td>
<td>○ Are accessible by all students through multiple entry points</td>
</tr>
<tr>
<td>Includes routines and norms</td>
<td>○ Are relevant to students’ lives and cultural experiences</td>
</tr>
<tr>
<td>Is thinking-focused vs. answer-seeking</td>
<td>○ Build on prior mathematical learning</td>
</tr>
<tr>
<td>Offers multiple modalities to engage in content learning and to demonstrate understanding</td>
<td>○ Demonstrate high cognitive demand</td>
</tr>
<tr>
<td>Includes explicit instruction of specific language targets</td>
<td>○ Offer multiple strategies for solutions</td>
</tr>
<tr>
<td>Provides participation techniques to include all learners</td>
<td>○ Allow for a language learning experience in addition to content</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Sensory Supports*</th>
<th>Graphic Supports*</th>
<th>Interactive Supports*</th>
<th>Verbal and Textual Supports</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real-life objects (realia) or concrete objects</td>
<td>Graphs</td>
<td>In a whole group</td>
<td>Labeling</td>
</tr>
<tr>
<td>Physical models</td>
<td>Charts</td>
<td>In a small group</td>
<td>Students’ native language</td>
</tr>
<tr>
<td>Manipulatives</td>
<td>Timelines</td>
<td>With a partner such as Turn-and-Talk</td>
<td>Modeling</td>
</tr>
<tr>
<td>Pictures &amp; photographs</td>
<td>Number lines</td>
<td>In pairs as a group (first, two pairs work independently, then they form a group of four)</td>
<td>Repetitions</td>
</tr>
<tr>
<td>Visual representations or models such as diagrams or drawings</td>
<td>Graphic organizers</td>
<td>In triads</td>
<td>Paraphrasing</td>
</tr>
<tr>
<td>Videos &amp; films</td>
<td>Graphing paper</td>
<td>Cooperative learning structures such as Think-Pair-Share</td>
<td>Summarizing</td>
</tr>
<tr>
<td>Newspapers or magazines</td>
<td></td>
<td>Interactive websites or software</td>
<td>Guiding questions</td>
</tr>
<tr>
<td>Gestures</td>
<td></td>
<td>With a mentor or coach</td>
<td>Clarifying questions</td>
</tr>
<tr>
<td>Physical movements</td>
<td></td>
<td></td>
<td>Probing questions</td>
</tr>
<tr>
<td>Music &amp; songs</td>
<td></td>
<td></td>
<td>Leveled questions such as What? When? Where? How? Why?</td>
</tr>
</tbody>
</table>

BUILDING EQUITY IN YOUR TEACHING PRACTICE

How do the essential questions highlight the connection between the big ideas of the unit and equity in your teaching practice?

CONTENT INTEGRATION
Teachers use examples and content from a variety of cultures & groups.

This unit/lesson is connected to other topics explored with students.

There are multiple viewpoints reflected in the content of this unit/lesson.

The materials and resources are reflective of the diverse identities and experiences of students.

The content affirms students, as well as exposes them to experiences other than their own.

KNOWLEDGE CONSTRUCTION
Teachers help students understand how knowledge is created and influenced by cultural assumptions, perspectives & biases.

This unit/lesson provides context to the history of privilege and oppression.

This unit/lesson addresses power relationships.

This unit/lesson helps students develop research and critical thinking skills.

This curriculum creates windows and mirrors* for students.

PREJUDICE REDUCTION
Teachers implement lessons and activities to assert positive images of ethnic groups & improve intergroup relations.

This unit/lesson helps students question and unpack biases & stereotypes.

This unit/lesson helps students examine, research and question information and sources.

The curriculum encourages discussion and understanding about the groups of people being represented.

This unit/lesson challenges dominant perspectives.

EQUITABLE PEDAGOGY
Teachers modify techniques and methods to facilitate the academic achievement of students from diverse backgrounds.

The instruction has been modified to meet the needs of each student.

Students feel respected and their cultural identities are valued.

Additional supports have been provided for students to become successful and independent learners.

Opportunities are provided for student to reflect on their learning and provide feedback.

EMPOWERING SCHOOL CULTURE
Using the other four dimensions to create a safe and healthy educational environment for all.

There are opportunities for students to connect with the community.

My classroom is welcoming and supportive for all students?

I am aware of and sensitive to the needs of my students and their families.

There are effective parent communication systems established. Parents can talk to me about issues as they arise in my classroom.

<table>
<thead>
<tr>
<th>Culturally Relevant Pedagogy Examples</th>
</tr>
</thead>
</table>
| • Integrate Relevant Word Problems: Contextualize equations using word problems that reference student interests and cultures.  
  Example: When learning to solve trigonometric equations, problems that relate to student interests such as music, sports and art enable the students to understand and relate to the concept in a more meaningful way. |
| • Everyone has a Voice: Create a classroom environment where students know that their contributions are expected and valued.  
  Example: Norms for sharing are established that communicate a growth mindset for mathematics. All students are capable of expressing mathematical thinking and contributing to the classroom community. Students learn new ways of looking at problem solving by working with and listening to each other. |
| • Run Problem Based Learning Scenarios: Encourage mathematical discourse among students by presenting problems that are relevant to them, the school and/or the community.  
  Example: Using a Place Based Education (PBE) model, students explore math concepts such as systems of equations while determining ways to address problems that are pertinent to their neighborhood, school or culture. |
| • Encourage Student Leadership: Create an avenue for students to propose problem solving strategies and potential projects.  
  Example: Students can learn to perform operations on vectors in the plane by creating problems together and deciding if the problems fit the necessary criteria. This experience will allow students to discuss and explore their current level of understanding by applying the concepts to relevant real-life experiences. |
| • Present New Concepts Using Student Vocabulary: Use student diction to capture attention and build understanding before using academic terms.  
  Example: Teach math vocabulary in various modalities for students to remember. Use multi-modal activities, analogies, realia, visual cues, graphic representations, gestures, pictures and cognates. Directly explain and model the idea of vocabulary words having multiple meanings. Students can create the Word Wall with their definitions and examples to foster ownership. |
## Differentiated Instruction

**Accommodate Based on Students Individual Needs: Strategies**

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<tr>
<th>Time/General</th>
<th>Processing</th>
<th>Comprehension</th>
<th>Recall</th>
</tr>
</thead>
<tbody>
<tr>
<td>Extra time for assigned tasks</td>
<td>Extra Response time</td>
<td>Precise processes for balanced math instruction model</td>
<td>Teacher-made checklist</td>
</tr>
<tr>
<td>Adjust length of assignment</td>
<td>Have students verbalize steps</td>
<td>Short manageable tasks</td>
<td>Use visual graphic organizers</td>
</tr>
<tr>
<td>Timeline with due dates for</td>
<td>Repeat, clarify or reword</td>
<td>Brief and concrete directions</td>
<td>Reference resources to</td>
</tr>
<tr>
<td>reports and projects</td>
<td>directions</td>
<td></td>
<td>promote independence</td>
</tr>
<tr>
<td>Communication system</td>
<td>Mini-breaks between tasks</td>
<td></td>
<td>Visual and verbal reminders</td>
</tr>
<tr>
<td>between home and school</td>
<td>Provide a warning for</td>
<td></td>
<td>Graphic organizers</td>
</tr>
<tr>
<td>Provide lecture notes/outline</td>
<td>transitions</td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>Reading partners</td>
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</tbody>
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<table>
<thead>
<tr>
<th>Assistive Technology</th>
<th>Tests/Quizzes/Grading</th>
<th>Behavior/Attention</th>
<th>Organization</th>
</tr>
</thead>
<tbody>
<tr>
<td>Computer/whiteboard</td>
<td>Extended time</td>
<td>Consistent daily structured</td>
<td>Individual daily planner</td>
</tr>
<tr>
<td>Tape recorder</td>
<td>Study guides</td>
<td>routine</td>
<td>Display a written agenda</td>
</tr>
<tr>
<td>Video Tape</td>
<td>Shortened tests</td>
<td>Simple and clear classroom rules</td>
<td>Note-taking assistance</td>
</tr>
<tr>
<td></td>
<td>Read directions aloud</td>
<td>Frequent feedback</td>
<td>Color code materials</td>
</tr>
</tbody>
</table>

<p>| Recollection                  |                                |                                                       |                               |
| Teacher-made checklist        | Use visual graphic organizers  |                                                       |                               |
|                                | Reference resources to         |                                                       |                               |
|                                | promote independence           |                                                       |                               |
|                                | Visual and verbal reminders    |                                                       |                               |
|                                | Graphic organizers             |                                                       |                               |</p>
<table>
<thead>
<tr>
<th>Differentiated Instruction</th>
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</thead>
<tbody>
<tr>
<td>Accommodate Based on Content Needs: Strategies</td>
</tr>
</tbody>
</table>

- Anchor charts to model strategies
- Review Algebra concepts to ensure students have the information needed to progress in understanding
- Pre-teach pertinent vocabulary
- Provide reference sheets that list formulas, step-by-step procedures, theorems, and modeling of strategies
- Word wall with visual representations of mathematical terms
- Teacher modeling of thinking processes involved in solving, graphing, and writing equations
- Introduce concepts embedded in real-life context to help students relate to the mathematics involved
- Record formulas, processes, and mathematical rules in reference notebooks
- Graphing calculator to assist with computations and graphing of trigonometric functions
- Utilize technology through interactive sites to represent nonlinear data
- Graphic organizers to help students interpret the meaning of terms in an expression or equation in context
- Translation dictionary
- Sentence stems to provide additional language support for ELL students.
Interdisciplinary Connections

Model interdisciplinary thinking to expose students to other disciplines.

Social Studies Connection:
Name of Task: INCA Dwelling NJSLS: 9.1.12.A.9
Shown below is a drawing of an Inca dwelling found in Machu Picchu, about 50 miles northwest of Cuzco, Peru. All that remains of the ancient city today are stone ruins.

1- Express the area of the triangular portion of the side of the dwelling as a function of \( \sin\left(\frac{\theta}{2}\right) \) and \( \cos\left(\frac{\theta}{2}\right) \)

2- Express the area as a function of \( \sin \theta \). Then solve for \( \theta \) assuming that the area is 132 square feet. 73

Science Connection:
Name of Task: Moving Boxes NJSLS: 5.2.12.E.1
In a warehouse a box is being pushed up a 15° inclined plane with a force of 2.5lb parallel to the inclined plane.

1- Find the component of the force.
2- Give an interpretation of the horizontal and vertical components of the force.
3- If the box is towed up the inclined plane with a force making a 33° with a horizontal line, Find the force \( w \) needed in order for the component of the force parallel to the incline plane to be 2.5 lb. Give the answer in component form

* Tasks can be found within the additional task folders
Enrichment

What is the purpose of Enrichment?

- The purpose of enrichment is to provide extended learning opportunities and challenges to students who have already mastered, or can quickly master, the basic curriculum. Enrichment gives the student more time to study concepts with greater depth, breadth, and complexity.
- Enrichment also provides opportunities for students to pursue learning in their own areas of interest and strengths.
- Enrichment keeps advanced students engaged and supports their accelerated academic needs.
- Enrichment provides the most appropriate answer to the question, “What do you do when the student already knows it?”

Enrichment is…

- Planned and purposeful
- *Different*, or differentiated, work – not just *more* work
- Responsive to students’ needs and situations
- A promotion of high-level thinking skills and making connections within content
- The ability to apply different or multiple strategies to the content
- The ability to synthesize concepts and make real world and cross-curricular connections.
- Elevated contextual complexity
- Sometimes independent activities, sometimes direct instruction
- Inquiry based or open ended assignments and projects
- Using supplementary materials in addition to the normal range of resources.
- Choices for students
- Tiered/Multi-level activities with flexible groups (may change daily or weekly)

Enrichment is not…

- Just for gifted students (some gifted students may need intervention in some areas just as some other students may need frequent enrichment)
- Worksheets that are more of the same (busywork)
- Random assignments, games, or puzzles not connected to the content areas or areas of student interest
- Extra homework
- A package that is the same for everyone
- Thinking skills taught in isolation
- Unstructured free time
Assessments

**Required District/State Assessments**
SGO Assessments

**Suggested Formative/Summative Classroom Assessments**
- Describe Learning Vertically
- Identify Key Building Blocks
- Make Connections (between and among key building blocks)
- Short/Extended Constructed Response Items
- Multiple-Choice Items (where multiple answer choices may be correct)
- Drag and Drop Items
- Use of Equation Editor
- Quizzes
- Journal Entries/Reflections/Quick-Writes
- Accountable talk
- Projects
- Portfolio
- Observation
- Graphic Organizers/Concept Mapping
- Presentations
- Role Playing
- Teacher-Student and Student-Student Conferencing
- Homework
### New Jersey Student Learning Standards

**F.TF.B.7:** Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.

**F.TF.C.9:** Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

**G.SRT.D.9:** Derive the formulas \( A = \frac{1}{2} ab \sin(C) \) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.

**G.SRT.D.10:** Prove the Laws of Sines and Cosines and use them to solve problems.

**G.SRT.D.11:** Understand and apply the law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g. surveying problems, resultant forces).

**N.VM.A.1:** Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitude (e.g., \( \mathbf{v}, |\mathbf{v}|, \|\mathbf{v}\|, \mathbf{v} \)).

**N.VM.A.2:** Find component of a vector by subtracting the coordinates of and initial point from the coordinates of a terminal point.

**N.VM.A.3:** Solve problems involving velocity and other quantities that can be represented by vectors.

**N.VM.B.4:** Add and Subtract vectors.

**N.VM.B.5:** Multiply a vector by a scalar.
Mathematical Practices

1. Make sense of problems and persevere in solving them.

2. Reason abstractly and quantitatively.

3. Construct viable arguments and critique the reasoning of others.

4. Model with mathematics.

5. Use appropriate tools strategically.

6. Attend to precision.

7. Look for and make use of structure.

8. Look for and express regularity in repeated reasoning.
<table>
<thead>
<tr>
<th>Course: Pre-Calculus Honors</th>
<th>Unit: 3</th>
<th>Topic: Analytic Trigonometry and Additional Topics in Trigonometry</th>
</tr>
</thead>
</table>

**NJSLS:**

**Unit Focus:**
- Solve all types of trigonometric equations including: equations of quadratic type, ones involving multiple angles, others using sum and difference, multiple angles, power reducing, half angle, product to sum and sum to product formulas.
- Find side lengths, angles, and areas of oblique triangles by using Law of Sines and the Law of Cosines.
- Perform operations (addition, subtraction, and multiplication by scalars) on vectors in the plane and solve applied problems using vectors.
- Represent vectors graphically and algebraically.

**New Jersey Student Learning Standard(s):**
F.TF.B.7: Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.

F.TF.C.9: Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

G.SRT.D.9: Derive the formulas $A = \frac{1}{2} ab \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.

**Student Learning Objective 1:** Use sum and difference formula to evaluate trigonometric identities, and solve trigonometric equations. Use multiple angle formulas to rewrite and evaluate trigonometric functions. Use power reducing formulas to rewrite and evaluate trigonometric functions. Use product to sum and sum to product formulas to rewrite and evaluate trigonometric functions.

<table>
<thead>
<tr>
<th>MPs</th>
<th>Skills, Strategies &amp; Concepts</th>
<th>Essential Understandings/Questions</th>
<th>Tasks/Activities</th>
</tr>
</thead>
<tbody>
<tr>
<td>MP 2</td>
<td>Use standard algebraic techniques to solve trigonometric equations.</td>
<td>Trigonometry can be used to indirectly measure oblique as well as right triangles.</td>
<td>INCA DWELLING</td>
</tr>
<tr>
<td>MP 3</td>
<td>Solve trigonometric equations of quadratic type.</td>
<td>How do you use fundamental trigonometric identities, sum and</td>
<td>Blue Prints</td>
</tr>
</tbody>
</table>

INCA DWELLING
Blue Prints
### New Jersey Student Learning Standard(s):  
**G.SRT.D.10**: Prove the Laws of Sines and Cosines and use them to solve problems.

**G.SRT.D.11**: Understand and apply the law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g. surveying problems, resultant forces).

**Student Learning Objective 2**: Use the law of Sine to solve oblique triangles (ASA or ASA and SSA), find their areas while modeling and solving real life problems. Use the law of Cosine to solve oblique triangles (SSS or SAS), find their areas while modeling and solving real life problems.

<table>
<thead>
<tr>
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</tr>
</thead>
</table>
| MP 6 | • Solve trigonometric equations involving multiple angles.  
• Use inverse trigonometric functions to solve trigonometric equations.  
• Use sum and difference formulas to evaluate trigonometric functions, verify trigonometric identities, and solve trigonometric equations. | difference formulas and Multiple-angle and Product-to-Sum Formulas in order to simplify and evaluate trigonometric functions and to verify trigonometric identities? |  |
|     |                               |                                   |                 |

- **MP 2**:  
  - Use laws of sine and cosine to solve oblique triangles and find their area.  
  - Find the area of oblique triangles using the sine function.  
  - Use the law of cosines to solve oblique triangles.  
  - Use Heron’s area formula to find area of triangles.

- **MP 7**:  
  - How do you use trigonometry and the Law of Sine and Law of Cosine to solve and find the areas of oblique triangles?

- **Immaculate Transport**  
  - Let Me Watch The Games  
  - The Law of Sines  
  - The Law of Cosines
New Jersey Student Learning Standard(s):
N.VM.A.1: Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitude (e.g., \( \mathbf{v} \), \(|\mathbf{v}|\), \(\|\mathbf{v}\|\), \(\mathbf{v}\)).

N.VM.A.2: Find component of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.

N.VM.A.3: Solve problems involving velocity and other quantities that can be represented by vectors.

N.VM.B.4: Add and subtract vectors.

N.VM.B.5: Multiply a vector by a scalar.

Student Learning Objective 3: Represent vectors as directed line segments. Perform basic vector operations and represent vectors graphically. Find the direction angles of vectors. Write vectors as the sum of two vector components. Write trigonometric forms of complex numbers. Multiply and divide complex numbers written in trigonometric form. Use DeMoivre’s Theorem to find powers of complex numbers and use vectors to model and solve real-life problems.

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</thead>
<tbody>
<tr>
<td>MP 1</td>
<td>• Write the component forms of vectors.</td>
<td>• How do you represent and perform operations with vector quantities?</td>
<td>Moving Boxes</td>
</tr>
<tr>
<td>MP 2</td>
<td>• Find the direction angles of vectors.</td>
<td>• How do you write a vector as a sum of two vector components?</td>
<td>Memorial Day Boating Trip</td>
</tr>
<tr>
<td>MP 7</td>
<td>• Find the resultant of two vectors.</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Find the dot product of two vectors and find angle between two vectors.</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Use vectors to model and solve real life solutions,</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
## Honor Project (must complete both)

<table>
<thead>
<tr>
<th>Project 1</th>
<th>Project 2</th>
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<tbody>
<tr>
<td><strong>Trigonome “tree”</strong></td>
<td><strong>Vector Project</strong></td>
</tr>
<tr>
<td><strong>Essential Question:</strong></td>
<td><strong>Essential Question:</strong></td>
</tr>
<tr>
<td>• How do you use trigonometry to find unknown side lengths and angle measures in right triangles?</td>
<td>• How can the effect of multiple vectors be determined?</td>
</tr>
<tr>
<td><strong>Skills:</strong></td>
<td><strong>Skills:</strong></td>
</tr>
<tr>
<td>• Solve equations involving sine and cosine using algebraic and graphical techniques.</td>
<td>• Write the component forms of vectors.</td>
</tr>
<tr>
<td>• Prove trigonometric identities using Pythagorean, reciprocal, and quotient identities.</td>
<td>• Find the direction angles of vectors.</td>
</tr>
<tr>
<td></td>
<td>• Find the resultant of two vectors.</td>
</tr>
</tbody>
</table>
# Unit 3 Vocabulary

- Co-function Identities
- conjugates
- Even/Odd Identities
- general solution
- Pythagorean Identities
- Quotient Identities
- Reciprocal Identities
- absolute value of a complex number
- angle between two nonzero vectors
- complex plane
- component form of a vector
- Demoiver’s Theorem
- directed line segment
- direction angle
- dot product
- force
- horizontal and vertical components
- imaginary axis
- initial point
- Law of Cosines
- Law of Sines
- linear combination
- magnitude
- modulus
- vector addition
- vector components
- nth root of a complex number
- oblique triangle
- orthogonal vectors
- parallelogram law
- projection
- real axis
- resultant
- scalar multiplication
- standard position
- standard unit vectors
- terminal point
- trigonometric form of a complex number
- unit vector
- work
- zero vector
<table>
<thead>
<tr>
<th>References &amp; Suggested Instructional Websites</th>
</tr>
</thead>
<tbody>
<tr>
<td>• <a href="http://www.mathwords.com/index_adv_alg_precal.htm">http://www.mathwords.com/index_adv_alg_precal.htm</a></td>
</tr>
<tr>
<td>• <a href="http://www.hershey.k12.pa.us/Page/3608">http://www.hershey.k12.pa.us/Page/3608</a></td>
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<td>• <a href="https://sites.google.com/site/dgrahamcalculus/trigpre-calculus/trig-pre-calculus-worksheets">https://sites.google.com/site/dgrahamcalculus/trigpre-calculus/trig-pre-calculus-worksheets</a></td>
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<td>• <a href="https://www.illustrativemathematics.org/">https://www.illustrativemathematics.org/</a></td>
</tr>
<tr>
<td>• The Data and Story Library (DASL). <a href="http://lib.stat.cmu.edu/DASL/">http://lib.stat.cmu.edu/DASL/</a></td>
</tr>
</tbody>
</table>
Field Trip Ideas

SIX FLAGS GREAT ADVENTURE: This educational event includes workbooks and special science and math related shows throughout the day. Your students will leave with a better understanding of real world applications of the material they have learned in the classroom. Each student will have the opportunity to experience different rides and attractions linking mathematical and scientific concepts to what they are experiencing.

www.sixflags.com

MUSEUM of MATHEMATICS: Mathematics illuminates the patterns that abound in our world. The National Museum of Mathematics strives to enhance public understanding and perception of mathematics. Its dynamic exhibits and programs stimulate inquiry, spark curiosity, and reveal the wonders of mathematics. The Museum’s activities lead a broad and diverse audience to understand the evolving, creative, human, and aesthetic nature of mathematics.

www.momath.org

LIBERTY SCIENCE CENTER: An interactive science museum and learning center located in Liberty State Park. The center, which first opened in 1993 as New Jersey's first major state science museum, has science exhibits, the largest IMAX Dome theater in the United States, numerous educational resources, and the original Hoberman sphere.

http://lsc.org/plan-your-visit/