Pre-Calculus Honors: Unit 2
Trigonometric Functions
Course Philosophy/Description

Pre-Calculus Honors course is for accelerated students. This course is designed for students who exhibit high interest and knowledge in math and science. In this course, students will extend topics introduced in Algebra II and learn to manipulate and apply more advanced functions and algorithms. Students extend their knowledge and understanding by solving open-ended real-world problems and thinking critically through the use of high level tasks and long-term projects. This course provides a mathematically sound foundation for students who intend to study Calculus.

The course covers mathematical topics ranging from Basics of Functions to Limits of Functions. It provides opportunities to the students to expand their knowledge base and understanding of mathematics in general. The overarching goal of the course is to build a solid foundation for the students who choose Mathematics, Engineering, Sciences, or Business as their college major and/or career options. The major topics in the course such as, Polynomials, Exponents, Trigonometry, Logarithms, Complex numbers, Series/sequences, and Limits help generate students’ inquiries about the mathematical nature, complexities, and applications of these topics in real-life situations. Students not only acquire new knowledge, but also deepen their topical and overall understanding of the content for future transfer to new situations or other disciplines.

Assessment results from this course may be used for the purpose of placements into Calculus/AP Calculus, Statistics, Physics/AP Physics, or other higher level courses.
**ESL Framework**

This ESL framework was designed to be used by bilingual, dual language, ESL and general education teachers. Bilingual and dual language programs use the home language and a second language for instruction. ESL teachers and general education or bilingual teachers may use this document to collaborate on unit and lesson planning to decide who will address certain components of the SLO and language objective. ESL teachers may use the appropriate leveled language objective to build lessons for ELLs which reflects what is covered in the general education program. In this way, whether it is a pull-out or push-in model, all teachers are working on the same Student Learning Objective connected to the Common Core standard. The design of language objectives are based on the alignment of the World-Class Instructional Design Assessment (WIDA) Consortium’s English Language Development (ELD) standards with the Common Core State Standards (CCSS). WIDA’s ELD standards advance academic language development across content areas ultimately leading to academic achievement for English learners. As English learners are progressing through the six developmental linguistic stages, this framework will assist all teachers who work with English learners to appropriately identify the language needed to meet the requirements of the content standard. At the same time, the language objectives recognize the cognitive demand required to complete educational tasks.

Even though listening and reading (receptive) skills differ from speaking and writing (expressive) skills across proficiency levels the cognitive function should not be diminished. For example, an Entering Level One student only has the linguistic ability to respond in single words in English with significant support from their home language. However, they could complete a Venn diagram with single words which demonstrates that they understand how the elements compare and contrast with each other or they could respond with the support of their home language (L1) with assistance from a teacher, para-professional, peer or a technology program.

[http://www.state.nj.us/education/modelcurriculum/ela/ELLOverview.pdf](http://www.state.nj.us/education/modelcurriculum/ela/ELLOverview.pdf)
<table>
<thead>
<tr>
<th>#</th>
<th>Student Learning Objective</th>
<th>NJSLS</th>
<th>Pre-Calculus with Limits Correlation</th>
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</table>
| 1 | • Recognize, evaluate and graph logarithmic and exponential functions with base \( a \) and \( e \).  
• Graph exponential functions.  
• Use exponential and logarithmic functions to model and solve real life problems. | F-LE.A.1, A.1c, A4 F-IF.C.7e | 3.1, 3.2 |
| 2 | • Use properties of logarithms to evaluate or rewrite logarithmic expressions.  
• Use properties of logarithms to expand or condense logarithmic expressions.  
• Solve exponential and logarithmic equations.  
• Use logarithmic functions to model and solve real-life problems. | F-IF.C.8b F-BF.B.5 | 3.3, 3.4, 3.5 |
| 3 | • Describe angles; convert between radian and degrees measures.  
• Identify a unit circle and use the unit circle to evaluate trigonometric functions. | F.TF.A.1 F.TF.A.2 | 4.1, 4.2, 4.3 |
| 4 | • Evaluate trigonometric functions using domain and range, reference angles, and using technology.  
• Sketch the graph of sine and cosine functions using amplitude, period and translations.  
• Sketch the graph of secant and cosecant functions using the graph of cosine and sine as aides. | F.TF.A.3 F.TF.B.5 | 4.4, 4.5, 4.6 |
| 5 | • Use the fundamental trigonometric identities.  
• Prove trigonometric identities using Pythagorean, reciprocal, and quotient identities. | F.TF.C.8 | 5.1, 5.2 |
| 6 | • Evaluate inverse and composite trigonometric functions and graph them.  
• Calculate both exact and approximate values for inverse sine and cosine. | F.TF.B.7 | 4.7 |
| 7 | Use trigonometric functions to model and solve real life problems. | F.TF.B.5 | 4.8 |
Research about Teaching and Learning Mathematics

Structure teaching of mathematical concepts and skills around problems to be solved (Checkly, 1997; Wood & Sellars, 1996; Wood & Sellars, 1997)
Encourage students to work cooperatively with others (Johnson & Johnson, 1975; Davidson, 1990)
Use group problem-solving to stimulate students to apply their mathematical thinking skills (Artzt & Armour-Thomas, 1992)
Students interact in ways that support and challenge one another’s strategic thinking (Artzt, Armour-Thomas, & Curcio, 2008)
Activities structured in ways allowing students to explore, explain, extend, and evaluate their progress (National Research Council, 1999)

There are three critical components to effective mathematics instruction (Shellard & Moyer, 2002):

- Teaching for conceptual understanding
- Developing children’s procedural literacy
- Promoting strategic competence through meaningful problem-solving investigations

Teachers should be:

- Demonstrating acceptance and recognition of students’ divergent ideas.
- Challenging students to think deeply about the problems they are solving, extending thinking beyond the solutions and algorithms required to solve the problem
- Influencing learning by asking challenging and interesting questions to accelerate students’ innate inquisitiveness and foster them to examine concepts further.
- Projecting a positive attitude about mathematics and about students’ ability to “do” mathematics

Students should be:

- Actively engaging in “doing” mathematics
- Solving challenging problems
- Investigating meaningful real-world problems
- Making interdisciplinary connections
- Developing an understanding of mathematical knowledge required to “do” mathematics and connect the language of mathematical ideas with numerical representations
- Sharing mathematical ideas, discussing mathematics with one another, refining and critiquing each other’s ideas and understandings
- Communicating in pairs, small group, or whole group presentations
- Using multiple representations to communicate mathematical ideas
- Using connections between pictures, oral language, written symbols, manipulative models, and real-world situations
- Using technological resources and other 21st century skills to support and enhance mathematical understanding
Mathematics is not a stagnant field of textbook problems; rather, it is a dynamic way of constructing meaning about the world around us, generating knowledge and understanding about the real world every day. Students should be metaphorically rolling up their sleeves and “doing mathematics” themselves, not watching others do mathematics for them or in front of them. (Protheroe, 2007)

Balanced Mathematics Instructional Model

Balanced math consists of three different learning opportunities; guided math, shared math, and independent math. Ensuring a balance of all three approaches will build conceptual understanding, problem solving, computational fluency, and procedural fluency. Building a balanced mathematical understanding is the focal point of developing mathematical proficiency. Students should frequently work on rigorous tasks, talk about the math, explain their thinking, justify their answer or process, build models with graphs or charts or manipulatives, and use technology.

When balanced math is used in the classroom it provides students opportunities to:

- solve problems
- make connections between math concepts and real-life situations
- communicate mathematical ideas (orally, visually and in writing)
- choose appropriate materials to solve problems
- reflect and monitor their own understanding of the math concepts
- practice strategies to build procedural and conceptual confidence

Teacher builds conceptual understanding by modeling through demonstration, explicit instruction, and think alouds, as well as guiding students as they practice math strategies and apply problem solving strategies. (whole group or small group instruction)

Teacher and students practice mathematics processes together through interactive activities, problem solving, and discussion. (whole group or small group instruction)

Students practice math strategies independently to build procedural and computational fluency. Teacher assesses learning and reteaches as necessary. (whole group instruction, small group instruction, or centers)
### Effective Pedagogical Routines/Instructional Strategies

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<th>Collaborative Problem Solving</th>
<th>Analyze Student Work</th>
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<tr>
<td>Connect Previous Knowledge to New Learning</td>
<td>Identify Student’s Mathematical Understanding</td>
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<td>Making Thinking Visible</td>
<td>Identify Student’s Mathematical Misunderstandings</td>
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<tr>
<td>DEVELOP AND DEMONSTRATE MATHEMATICAL PRACTICES</td>
<td>Interviews</td>
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<td>INQUIRY-ORIENTED AND EXPLORATORY APPROACH</td>
<td>Role Playing</td>
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<tr>
<td>Multiple Solution Paths and Strategies</td>
<td>DIAGRAMS, CHARTS, TABLES, AND GRAPHS</td>
</tr>
<tr>
<td>USE OF MULTIPLE REPRESENTATIONS</td>
<td>ANTICIPATE LIKELY AND POSSIBLE STUDENT RESPONSES</td>
</tr>
<tr>
<td>EXPLAIN THE RATIONALE OF YOUR MATH WORK</td>
<td>COLLECT DIFFERENT STUDENT APPROACHES</td>
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<tr>
<td>QUICK WRITES</td>
<td>MULTIPLE RESPONSE STRATEGIES</td>
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<tr>
<td>PAIR/Trio SHARING</td>
<td>ASKING ASSESSING AND ADVANCING QUESTIONS</td>
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<tr>
<td>TURN AND TALK</td>
<td>REVOICING</td>
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<td>CHARTING</td>
<td>MARKING</td>
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<td>GALLERY WALKS</td>
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<td>SMALL GROUP AND WHOLE CLASS DISCUSSIONS</td>
<td>CHALLENGING</td>
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<tr>
<td>STUDENT MODELING</td>
<td>PRESSING FOR ACCURACY AND REASONING</td>
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<tr>
<td>Maintain the Cognitive Demand</td>
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Educational Technology

Standards


➢ Technology Operations and Concepts
  • Construct a spreadsheet workbook with multiple worksheets, rename tabs to reflect the data on the worksheet, and use mathematical or logical functions, charts and data from all worksheets to convey the results.

  Example: Using digital tools such as math software, TI-83/84/89, students are able to easily draw any trigonometric graphs and explore them by tracing or using automatic procedures to explore key points, noting periodicity, amplitude, and so on.

➢ Digital Citizenship
  • Analyze the capabilities and limitations of current and emerging technology resources and assess their potential to address personal, social, lifelong learning and career needs.

  Example: Students will be able to use critical thinking skills to plan and conduct, research, manage projects, solve problems, and make informed decisions using appropriate digital tools and resource.

➢ Research and Information Literacy
  • Produce a position statement about a real world problem by developing a systematic plan of investigation with peers and experts synthesizing information from multiple sources.

  Example: Students will be able to produce a position statement about the real world problem by developing a systematic plan of investigation with peers and experts synthesizing information from multiple source.

➢ Critical Thinking, Problem Solving, Decision Making
  • Evaluate the strengths and limitations of emerging technologies and their impact on educational, career, personal and or social needs.
Example: A problem-solving approach will allow students to construct their own ideas about mathematics and to take responsibility for their own learning.

Career Ready Practices

Career Ready Practices describe the career-ready skills that all educators in all content areas should seek to develop in their students. They are practices that have been linked to increase college, career, and life success. Career Ready Practices should be taught and reinforced in all career exploration and preparation programs with increasingly higher levels of complexity and expectation as a student advances through a program of study.

- CRP2. Apply appropriate academic and technical skills.
Career Ready Practices

Career-ready individuals readily access and use the knowledge and skills acquired through experience and education to be more productive. They make connections between abstract concepts with real-world applications, and they make correct insights about when it is appropriate to apply the use of an academic skill in a workplace situation.

**Example:** Students will use technology to acquire, manipulate, analyze and report data, display and communicate STEM information and apply science and mathematical concepts to the development of plans, processes and projects that address real world problems.

- **CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.**
  Career-ready individuals readily recognize problems in the workplace, understand the nature of the problem, and devise effective plans to solve the problem. They are aware of problems when they occur and take action quickly to address the problem; they thoughtfully investigate the root cause of the problem prior to introducing solutions. They carefully consider the options to solve the problem. Once a solution is agreed upon, they follow through to ensure the problem is solved, whether through their own actions or the actions of others.

  **Example:** Throughout this unit, students work to improve their critical-thinking skills and problem-solving techniques.

- **CRP11. Use technology to enhance productivity.**
  Career-ready individuals find and maximize the productive value of existing and new technology to accomplish workplace tasks and solve workplace problems. They are flexible and adaptive in acquiring new technology. They are proficient with ubiquitous technology applications. They understand the inherent risks—personal and organizational—of technology applications, and they take actions to prevent or mitigate these risks.

  **Example:** Students will use digital tools such as TI-83/84/89, Interactive Whiteboard, etc. to access, manage, evaluate, and synthesize information in order to solve problems individually and collaboratively to create and communicate knowledge.

WIDA Proficiency Levels

At the given level of English language proficiency, English language learners will process, understand, produce or use

- Specialized or technical language reflective of the content areas at grade level
<table>
<thead>
<tr>
<th>Level</th>
<th>Description</th>
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</thead>
</table>
| **6- Reaching** | A variety of sentence lengths of varying linguistic complexity in extended oral or written discourse as required by the specified grade level  
Oral or written communication in English comparable to proficient English peers |
| **5- Bridging** | Specialized or technical language of the content areas  
A variety of sentence lengths of varying linguistic complexity in extended oral or written discourse, including stories, essays or reports  
Oral or written language approaching comparability to that of proficient English peers when presented with grade level material. |
| **4- Expanding** | Specific and some technical language of the content areas  
A variety of sentence lengths of varying linguistic complexity in oral discourse or multiple, related sentences or paragraphs  
Oral or written language with minimal phonological, syntactic or semantic errors that may impede the communication, but retain much of its meaning, when presented with oral or written connected discourse, with sensory, graphic or interactive support |
| **3- Developing** | General and some specific language of the content areas  
Expanded sentences in oral interaction or written paragraphs  
Oral or written language with phonological, syntactic or semantic errors that may impede the communication, but retain much of its meaning, when presented with oral or written, narrative or expository descriptions with sensory, graphic or interactive support |
| **2- Beginning** | General language related to the content area  
Phrases or short sentences  
Oral or written language with phonological, syntactic, or semantic errors that often impede the communication when presented with one to multiple-step commands, directions, or a series of statements with sensory, graphic or interactive support |
| **1- Entering** | Pictorial or graphic representation of the language of the content areas  
Words, phrases or chunks of language when presented with one-step commands directions, WH-, choice or yes/no questions, or statements with sensory, graphic or interactive support |
# Language Development Supports For English Language Learners
To Increase Comprehension and Communication Skills

## Environment
- Welcoming and stress-free
- Respectful of linguistic and cultural diversity
- Honors students’ background knowledge
- Sets clear and high expectations
- Includes routines and norms
- Is thinking-focused vs. answer-seeking
- Offers multiple modalities to engage in content learning and to demonstrate understanding
- Includes explicit instruction of specific language targets
- Provides participation techniques to include all learners
- Integrates learning centers and games in a meaningful way
- Provides opportunities to practice and refine receptive and productive skills in English as a new language
- Integrates meaning and purposeful tasks/activities that:
  - Are accessible by all students through multiple entry points
  - Are relevant to students’ lives and cultural experiences
  - Build on prior mathematical learning
  - Demonstrate high cognitive demand
  - Offer multiple strategies for solutions
  - Allow for a language learning experience in addition to content

## Sensory Supports*
- Real-life objects (realia) or concrete objects
- Physical models
- Manipulatives
- Pictures & photographs
- Visual representations or models such as diagrams or drawings
- Videos & films
- Newspapers or magazines
- Gestures
- Physical movements
- Music & songs

## Graphic Supports*
- Graphs
- Charts
- Timelines
- Number lines
- Graphic organizers
- Graphing paper

## Interactive Supports*
- In a whole group
- In a small group
- With a partner such as Turn-and-Talk
- In pairs as a group (first, two pairs work independently, then they form a group of four)
- In triads
- Cooperative learning structures such as Think-Pair-Share
- Interactive websites or software
- With a mentor or coach

## Verbal and Textual Supports
- Labeling
- Students’ native language
- Modeling
- Repetitions
- Paraphrasing
- Summarizing
- Guiding questions
- Clarifying questions
- Probing questions
- Leveled questions such as What? Where? How? Why?
- Questioning prompts & cues
- Word Banks
- Sentence starters
- Sentence frames
- Discussion frames
- Talk moves, including Wait Time

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# Building Equity in Your Teaching Practice

How do the essential questions highlight the connection between the big ideas of the unit and equity in your teaching practice?

<table>
<thead>
<tr>
<th><strong>Content Integration</strong></th>
<th><strong>Knowledge Construction</strong></th>
<th><strong>Prejudice Reduction</strong></th>
<th><strong>Equitable Pedagogy</strong></th>
<th><strong>Empowering School Culture</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Teachers use examples and content from a variety of cultures &amp; groups.</td>
<td>Teachers help students understand how knowledge is created and influenced by cultural assumptions, perspectives &amp; biases.</td>
<td>Teachers implement lessons and activities to assert positive images of ethnic groups &amp; improve intergroup relations.</td>
<td>Teachers modify techniques and methods to facilitate the academic achievement of students from diverse backgrounds.</td>
<td>Using the other four dimensions to create a safe and healthy educational environment for all.</td>
</tr>
</tbody>
</table>

### This unit / lesson is connected to other topics explored with students.

There are multiple viewpoints reflected in the content of this unit / lesson.

The materials and resources are reflective of the diverse identities and experiences of students.

The content affirms students, as well as exposes them to experiences other than their own.

### This unit / lesson provides context to the history of privilege and oppression.

This unit / lesson addresses power relationships.

This unit / lesson help students to develop research and critical thinking skills.

This curriculum creates windows and mirrors* for students.

### This unit / lesson help students question and unpack biases & stereotypes.

This unit / lesson help students examine, research and question information and sources.

The curriculum encourage discussion and understanding about the groups of people being represented.

This unit / lesson challenges dominant perspectives.

### The instruction has been modified to meet the needs of each student.

Students feel respected and their cultural identities are valued.

Additional supports have been provided for students to become successful and independent learners.

Opportunities are provided for student to reflect on their learning and provide feedback.

### There are opportunities for students to connect with the community.

My classroom is welcoming and supportive for all students?

I am aware of and sensitive to the needs of my students and their families.

There are effective parent communication systems established. Parents can talk to me about issues as they arise in my classroom.

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### Culturally Relevant Pedagogy Examples

- **Integrate Relevant Word Problems:** Contextualize equations using word problems that reference student interests and cultures.
  
  **Example:** When learning to interpret functions, problems that relate to student interests such as music, sports, and art enable the students to understand and relate to the concept in a more meaningful way.

- **Everyone has a Voice:** Create a classroom environment where students know that their contributions are expected and valued.
  
  **Example:** Norms for sharing are established that communicate a growth mindset for mathematics. All students are capable of expressing mathematical thinking and contributing to the classroom community. Students learn new ways of looking at problem solving by working with and listening to each other.

- **Run Problem Based Learning Scenarios:** Encourage mathematical discourse among students by presenting problems that are relevant to them, the school and/or the community.
  
  **Example:** Using a Place Based Education (PBE) model, students explore math concepts such as systems of equations while determining ways to address problems that are pertinent to their neighborhood, school, or culture.

- **Encourage Student Leadership:** Create an avenue for students to propose problem solving strategies and potential projects.
  
  **Example:** Students can learn to write and interpret exponential and logarithmic functions by creating problems together and deciding if the problems fit the necessary criteria. This experience will allow students to discuss and explore their current level of understanding by applying the concepts to relevant real-life experiences.

- **Present New Concepts Using Student Vocabulary:** Use student diction to capture attention and build understanding before using academic terms.
  
  **Example:** Teach math vocabulary in various modalities for students to remember. Use multi-modal activities, analogies, realia, visual cues, graphic representations, gestures, pictures, and cognates. Directly explain and model the idea of vocabulary words having multiple meanings. Students can create the Word Wall with their definitions and examples to foster ownership.
## Differentiated Instruction

Accommodate Based on Students Individual Needs: Strategies

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<tr>
<th>Time/General</th>
<th>Processing</th>
<th>Comprehension</th>
<th>Recall</th>
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</thead>
<tbody>
<tr>
<td>Extra time for assigned tasks</td>
<td>Extra Response time</td>
<td>Precise processes for balanced math instruction model</td>
<td>Teacher-made checklist</td>
</tr>
<tr>
<td>Adjust length of assignment</td>
<td>Have students verbalize steps</td>
<td>Short manageable tasks</td>
<td>Use visual graphic organizers</td>
</tr>
<tr>
<td>Timeline with due dates for reports and projects</td>
<td>Repeat, clarify or reword directions</td>
<td>Brief and concrete directions</td>
<td>Reference resources to promote independence</td>
</tr>
<tr>
<td>Communication system between home and school</td>
<td>Mini-breaks between tasks</td>
<td>Provide immediate feedback</td>
<td>Visual and verbal reminders</td>
</tr>
<tr>
<td>Provide lecture notes/outline</td>
<td>Provide a warning for transitions</td>
<td>Small group instruction</td>
<td>Graphic organizers</td>
</tr>
<tr>
<td></td>
<td>Reading partners</td>
<td>Emphasize multi-sensory learning</td>
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<tr>
<th>Assistive Technology</th>
<th>Tests/Quizzes/Grading</th>
<th>Behavior/Attention</th>
<th>Organization</th>
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<tr>
<td>Computer/whiteboard</td>
<td>Extended time</td>
<td>Consistent daily structured routine</td>
<td>Individual daily planner</td>
</tr>
<tr>
<td>Tape recorder</td>
<td>Study guides</td>
<td>Simple and clear classroom rules</td>
<td>Display a written agenda</td>
</tr>
<tr>
<td>Video Tape</td>
<td>Shortened tests</td>
<td>Frequent feedback</td>
<td>Note-taking assistance</td>
</tr>
<tr>
<td></td>
<td>Read directions aloud</td>
<td></td>
<td>Color code materials</td>
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</tbody>
</table>

## Assistive Technology
- Computer/whiteboard
- Tape recorder
- Video Tape

## Tests/Quizzes/Grading
- Extended time
- Study guides
- Shortened tests
- Read directions aloud

## Behavior/Attention
- Consistent daily structured routine
- Simple and clear classroom rules
- Frequent feedback

## Organization
- Individual daily planner
- Display a written agenda
- Note-taking assistance
- Color code materials
**Differentiated Instruction**

**Accommodate Based on Content Needs: Strategies**

- Anchor charts to model strategies
- Review Algebra concepts to ensure students have the information needed to progress in understanding
- Pre-teach pertinent vocabulary
- Provide reference sheets that list formulas, step-by-step procedures, theorems, and modeling of strategies
- Word wall with visual representations of mathematical terms
- Teacher modeling of thinking processes involved in solving, graphing, and writing equations
- Introduce concepts embedded in real-life context to help students relate to the mathematics involved
- Record formulas, processes, and mathematical rules in reference notebooks
- Graphing calculator to assist with computations and graphing of trigonometric functions
- Utilize technology through interactive sites to represent nonlinear data
- Graphic organizers to help students interpret the meaning of terms in an expression or equation in context
- Translation dictionary
- Sentence stems to provide additional language support for ELL students.
Interdisciplinary Connections

Model interdisciplinary thinking to expose students to other disciplines.

Art connection:
- **Name of Task: Math Music**  
  **NJSLS: 1.1, 1.3**

A pure tone produces a sine wave when shown on an oscilloscope. When an instrument is played, the tone is not pure. For instance, when a guitar string or piano string vibrates it does not produce a simple sine wave. It does produce other, less distinguishable harmonious waves of higher pitch called harmonic waves. For instance, \( y = \sin (2x) \) is called the second harmonic, \( y = \sin (3x) \) is called the third harmonic, and \( y = \sin (4x) \) is called the fourth harmonic.

Science Connection:
- **Name of Task: Rabbits, Rabbits Everywhere**  
  **NJSLS: MS-LS4-4**

The rabbit population in a national park rises and falls throughout the year. The population is at its approximate minimum of 6000 rabbits in December. As the weather gets warmer and food becomes more available, the population grows to its approximate maximum of 16,000 rabbits in June. The function describing the rabbit population is \( f(x) = 5000 \sin \left( \frac{\pi}{6} x - \frac{\pi}{2} \right) + 11,000 \) where \( x \) is the time in months and \( f(x) \) is the rabbit population.

- **Name of Task: Speed of CD-RW**  
  **NJSLS: MS-PS3-1.**

A CD-RW has diameter of 120 millimeters. When playing audio, the angular speed varies to keep the linear speed constant where the disc is being read. When reading along the outer edge of the disc, the angular speed is about 200 RPM (revolutions per minute).

1- Find the linear speed.
2- What would the angular speed be when you reach half of the CD?
3- When being burned in this writable CD-R drive, the angular speed of the CD is often much faster than when playing audio, but the angular speed still varies to keep the linear speed constant where the disc is being written. When writing along the outer edge of the disc, the angular speed of one drive is about 4800 RPM (revolutions per minute). Find the linear speed.

* Tasks can be found within the additional task folders.
Enrichment

What is the purpose of Enrichment?

- The purpose of enrichment is to provide extended learning opportunities and challenges to students who have already mastered, or can quickly master, the basic curriculum. Enrichment gives the student more time to study concepts with greater depth, breadth, and complexity.
- Enrichment also provides opportunities for students to pursue learning in their own areas of interest and strengths.
- Enrichment keeps advanced students engaged and supports their accelerated academic needs.
- Enrichment provides the most appropriate answer to the question, “What do you do when the student already knows it?”

Enrichment is…

- Planned and purposeful
- Different, or differentiated, work – not just more work
- Responsive to students’ needs and situations
- A promotion of high-level thinking skills and making connections within content
- The ability to apply different or multiple strategies to the content
- The ability to synthesize concepts and make real world and cross-curricular connections.
- Elevated contextual complexity
- Sometimes independent activities, sometimes direct instruction
- Inquiry based or open ended assignments and projects
- Using supplementary materials in addition to the normal range of resources.
- Choices for students
- Tiered/Multi-level activities with flexible groups (may change daily or weekly)

Enrichment is not…

- Just for gifted students (some gifted students may need intervention in some areas just as some other students may need frequent enrichment)
- Worksheets that are more of the same (busywork)
- Random assignments, games, or puzzles not connected to the content areas or areas of student interest
- Extra homework
- A package that is the same for everyone
- Thinking skills taught in isolation
- Unstructured free time
Assessments

Required District/State Assessments
SGO Assessments

Suggested Formative/Summative Classroom Assessments

- Describe Learning Vertically
- Identify Key Building Blocks
- Make Connections (between and among key building blocks)
- Short/Extended Constructed Response Items
- Multiple-Choice Items (where multiple answer choices may be correct)
- Drag and Drop Items
- Use of Equation Editor
- Quizzes
- Journal Entries/Reflections/Quick-Writes
- Accountable talk
- Projects
- Portfolio
- Observation
- Graphic Organizers/Concept Mapping
- Presentations
- Role Playing
- Teacher-Student and Student-Student Conferencing
- Homework
New Jersey Student Learning Standards

F.TF.A.1: Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.

F.TF.A.2: Explain how the unit circle in the coordinate plane enables the extension of trigonometric function to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

F.TF.A.3: Use special triangles to determine geometrically the values of sine, cosine, tangent for \( \pi/3, \pi/4, \) and \( \pi/6, \) and to use the unit circle to express the values of sine, cosine, and tangent for \( \pi - x, \pi + x, \) and \( 2\pi - x \) in terms of their values for \( x, \) where \( x \) is any real number.

F.TF.B.5: Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.

F.TF.B.7: Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.

F.TF.C.8: Prove the Pythagorean identity \( \sin^2 \theta + \cos^2 \theta = 1 \) and use it to find \( \sin(\theta), \cos(\theta), \tan(\theta), \) and the quadrant of the angle.

F.LE.A.1: Distinguish between situations that can be modeled with linear functions and with exponential functions.

F.LE.A.1c: Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

F.LE.A.4: For exponential models, express as a logarithm the solutions to \( ab^x = d \) where \( a, c, \) and \( d \) are numbers and the base \( b \) is 2, 10, or \( e; \) evaluate the logarithm using technology.

F.IF.C.7e: Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period midline, and amplitude.

F.IF.C.8b: Use the properties of exponents to interpret expressions for exponential functions.

F.BF.B.5: Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.
## Mathematical Practices

1. Make sense of problems and persevere in solving them.

2. Reason abstractly and quantitatively.

3. Construct viable arguments and critique the reasoning of others.

4. Model with mathematics.

5. Use appropriate tools strategically.

6. Attend to precision.

7. Look for and make use of structure.

8. Look for and express regularity in repeated reasoning.
# Course: Pre-Calculus Honors

## Unit: 2

## Topic: Logarithmic and Trigonometric Functions

**NJSLS:**

F.TF.A.1, F.TF.A.2, F.TF.A.3, F.TF.B.5, F.TF.B.7, F.TF.C.8, F.LE.A.1, F.LE.A.1c, F.LE.A.4, F.IF.C.7e, F.IF.C.8b, F.BF.B.5

**Unit Focus:**

- Explain the relationship between logarithms and exponents.
- Rewrite and solve exponential and logarithmic functions.
- Write and interpret exponential and logarithmic functions.
- Determine an equivalent form of a logarithmic expression using the laws of logarithms.
- Relate the unit to right triangles and trigonometry as well as the relationships between the six trigonometric ratios, their reciprocals and their inverses.
- Evaluate and graph the trigonometric functions, their inverses, and their reciprocals.
- Use trigonometric ratios to solve problems in a variety of context, such as mechanics, biology, and navigation.
- Learn strategies for simplifying expressions and solving equations by using trigonometric identities.

**New Jersey Student Learning Standard(s):**

**F.LE.A.1:** Distinguish between situations that can be modeled with linear functions and with exponential functions.

**F.LE.A.1c:** Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

**F.LE.A.4:** For exponential models, express as a logarithm the solutions to $ab^ct = d$ where $a$, $c$, and $d$ are numbers and the base $b$ is 2, 10, or $e$; evaluate the logarithm using technology.

**F.IF.C.7e:** Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period midline, and amplitude.

**Student Learning Objective 1:** Recognize, evaluate and graph logarithmic and exponential functions with base $a$ and $e$. Graph exponential functions. Use exponential and logarithmic functions to model and solve real life problems.
| MPs | Skills, Strategies & Concepts | Essential Understandings/ Questions | Tasks/Activities |
|-----|-----------------------------|-------------------------------------|----------------|---|
| MP 2 | • Evaluate exponential functions.  
      • Define the number $e$.  
      • Analyze Log as the inverse of exponential functions.  
      • Graph logs and exponential functions.  
      • Determine the domain of a logarithmic function. | • Why is the number $e$ important?  
      • What is the relationship between an exponential expression and a logarithm? | Fruit Fly Colony |
| MP 3 | • Create models and use arguments to help in solving real life problems. | | Student Loan Payback |
| MP 6 | | | |

**New Jersey Student Learning Standard(s):**

**F.IF.C.8b:** Use the properties of exponents to interpret expressions for exponential functions.

**F.BF.B.5:** Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

**Student Learning Objective 2:** Use properties of logarithms to evaluate or rewrite logarithmic expressions. Use properties of logarithms to expand or condense logarithmic expressions. Solve exponential and logarithmic equations. Use logarithmic functions to model and solve real-life problems.

| MPs | Skills, Strategies & Concepts | Essential Understandings/ Questions | Tasks/Activities |
|-----|-----------------------------|-------------------------------------|----------------|---|
| MP 2 | • Expand and condense logarithmic expressions.  
      • Create models and use arguments to help in solving real life problems. | • How do I use exponential functions to model exponential behavior?  
      • Logarithms can be solved by rewriting them as exponents, and vice versa.  
      Exponential formulas Like: $A = Pert$ can be used to solve growth problems in science and business. The number “$e$” is | Drug Absorption Rate  
      Deer In The Neighborhood |
New Jersey Student Learning Standard(s):
F.TF.A.1: Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.

F.TF.A.2: Explain how the unit circle in the coordinate plane enables the extension of trigonometric function to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

**Student Learning Objective 3:** Describe angles; convert between radian and degrees measures. Identify a unit circle and use the unit circle to evaluate trigonometric functions.

<table>
<thead>
<tr>
<th>MPs</th>
<th>Skills, Strategies &amp; Concepts</th>
<th>Essential Understandings/ Questions</th>
<th>Tasks/Activities</th>
</tr>
</thead>
<tbody>
<tr>
<td>MP 2</td>
<td>• Create a unit circle from its relationship to right triangle trigonometry.</td>
<td>• Trigonometry allows for indirect measurement of triangles when given limited information.</td>
<td>Speed of CD-RW</td>
</tr>
<tr>
<td></td>
<td>• Identify co-terminal and reference angles using degrees.</td>
<td>• The unit circle is a gateway between geometry and algebra.</td>
<td>Marla’s Track</td>
</tr>
<tr>
<td></td>
<td>• Use the unit circle to evaluate all six trigonometric relations; use the calculator to evaluate each relation.</td>
<td>• What relationship exists between a triangle and the unit circle?</td>
<td>Ferris Wheel</td>
</tr>
<tr>
<td></td>
<td>• Identify co-terminal and reference angles using degrees.</td>
<td>• How are trigonometric functions evaluated in the quadrants?</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Evaluate trigonometric functions of any angle using reference angles.</td>
<td>• How can you find the measure of an angle in radians?</td>
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</tr>
<tr>
<td></td>
<td>• Relate right triangle trigonometry to the ratio of sides of a triangle relating it to the algorithm used in a calculator.</td>
<td>• How can you use the unit circle to define the trigonometric functions of any angle?</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>• How do you describe angles and angular movement?</td>
<td></td>
</tr>
</tbody>
</table>
New Jersey Student Learning Standard(s):

F.TF.A.3: Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$, and $\pi/6$, and to use the unit circle to express the values of sine, cosine, and tangent for $\pi - x$, $\pi + x$, and $2\pi - x$ in terms of their values for $x$, where $x$ is any real number.

F.TF.B.5: Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.

Student Learning Objective 4: Evaluate trigonometric functions using domain and range, reference angles, and using technology. Sketch the graph of sine and cosine functions using amplitude, period and translations.

<table>
<thead>
<tr>
<th>MPs</th>
<th>Skills, Strategies &amp; Concepts</th>
<th>Essential Understandings/ Questions</th>
<th>Tasks/Activities</th>
</tr>
</thead>
</table>
| MP 2 | • Use domain and period to evaluate sine and cosine functions.  
     • Use a calculator to evaluate trigonometric functions.  
     • Evaluate trigonometric functions of acute angles.  
     • Use amplitude and period to help sketch the graphs of sine and cosine functions. | • Angles are the domain elements of the trigonometric functions.  
     • How do you evaluate and graph trigonometric functions by using the unit circle?  
     • How do you use trigonometry to solve right triangles, and any angle?  
     • How do you sketch the graphs of cosine and sine functions?  
     • What are the characteristics of the graphs of the sine and cosine functions? | Rabbits, Rabbits Everywhere!  
     It Is Too Hot  
     Seeing Music |
| MP 7 | | | |

MP 7

Rabbits, Rabbits Everywhere!
It Is Too Hot
Seeing Music
New Jersey Student Learning Standard(s):
F.TF.C.8: Prove the Pythagorean identity $\sin^2 \theta + \cos^2 \theta = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, $\tan(\theta)$, and the quadrant of the angle.

**Student Learning Objective 5:** Use the fundamental trigonometric identities. Prove trigonometric identities using Pythagorean, reciprocal, and quotient identities.

<table>
<thead>
<tr>
<th>MPs</th>
<th>Skills, Strategies &amp; Concepts</th>
<th>Essential Understandings/ Questions</th>
<th>Tasks/Activities</th>
</tr>
</thead>
<tbody>
<tr>
<td>MP 1</td>
<td>• Use the fundamental trigonometric identities.</td>
<td>• How are all trigonometric functions related?</td>
<td>Trigonometric Identities</td>
</tr>
<tr>
<td>MP 2</td>
<td></td>
<td>• How can you verify a trigonometric identity?</td>
<td></td>
</tr>
<tr>
<td>MP 7</td>
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</tbody>
</table>

New Jersey Student Learning Standard(s):
F.TF.B.7: Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.

**Student Learning Objective 6:** Evaluate inverse and composite trigonometric functions and graph them.

<table>
<thead>
<tr>
<th>MPs</th>
<th>Skills, Strategies &amp; Concepts</th>
<th>Essential Understandings/ Questions</th>
<th>Tasks/Activities</th>
</tr>
</thead>
<tbody>
<tr>
<td>MP 1</td>
<td>• Evaluate inverse trigonometric functions.</td>
<td>• How do you evaluate and graph the inverses of trigonometric functions?</td>
<td>Math Music</td>
</tr>
<tr>
<td>MP 7</td>
<td>• Evaluate composite trigonometric functions.</td>
<td></td>
<td>My Beautiful Sand Castle</td>
</tr>
</tbody>
</table>
New Jersey Student Learning Standard(s):
F.TF.B.5: Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.

**Student Learning Objective 7:** Use trigonometric functions to model and solve real life problems.

<table>
<thead>
<tr>
<th>MPs</th>
<th>Skills, Strategies &amp; Concepts</th>
<th>Essential Understandings/Questions</th>
<th>Tasks/Activities</th>
</tr>
</thead>
<tbody>
<tr>
<td>MP 1</td>
<td>• Use the fundamental trigonometric identities to evaluate trigonometric functions, simplify expressions and rewrite trigonometric expressions.</td>
<td>• How are trigonometric functions related to anything in real world?</td>
<td>How Tall Is This Cliff?</td>
</tr>
<tr>
<td>MP 2</td>
<td></td>
<td>• How do you use trigonometric functions to solve real life problems?</td>
<td>Amusement Rides</td>
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<tr>
<td>MP 5</td>
<td></td>
<td>• What does evaluating a trig function at a given angle mean in real life?</td>
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<tr>
<td></td>
<td></td>
<td>• How can sinusoids be used to model real-world phenomena like tides, the motion of a roller coaster, the motion of a piston, etc.?</td>
<td></td>
</tr>
</tbody>
</table>
## Honors Project (must complete all)

<table>
<thead>
<tr>
<th>Project 1</th>
<th>Project 2</th>
<th>Project 3</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Function Composition</strong></td>
<td><strong>String Functions</strong></td>
<td><strong>Find your Trig Functions</strong></td>
</tr>
</tbody>
</table>
| **Essential Question:**  
  • What is function composition?  
  • How do we decide on the order of composite functions?  
  • How do you combine two functions to form a new function?  
| **Essential Question:**  
  • How can we model a real world situation with a trigonometric function?  
  • How is the amplitude, midline, period, and frequency of a trigonometric function related to the transformation of the parent graph?  
  • How do the parent graphs of the six major trig functions help identifying trigonometric shapes in real life?  
  • Do sinusoidal exist in nature and/or in real life?  
  • How can we model a real world situation with a trigonometric function?  
| **Skills:**  
  • Utilize the graphing calculator accurately to check your results.  
| **Skills:**  
  • Understand functions can be used to model periodic phenomena.  
  • Interpret functions in context.  
| **Skills:**  
  • Emphasize the mathematical practices as you address the standards in this project.  

## Unit 2 Vocabulary

<table>
<thead>
<tr>
<th>Left Column</th>
<th>Right Column</th>
</tr>
</thead>
<tbody>
<tr>
<td>- acute angle</td>
<td>- lower bound</td>
</tr>
<tr>
<td>- adjacent side</td>
<td>- multiplicity</td>
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<tr>
<td>- amplitude</td>
<td>- natural base</td>
</tr>
<tr>
<td>- angle</td>
<td>- opposite side</td>
</tr>
<tr>
<td>- angle of depression</td>
<td>- period</td>
</tr>
<tr>
<td>- angle of elevation</td>
<td>- periodic function</td>
</tr>
<tr>
<td>- angular speed</td>
<td>- phase shift</td>
</tr>
<tr>
<td>- arc</td>
<td>- positive angles</td>
</tr>
<tr>
<td>- bearing</td>
<td>- radian</td>
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<tr>
<td>- central angle</td>
<td>- radius</td>
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<tr>
<td>- complementary angles</td>
<td>- reciprocal functions</td>
</tr>
<tr>
<td>- coterminal angles</td>
<td>- reference angle</td>
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<tr>
<td>- cosecant</td>
<td>- secant</td>
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<tr>
<td>- cosine</td>
<td>- natural exponential function</td>
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<tr>
<td>- cotangent</td>
<td>- natural logarithmic function</td>
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<tr>
<td>- cycle</td>
<td>- natural logarithmic model</td>
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<tr>
<td>- damped trigonometric function</td>
<td>- negative angle</td>
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<tr>
<td>- degrees</td>
<td>- obtuse angle</td>
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<tr>
<td>- displacement</td>
<td>- sine</td>
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<tr>
<td>- equilibrium</td>
<td>- solving right triangles</td>
</tr>
<tr>
<td>- frequency</td>
<td>- standard position</td>
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<td>- harmonic motion</td>
<td>- supplementary angles</td>
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<tr>
<td>- hypotenuse</td>
<td>- tangent</td>
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<tr>
<td>- initial side of and angle</td>
<td>- terminal side of and angle</td>
</tr>
<tr>
<td>- inverse(arc) trigonometric functions</td>
<td>- trigonometry</td>
</tr>
<tr>
<td>- linear speed</td>
<td>- unit circle</td>
</tr>
<tr>
<td>- logarithmic function</td>
<td>- vertex</td>
</tr>
<tr>
<td>- logistic growth model</td>
<td></td>
</tr>
</tbody>
</table>
## References & Suggested Instructional Websites

- [http://www.mathwords.com/index_adv_alg_precal.htm](http://www.mathwords.com/index_adv_alg_precal.htm)
- [http://www.hershey.k12.pa.us/Page/3608](http://www.hershey.k12.pa.us/Page/3608)
- [https://sites.google.com/a/evergreenps.org/ms-griffin-s-math-classes/updates](https://sites.google.com/a/evergreenps.org/ms-griffin-s-math-classes/updates)
- [https://sites.google.com/site/dgrahamcalculus/trigpre-calculus/trig-pre-calculus-worksheets](https://sites.google.com/site/dgrahamcalculus/trigpre-calculus/trig-pre-calculus-worksheets)
- [https://www.ixl.com/math/precalculus](https://www.ixl.com/math/precalculus)
- [https://www.illustrativemathematics.org/](https://www.illustrativemathematics.org/)
- The Data and Story Library (DASL). [http://lib.stat.cmu.edu/DASL/](http://lib.stat.cmu.edu/DASL/)


Field Trip Ideas

**SIX FLAGS GREAT ADVENTURE:** This educational event includes workbooks and special science and math related shows throughout the day. Your students will leave with a better understanding of real world applications of the material they have learned in the classroom. Each student will have the opportunity to experience different rides and attractions linking mathematical and scientific concepts to what they are experiencing.

[www.sixflags.com](http://www.sixflags.com)

**MUSEUM of MATHEMATICS:** Mathematics illuminates the patterns that abound in our world. The National Museum of Mathematics strives to enhance public understanding and perception of mathematics. Its dynamic exhibits and programs stimulate inquiry, spark curiosity, and reveal the wonders of mathematics. The Museum’s activities lead a broad and diverse audience to understand the evolving, creative, human, and aesthetic nature of mathematics.

[www.momath.org](http://www.momath.org)

**LIBERTY SCIENCE CENTER:** An interactive science museum and learning center located in Liberty State Park. The center, which first opened in 1993 as New Jersey's first major state science museum, has science exhibits, the largest IMAX Dome theater in the United States, numerous educational resources, and the original *Hoberman sphere*.

[http://lsc.org/plan-your-visit/](http://lsc.org/plan-your-visit/)