Pre-Calculus Honors: Unit 1

Functions and Their Graphs

Polynomial, Rational, and Exponential Functions
Course Philosophy/Description

Pre-Calculus Honors course is for accelerated students. This course is designed for students who exhibit high interest and knowledge in math and science. In this course, students will extend topics introduced in Algebra II and learn to manipulate and apply more advanced functions and algorithms. Students extend their knowledge and understanding by solving open-ended real-world problems and thinking critically through the use of high level tasks and long-term projects. This course provides a mathematically sound foundation for students who intend to study Calculus.

The course covers mathematical topics ranging from Basics of Functions to Limits of Functions. It provides opportunities to the students to expand their knowledge base and understanding of mathematics in general. The overarching goal of the course is to build a solid foundation for the students who choose Mathematics, Engineering, Sciences, or Business as their college major and/or career options. The major topics in the course such as, Polynomials, Exponents, Trigonometry, Logarithms, Complex numbers, Series/sequences, and Limits help generate students’ inquiries about the mathematical nature, complexities, and applications of these topics in real-life situations. Students not only acquire new knowledge, but also deepen their topical and overall understanding of the content for future transfer to new situations or other disciplines.

Assessment results from this course may be used for the purpose of placements into Calculus/AP Calculus, Statistics, Physics/AP Physics, or other higher level courses.
This ESL framework was designed to be used by bilingual, dual language, ESL and general education teachers. Bilingual and dual language programs use the home language and a second language for instruction. ESL teachers and general education or bilingual teachers may use this document to collaborate on unit and lesson planning to decide who will address certain components of the SLO and language objective. ESL teachers may use the appropriate leveled language objective to build lessons for ELLs which reflects what is covered in the general education program. In this way, whether it is a pull-out or push-in model, all teachers are working on the same Student Learning Objective connected to the Common Core standard. The design of language objectives are based on the alignment of the World-Class Instructional Design Assessment (WIDA) Consortium’s English Language Development (ELD) standards with the Common Core State Standards (CCSS). WIDA’s ELD standards advance academic language development across content areas ultimately leading to academic achievement for English learners. As English learners are progressing through the six developmental linguistic stages, this framework will assist all teachers who work with English learners to appropriately identify the language needed to meet the requirements of the content standard. At the same time, the language objectives recognize the cognitive demand required to complete educational tasks. Even though listening and reading (receptive) skills differ from speaking and writing (expressive) skills across proficiency levels the cognitive function should not be diminished. For example, an Entering Level One student only has the linguistic ability to respond in single words in English with significant support from their home language. However, they could complete a Venn diagram with single words which demonstrates that they understand how the elements compare and contrast with each other or they could respond with the support of their home language (L1) with assistance from a teacher, para-professional, peer or a technology program.

http://www.state.nj.us/education/modelcurriculum/ela/ELLOverview.pdf
<table>
<thead>
<tr>
<th>#</th>
<th>Student Learning Objective</th>
<th>NJSLS</th>
<th>Pre-Calculus with Limits Correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>• Write linear equations in various forms and graph linear functions.</td>
<td>A-CED.A.2</td>
<td>1.2, 1.3, 1.4</td>
</tr>
<tr>
<td></td>
<td>• Identify functions, find their domain and range.</td>
<td>A-REI.D.10</td>
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<td></td>
<td>• Use function notation and evaluate functions abstractly and graphically.</td>
<td>F-IF.C.7.a</td>
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<td></td>
<td>• Use functions to model and solve real life problems.</td>
<td>F-IF.A.1</td>
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<tr>
<td></td>
<td></td>
<td>F-IF.A.2</td>
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<tr>
<td>2</td>
<td>• Analyze functions by determining intervals on which they are increasing, decreasing, or constant and determining their relative minimum and relative maximum values.</td>
<td>F-IF.B.4</td>
<td>1.5, 1.6</td>
</tr>
<tr>
<td></td>
<td>• Identify even and odd functions.</td>
<td>A-SSE.B.3b</td>
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<td></td>
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<td>F-IF.C.7a</td>
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<tr>
<td>3</td>
<td>• Recognize graph of common functions, and use vertical, horizontal shifts, reflections, and non-rigid transformations to graph them.</td>
<td>F-IF.C.7</td>
<td>1.7</td>
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<td>F-BF.B.3</td>
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<tr>
<td>4</td>
<td>• Add, subtract, multiply, and divide functions.</td>
<td>F-BF.A.1b, 1c</td>
<td>1.8, 1.9, 1.10</td>
</tr>
<tr>
<td></td>
<td>• Find composition of functions.</td>
<td>F-BF.B.4b, 4c</td>
<td></td>
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<td></td>
<td>• Determine if graphs are one-to-one and find the inverse of functions.</td>
<td>F-BF.B.5</td>
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<tr>
<td>5</td>
<td>• Construct scatter plots and interpret data from linear models.</td>
<td>S-ID.B.6a-6c</td>
<td>N/A</td>
</tr>
<tr>
<td></td>
<td>• Use scatter plots and a graphing utility to find model for data.</td>
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<tr>
<td>6</td>
<td>• Write quadratic functions in standard and vertex form and use results to sketch graphs of functions.</td>
<td>A-REI.B.4a</td>
<td>2.1</td>
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<td></td>
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<td>F-IF.C.7a</td>
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</tbody>
</table>
| 7  | • Analyze polynomial functions by finding all zeros including complex ones using the Fundamental Theorem of Algebra, factoring, and graphing them, both manually and using technology. | A-SSE.B.3a  
A-APR.B.3  
F-IF.C.7, 8a  
N-CN.C.7, C.9  
A-REI.B.4b | 2.2 |
| 8  | • Use long division and synthetic division to divide polynomials. | A-APR.D.6 | 2.3 |
| 9  | • Analyze rational functions by finding all horizontal, vertical and slanted asymptotes and graphing them, both manually and using technology.  
• Use polynomial and rational functions to model and solve real-life problems including ones with minimum and maximum values. | F.IF.C.7a, 7d  
F-BF.A.1  
S-ID.B.6a  
A-SSE.B.3b  
F-IF.B.4 | 2.5, 2.6, 2.7 |
Research about Teaching and Learning Mathematics

Structure teaching of mathematical concepts and skills around problems to be solved (Checkly, 1997; Wood & Sellars, 1996; Wood & Sellars, 1997)

Encourage students to work cooperatively with others (Johnson & Johnson, 1975; Davidson, 1990)

Use group problem-solving to stimulate students to apply their mathematical thinking skills (Artzt & Armour-Thomas, 1992)

Students interact in ways that support and challenge one another’s strategic thinking (Artzt, Armour-Thomas, & Curcio, 2008)

Activities structured in ways allowing students to explore, explain, extend, and evaluate their progress (National Research Council, 1999)

There are three critical components to effective mathematics instruction (Shellard & Moyer, 2002):

- Teaching for conceptual understanding
- Developing children’s procedural literacy
- Promoting strategic competence through meaningful problem-solving investigations

Teachers should be:

- Demonstrating acceptance and recognition of students’ divergent ideas.
- Challenging students to think deeply about the problems they are solving, extending thinking beyond the solutions and algorithms required to solve the problem
- Influencing learning by asking challenging and interesting questions to accelerate students’ innate inquisitiveness and foster them to examine concepts further.
- Projecting a positive attitude about mathematics and about students’ ability to “do” mathematics

Students should be:

- Actively engaging in “doing” mathematics
- Solving challenging problems
- Investigating meaningful real-world problems
- Making interdisciplinary connections
- Developing an understanding of mathematical knowledge required to “do” mathematics and connect the language of mathematical ideas with numerical representations
- Sharing mathematical ideas, discussing mathematics with one another, refining and critiquing each other’s ideas and understandings
- Communicating in pairs, small group, or whole group presentations
- Using multiple representations to communicate mathematical ideas
- Using connections between pictures, oral language, written symbols, manipulative models, and real-world situations
- Using technological resources and other 21st century skills to support and enhance mathematical understanding
Mathematics is not a stagnate field of textbook problems; rather, it is a dynamic way of constructing meaning about the world around us, generating knowledge and understanding about the real world every day. Students should be metaphorically rolling up their sleeves and “doing mathematics” themselves, not watching others do mathematics for them or in front of them. (Protheroe, 2007)

Balanced Mathematics Instructional Model

Balanced math consists of three different learning opportunities; guided math, shared math, and independent math. Ensuring a balance of all three approaches will build conceptual understanding, problem solving, computational fluency, and procedural fluency. Building a balanced mathematical understanding is the focal point of developing mathematical proficiency. Students should frequently work on rigorous tasks, talk about the math, explain their thinking, justify their answer or process, build models with graphs or charts or manipulatives, and use technology.

When balanced math is used in the classroom it provides students opportunities to:

- solve problems
- make connections between math concepts and real-life situations
- communicate mathematical ideas (orally, visually and in writing)
- choose appropriate materials to solve problems
- reflect and monitor their own understanding of the math concepts
- practice strategies to build procedural and conceptual confidence

Teacher builds conceptual understanding by modeling through demonstration, explicit instruction, and think alouds, as well as guiding students as they practice math strategies and apply problem solving strategies. (whole group or small group instruction)

Teacher and students practice mathematics processes together through interactive activities, problem solving, and discussion. (whole group or small group instruction)

Students practice math strategies independently to build procedural and computational fluency. Teacher assesses learning and reteaches as necessary. (whole group instruction, small group instruction, or centers)
<table>
<thead>
<tr>
<th>Effective Pedagogical Routines/Instructional Strategies</th>
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<tbody>
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<td>Collaborative Problem Solving</td>
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<td>Connect Previous Knowledge to New Learning</td>
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<td>Making Thinking Visible</td>
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<tr>
<td>Develop and Demonstrate Mathematical Practices</td>
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<td>Inquiry-Oriented and Exploratory Approach</td>
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<td>Multiple Solution Paths and Strategies</td>
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<tr>
<td>Use of Multiple Representations</td>
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<td>Explain the Rationale of your Math Work</td>
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<td>Quick Writes</td>
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<td>Pair/Trio Sharing</td>
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<tr>
<td>Turn and Talk</td>
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<tr>
<td>Charting</td>
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<td>Gallery Walks</td>
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<td>Small Group and Whole Class Discussions</td>
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<td>Student Modeling</td>
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<td>Analyze Student Work</td>
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<tr>
<td>Identify Student’s Mathematical Understanding</td>
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<tr>
<td>Identify Student’s Mathematical Misunderstandings</td>
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<tr>
<td>Interviews</td>
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<tr>
<td>Role Playing</td>
</tr>
<tr>
<td>Diagrams, Charts, Tables, and Graphs</td>
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<tr>
<td>Anticipate Likely and Possible Student Responses</td>
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<td>Collect Different Student Approaches</td>
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<td>Multiple Response Strategies</td>
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<td>Asking Assessing and Advancing Questions</td>
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<td>Revoicing</td>
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<td>Marking</td>
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<tr>
<td>Recapping</td>
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<tr>
<td>Challenging</td>
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<tr>
<td>Pressing for Accuracy and Reasoning</td>
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<td>Maintain the Cognitive Demand</td>
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## Educational Technology

### Standards


<table>
<thead>
<tr>
<th>Technology Operations and Concepts</th>
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<tr>
<td>Construct a spreadsheet workbook with multiple worksheets, rename tabs to reflect the data on the worksheet, and use mathematical or logical functions, charts and data from all worksheets to convey the results.</td>
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</table>

**Example:** Students will be able to define functions by a recursive process, which can be displayed effectively using a spreadsheet or other technology.

<table>
<thead>
<tr>
<th>Digital Citizenship</th>
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<tbody>
<tr>
<td>Analyze the capabilities and limitations of current and emerging technology resources and assess their potential to address personal, social, lifelong learning and career needs.</td>
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</table>

**Example:** Students will be able to use critical thinking skills to plan and conduct, research, manage projects, solve problems, and make informed decisions using appropriate digital tools and resource.

<table>
<thead>
<tr>
<th>Research and Information Literacy</th>
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<tbody>
<tr>
<td>Produce a position statement about a real world problem by developing a systematic plan of investigation with peers and experts synthesizing information from multiple sources.</td>
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</tbody>
</table>

**Example:** Students will be able to produce a position statement about the real world problem by developing a systematic plan of investigation with peers and experts synthesizing information from multiple sources.

<table>
<thead>
<tr>
<th>Critical Thinking, Problem Solving, Decision Making</th>
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<tbody>
<tr>
<td>Evaluate the strengths and limitations of emerging technologies and their impact on educational, career, personal and or social needs.</td>
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</tbody>
</table>

**Example:** A problem-solving approach will allow students to construct their own ideas about mathematics and to take responsibility for their own learning.
Career Ready Practices

Career Ready Practices describe the career-ready skills that all educators in all content areas should seek to develop in their students. They are practices that have been linked to increase college, career, and life success. Career Ready Practices should be taught and reinforced in all career exploration and preparation programs with increasingly higher levels of complexity and expectation as a student advances through a program of study.

- **CRP2. Apply appropriate academic and technical skills.**
  Career-ready individuals readily access and use the knowledge and skills acquired through experience and education to be more productive. They make connections between abstract concepts with real-world applications, and they make correct insights about when it is appropriate to apply the use of an academic skill in a workplace situation

  **Example:** Students will use technology to acquire, manipulate, analyze and report data, display and communicate STEM information and apply science and mathematical concepts to the development of plans, processes and projects that address real world problems.

- **CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.**
  Career-ready individuals readily recognize problems in the workplace, understand the nature of the problem, and devise effective plans to solve the problem. They are aware of problems when they occur and take action quickly to address the problem; they thoughtfully investigate the root cause of the problem prior to introducing solutions. They carefully consider the options to solve the problem. Once a solution is agreed upon, they follow through to ensure the problem is solved, whether through their own actions or the actions of others.

  **Example:** Throughout this unit, students work to improve their critical-thinking skills and problem-solving techniques.

- **CRP11. Use technology to enhance productivity.**
  Career-ready individuals find and maximize the productive value of existing and new technology to accomplish workplace tasks and solve workplace problems. They are flexible and adaptive in acquiring new technology. They are proficient with ubiquitous technology applications. They understand the inherent risks-personal and organizational-of technology applications, and they take actions to prevent or mitigate these risks.

  **Example:** Students will use digital tools such as TI-83/84/89, Interactive Whiteboard, etc. to access, manage, evaluate, and synthesize information in order to solve problems individually and collaboratively to create and communicate knowledge.
## WIDA Proficiency Levels

At the given level of English language proficiency, English language learners will process, understand, produce or use:

<table>
<thead>
<tr>
<th>Level</th>
<th>Proficiency Description</th>
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</thead>
</table>
| 6- Reaching | Specialized or technical language reflective of the content areas at grade level  
- A variety of sentence lengths of varying linguistic complexity in extended oral or written discourse as required by the specified grade level  
- Oral or written communication in English comparable to proficient English peers |
| 5- Bridging | Specialized or technical language of the content areas  
- A variety of sentence lengths of varying linguistic complexity in extended oral or written discourse, including stories, essays or reports  
- Oral or written language approaching comparability to that of proficient English peers when presented with grade level material. |
| 4- Expanding | Specific and some technical language of the content areas  
- A variety of sentence lengths of varying linguistic complexity in oral discourse or multiple, related sentences or paragraphs  
- Oral or written language with minimal phonological, syntactic or semantic errors that may impede the communication, but retain much of its meaning, when presented with oral or written connected discourse, with sensory, graphic or interactive support |
| 3- Developing | General and some specific language of the content areas  
- Expanded sentences in oral interaction or written paragraphs  
- Oral or written language with phonological, syntactic or semantic errors that may impede the communication, but retain much of its meaning, when presented with oral or written, narrative or expository descriptions with sensory, graphic or interactive support |
| 2- Beginning | General language related to the content area  
- Phrases or short sentences  
- Oral or written language with phonological, syntactic, or semantic errors that often impede the communication when presented with one to multiple-step commands, directions, or a series of statements with sensory, graphic or interactive support |
| 1- Entering | Pictorial or graphic representation of the language of the content areas  
- Words, phrases or chunks of language when presented with one-step commands directions, WH-, choice or yes/no questions, or statements with sensory, graphic or interactive support |
# Language Development Supports For English Language Learners To Increase Comprehension and Communication Skills

## Environment

- Welcoming and stress-free
- Respectful of linguistic and cultural diversity
- Honors students’ background knowledge
- Sets clear and high expectations
- Includes routines and norms
- Is thinking-focused vs. answer-seeking
- Offers multiple modalities to engage in content learning and to demonstrate understanding
- Includes explicit instruction of specific language targets
- Provides participation techniques to include all learners
- Integrates learning centers and games in a meaningful way
- Provides opportunities to practice and refine receptive and productive skills in English as a new language
- Integrates meaning and purposeful tasks/activities that:
  - Are accessible by all students through multiple entry points
  - Are relevant to students’ lives and cultural experiences
  - Build on prior mathematical learning
  - Demonstrate high cognitive demand
  - Offer multiple strategies for solutions
  - Allow for a language learning experience in addition to content

## Sensory Supports*

- Real-life objects (realia) or concrete objects
- Physical models
- Manipulatives
- Pictures & photographs
- Visual representations or models such as diagrams or drawings
- Videos & films
- Newspapers or magazines
- Gestures
- Physical movements
- Music & songs

## Graphic Supports*

- Graphs
- Charts
- Timelines
- Number lines
- Graphic organizers
- Graphing paper

## Interactive Supports*

- In a whole group
- In a small group
- With a partner such as Turn-and-Talk
- In pairs as a group (first, two pairs work independently, then they form a group of four)
- In triads
- Cooperative learning structures such as Think-Pair-Share
- Interactive websites or software
- With a mentor or coach

## Verbal and Textual Supports

- Labeling
- Students’ native language
- Modeling
- Repetitions
- Paraphrasing
- Summarizing
- Guiding questions
- Clarifying questions
- Probing questions
- Leveled questions such as What? When? Where? How? Why?
- Questioning prompts & cues
- Word Banks
- Sentence starters
- Sentence frames
- Discussion frames
- Talk moves, including Wait Time

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**BUILDING EQUITY IN YOUR TEACHING PRACTICE**

How do the essential questions highlight the connection between the big ideas of the unit and equity in your teaching practice?

<table>
<thead>
<tr>
<th>CONTENT INTEGRATION</th>
<th>KNOWLEDGE CONSTRUCTION</th>
<th>PREJUDICE REDUCTION</th>
<th>EQUITABLE PEDAGOGY</th>
<th>EMPOWERING SCHOOL CULTURE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Teachers use examples and content from a variety of cultures &amp; groups.</td>
<td>Teachers help students understand how knowledge is created and influenced by cultural assumptions, perspectives &amp; biases.</td>
<td>Teachers implement lessons and activities to assert positive images of ethnic groups &amp; improve intergroup relations.</td>
<td>Teachers modify techniques and methods to facilitate the academic achievement of students from diverse backgrounds.</td>
<td>Using the other four dimensions to create a safe and healthy educational environment for all.</td>
</tr>
</tbody>
</table>

This unit / lesson is connected to other topics explored with students.

This unit / lesson provides context to the history of privilege and oppression.

This unit / lesson addresses power relationships.

This unit / lesson helps students to develop research and critical thinking skills.

This curriculum creates windows and mirrors* for students.

This unit / lesson helps students question and unpack biases & stereotypes.

This unit / lesson helps students examine, research and question information and sources.

This unit / lesson challenges dominant perspectives.

The instruction has been modified to meet the needs of each student.

Students feel respected and their cultural identities are valued.

Additional supports have been provided for students to become successful and independent learners.

Opportunities are provided for students to reflect on their learning and provide feedback.

There are opportunities for students to connect with the community.

My classroom is welcoming and supportive for all students?

I am aware of and sensitive to the needs of my students and their families.

There are effective parent communication systems established. Parents can talk to me about issues as they arise in my classroom.

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Culturally Relevant Pedagogy Examples

- **Integrate Relevant Word Problems:** Contextualize equations using word problems that reference student interests and cultures.
  
  **Example:** When learning to interpret functions, problems that relate to student interests such as music, sports and art enable the students to understand and relate to the concept in a more meaningful way.

- **Everyone has a Voice:** Create a classroom environment where students know that their contributions are expected and valued.
  
  **Example:** Norms for sharing are established that communicate a growth mindset for mathematics. All students are capable of expressing mathematical thinking and contributing to the classroom community. Students learn new ways of looking at problem solving by working with and listening to each other.

- **Run Problem Based Learning Scenarios:** Encourage mathematical discourse among students by presenting problems that are relevant to them, the school and/or the community.
  
  **Example:** Using a Place Based Education (PBE) model, students explore math concepts such as systems of equations while determining ways to address problems that are pertinent to their neighborhood, school or culture.

- **Encourage Student Leadership:** Create an avenue for students to propose problem solving strategies and potential projects.
  
  **Example:** Students can learn to construct and compare non-linear functions by creating problems together and deciding if the problems fit the necessary criteria. This experience will allow students to discuss and explore their current level of understanding by applying the concepts to relevant real-life experiences.

- **Present New Concepts Using Student Vocabulary:** Use student diction to capture attention and build understanding before using academic terms.
  
  **Example:** Teach math vocabulary in various modalities for students to remember. Use multi-modal activities, analogies, realia, visual cues, graphic representations, gestures, pictures and cognates. Directly explain and model the idea of vocabulary words having multiple meanings. Students can create the Word Wall with their definitions and examples to foster ownership.
## Differentiated Instruction

### Accommodate Based on Students Individual Needs: Strategies

<table>
<thead>
<tr>
<th><strong>Time/General</strong></th>
<th><strong>Processing</strong></th>
<th><strong>Comprehension</strong></th>
<th><strong>Recall</strong></th>
<th><strong>Assistive Technology</strong></th>
<th><strong>Tests/Quizzes/Grading</strong></th>
<th><strong>Behavior/Attention</strong></th>
<th><strong>Organization</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Extra time for assigned tasks</td>
<td>Extra Response time</td>
<td>Precise processes for balanced math instruction model</td>
<td>Teacher-made checklist</td>
<td>Computer/whiteboard</td>
<td>Extended time</td>
<td>Consistent daily structured routine</td>
<td>Individual daily planner</td>
</tr>
<tr>
<td>Adjust length of assignment</td>
<td>Have students verbalize steps</td>
<td>Short manageable tasks</td>
<td>Use visual graphic organizers</td>
<td>Tape recorder</td>
<td>Study guides</td>
<td>Simple and clear classroom rules</td>
<td>Display a written agenda</td>
</tr>
<tr>
<td>Timeline with due dates for reports and projects</td>
<td>Repeat, clarify or reword directions</td>
<td>Brief and concrete directions</td>
<td>Reference resources to promote independence</td>
<td>Video Tape</td>
<td>Shortened tests</td>
<td>Provide immediate feedback</td>
<td>Note-taking assistance</td>
</tr>
<tr>
<td>Communication system between home and school</td>
<td>Mini-breaks between tasks</td>
<td>Provide immediate feedback</td>
<td>Visual and verbal reminders</td>
<td></td>
<td>Provide a warning for transitions</td>
<td>Small group instruction</td>
<td>Color code materials</td>
</tr>
<tr>
<td>Provide lecture notes/outline</td>
<td>Partnering</td>
<td>Emphasize multi-sensory learning</td>
<td>Graphic organizers</td>
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</table>
**Differentiated Instruction**

**Accommodate Based on Content Needs: Strategies**

- Anchor charts to model strategies
- Review Algebra concepts to ensure students have the information needed to progress in understanding
- Pre-teach pertinent vocabulary
- Provide reference sheets that list formulas, step-by-step procedures, theorems, and modeling of strategies
- Word wall with visual representations of mathematical terms
- Teacher modeling of thinking processes involved in solving, graphing, and writing equations
- Introduce concepts embedded in real-life context to help students relate to the mathematics involved
- Record formulas, processes, and mathematical rules in reference notebooks
- Graphing calculator to assist with computations and graphing of trigonometric functions
- Utilize technology through interactive sites to represent nonlinear data
- Graphic organizers to help students interpret the meaning of terms in an expression or equation in context
- Translation dictionary
- Sentence stems to provide additional language support for ELL students.
Interdisciplinary Connections

Model interdisciplinary thinking to expose students to other disciplines.

Social Studies and ELA Literacy Connection:

- **Name of Task: Americans’ spending:** NJCCCS: 9.1.12.A.9; W.11-12.1

From July 1998 to July 1999, Americans' spending rose from 5.82 trillion dollars to 6.20 trillion dollars

a. Let \( x = 0 \) represent July 1998, \( x = 1 \) represent August 1998, ..., and \( x = 12 \) represent July 1999. Write a linear equation for Americans' spending in terms of the month \( x \)

b. Use the equation in (a) to predict Americans' spending in July 2002.

c. Based on the model created in (a) when would the aggregate expenditure exceed 10 trillion dollars?

d. What part of the US GDP is spent by the Americans in 2013?

- **Name of Task: Publishing Cost:** NJCCCS: 9.1.12.A.4; W.11-12.1

A publishing company estimates that the average cost (in dollars) for one copy of a new scenic calendar it plans to produce can be approximated by the function

\[
C(x) = \frac{2.25x + 275}{x}
\]

Where \( x \) is the number of calendars printed.

a. Find the average cost per calendar when the company prints 100 calendars.

b. Identify the domain and range of this function.

c. After analyzing the function, Alex said that this company should not be allowed to publish zero calendars. As a result, the company has no option to shut down and go out of business. Write an argument to support or reject Alex’s conclusion.

Science Connection:

- **Name of Task: Myoglobin and Hemoglobin:** NJCCCS: HS-LS1-2; HS-LS1-4
Interdisciplinary Connections

Myoglobin and hemoglobin are oxygen-carrying molecules in the human body. Hemoglobin is found inside red blood cells, which flow from the lungs to the muscles through the bloodstream. Myoglobin is found in muscle cells. The function \( Y = M(p) = \frac{p}{1+p} \) calculates the fraction of myoglobin saturated with oxygen at a given pressure \( p \) Torrs. For example, at a pressure of 1 Torr, \( M(1) = 0.5 \), which means half of the myoglobin (i.e. 50%) is oxygen saturated. (Note: More precisely, you need to use something called the “partial pressure”, but the distinction is not important for this problem.) Likewise, the function calculates the fraction of hemoglobin saturated with oxygen at a given pressure \( p \). [UW]

a. The graphs of \( M(P) \) and \( H(P) \) are given here on the domain \( 0 \leq p \leq 100 \)

![Graph](image)

Which is which?

b. If the pressure in the lungs is 100 Torrs, what is the level of oxygen saturation of the hemoglobin in the lungs?

c. The pressure in an active muscle is 20 Torrs. What is the level of oxygen saturation of myoglobin in an active muscle? What is the level of hemoglobin in an active muscle?

d. Define the efficiency of oxygen transport at a given pressure \( p \) to be \( M(p) - H(p) \). What is the oxygen transport efficiency at 20 Torrs? At 40 Torrs? At 60 Torrs? Sketch the graph of \( M(p) - H(p) \); are there conditions under which transport efficiency is maximized (explain)?
Interdisciplinary Connections

Business Connection:

- **Name of Task: Minimize the metal in a can: NJCCCS: 9.1.12.A.4; W.11-12.1**

A manufacturer wants to manufacture a metal can that holds 1000 cm$^3$ of oil. The can is in the shape of a right cylinder with a radius $r$ and height $h$. Assume the thickness of the material used to make the metal can is negligible.

For each question, include correct units of measurement and round your answers to the nearest tenth. Using your knowledge of volume and surface area of a right cylinder, write a function $S(r)$ that represents the surface area of the cylindrical can in terms of the radius, $r$, of its base. Show in detail your algebraic thinking.

1. Sketch the graph of $S(r)$ and show key features of the graph. State any restriction on the value of $r$ so that it represents the physical model of the can.
2. What dimensions will minimize the quantity of metal needed to manufacture the cylindrical can? Show in detail your mathematical solution.
3. Calculate the minimum value of the function $S(r)$ and interpret the result in the context of the physical model. Show the mathematical steps you used to obtain the answer.

- **Name of Task: Chemco Manufacturing: NJSLS: 9.1.12.A.4; W.11-12.1**

Chemco Manufacturing estimates that its profit $P$ in hundreds of dollars is $P = -4x^2 + 40x + 3$ where $x$ is the number of units produced in thousands.

a. How many units must be produced to obtain the maximum profit?
b. Graph the profit function and identify its vertex.
c. An increase in productivity increased profit by $7 at each quantity sold. What kind of a transformation would model this situation? Show your work graphically and algebraically.
d. A decrease in marginal cost lead to a 4 units increase in the optimum level of production. What kind of a transformation would model this situation? Show your work graphically and algebraically.

* Tasks can be found within the additional task folders*
Enrichment

What is the purpose of Enrichment?

- The purpose of enrichment is to provide extended learning opportunities and challenges to students who have already mastered, or can quickly master, the basic curriculum. Enrichment gives the student more time to study concepts with greater depth, breadth, and complexity.
- Enrichment also provides opportunities for students to pursue learning in their own areas of interest and strengths.
- Enrichment keeps advanced students engaged and supports their accelerated academic needs.
- Enrichment provides the most appropriate answer to the question, “What do you do when the student already knows it?”

Enrichment is…

- Planned and purposeful
- Different, or differentiated, work – not just more work
- Responsive to students’ needs and situations
- A promotion of high-level thinking skills and making connections within content
- The ability to apply different or multiple strategies to the content
- The ability to synthesize concepts and make real world and cross-curricular connections.
- Elevated contextual complexity
- Sometimes independent activities, sometimes direct instruction
- Inquiry based or open ended assignments and projects
- Using supplementary materials in addition to the normal range of resources.
- Choices for students
- Tiered/Multi-level activities with flexible groups (may change daily or weekly)

Enrichment is not…

- Just for gifted students (some gifted students may need intervention in some areas just as some other students may need frequent enrichment)
- Worksheets that are more of the same (busywork)
- Random assignments, games, or puzzles not connected to the content areas or areas of student interest
- Extra homework
- A package that is the same for everyone
- Thinking skills taught in isolation
- Unstructured free time
Assessments

**Required District/State Assessments**
SGO Assessments

**Suggested Formative/Summative Classroom Assessments**
- Describe Learning Vertically
- Identify Key Building Blocks
- Make Connections (between and among key building blocks)
- Short/Extended Constructed Response Items
- Multiple-Choice Items (where multiple answer choices may be correct)
- Drag and Drop Items
- Use of Equation Editor
- Quizzes
- Journal Entries/Reflections/Quick-Writes
- Accountable talk
- Projects
- Portfolio
- Observation
- Graphic Organizers/ Concept Mapping
- Presentations
- Role Playing
- Teacher-Student and Student-Student Conferencing
- Homework
New Jersey Student Learning Standards

A.CED.A.2: Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

A.APR.B.3: Identity zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

A.APR.D.6: Rewrite simple rational expressions in different forms; write \( \frac{a(x)}{b(x)} \) in the form \( q(x) + \frac{r(x)}{b(x)} \), where \( a(x) \), \( b(x) \), \( q(x) \), and \( r(x) \) are polynomials with the degree of \( r(x) \) less than the degree of \( b(x) \), using inspection, long division, or, for the more complicated examples, a computer algebra system.

A.REI.D.10: Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

A.REI.B.4a: Use the method of completing the square to transform any quadratic equation in \( x \) into an equation of the form \((x - p)^2 = q\) that has the same solutions. Derive the quadratic formula from this form.

A.REI.B.4b: Solve quadratic equations by inspection (e.g., for \( x^2 = 49 \)), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as \( a \pm bi \) for real numbers \( a \) and \( b \).

A.SSE.B.3a: Factor a quadratic expression to find real the zeros of the function it defines

A.SSE.B.3b: Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.

F.BF.A.1: Write a function that describes a relationship between two quantities.

F.BF.B.5: Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

F.IF.A.1: Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If \( f \) is a function and \( x \) is an element of its domain, then \( f(x) \) denotes the output of \( f \) corresponding to the input \( x \). The graph of \( f \) is the graph of the equation \( y = f(x) \).

F.IF.A.2: Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
<table>
<thead>
<tr>
<th>New Jersey Student Learning Standards</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>F.IF.B.4</strong>: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.</td>
</tr>
<tr>
<td><strong>F.IF.C.7</strong>: Graph function expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.</td>
</tr>
<tr>
<td><strong>F.IF.C.7a</strong>: Graph linear and quadratic functions and show intercepts, maxima and minima.</td>
</tr>
<tr>
<td><strong>F.IF.C.7d</strong>: Graph rational function, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.</td>
</tr>
<tr>
<td><strong>F.IF.C.8a</strong>: Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.</td>
</tr>
<tr>
<td><strong>F.BF.A.1b</strong>: Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.</td>
</tr>
<tr>
<td><strong>F.BF.A.1c</strong>: Compose functions. For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.</td>
</tr>
<tr>
<td><strong>F.BF.B.4</strong>: Find inverse functions</td>
</tr>
<tr>
<td><strong>N.CN.C.7</strong>: Solve quadratic equations with real coefficients that have complex solutions.</td>
</tr>
<tr>
<td><strong>N.CN.C.9</strong>: Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.</td>
</tr>
<tr>
<td><strong>S.ID.B.6a</strong>: Fit a function to the data; use function fitted to data to solve problems in the context of the data. Use given function or choose a function suggested by the context. <em>Emphasize linear, quadratic, and exponential models.</em></td>
</tr>
<tr>
<td><strong>S.ID.B.6b</strong>: Informally assess the fit of a function by plotting and analyzing residuals.</td>
</tr>
<tr>
<td><strong>S.ID.B.6c</strong>: Fit a linear function for a scatter plot that suggests a linear association.</td>
</tr>
</tbody>
</table>
### Mathematical Practices

1. Make sense of problems and persevere in solving them.

2. Reason abstractly and quantitatively.

3. Construct viable arguments and critique the reasoning of others.

4. Model with mathematics.

5. Use appropriate tools strategically.

6. Attend to precision.

7. Look for and make use of structure.

8. Look for and express regularity in repeated reasoning.
### Course: Pre-Calculus Honors

<table>
<thead>
<tr>
<th>Unit: 1</th>
<th>Topic: Functions and Their Graphs</th>
</tr>
</thead>
</table>

#### NJSLS:

#### Unit Focus:
- Understand the concept of a function and use function notation
- Interpret functions that arise in applications in terms of the context
- Understand the concept of a non-linear function and use function notation
- Interpret non-linear functions that arise in applications in terms of the context
- Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function
- Understand that the zeroes of polynomials are related to the factors of the polynomials and that the zeroes represent one key feature used in graphing polynomial.

#### New Jersey Student Learning Standard(s):

**A.CED.A.2:** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

**A.REI.D.10:** Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

**F.IF.C.7a:** Graph linear and quadratic functions and show intercepts, maxima and minima.

**F.IF.A.1:** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).

**F.IF.A.2:** Use function notations, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

**Student Learning Objective 1:** Write linear equations in various forms and graph linear functions. Identify functions, find their domain and range. Use function notation and evaluate functions abstractly and graphically. Use functions to model and solve real life problems.
<table>
<thead>
<tr>
<th>MPs</th>
<th>Skills, Strategies &amp; Concepts</th>
<th>Essential Understandings/Questions</th>
<th>Tasks/Activities</th>
</tr>
</thead>
<tbody>
<tr>
<td>MP 1</td>
<td>• Find and use the slope of a line to write and graph linear functions.</td>
<td>• What is the general equation of a linear function?</td>
<td>Americans Spending</td>
</tr>
<tr>
<td>MP 2</td>
<td>• Determine if a relationship represents a function.</td>
<td>• How can domain and range of a function be found graphically and algebraically?</td>
<td>Grade of Highway</td>
</tr>
<tr>
<td>MP 3</td>
<td>• Evaluate functions and find their domain and range.</td>
<td>• How can you determine if a relation is a function?</td>
<td>Publishing Cost</td>
</tr>
<tr>
<td>MP 4</td>
<td>• Graph the basic parent functions; including linear, absolute value, quadratic, square root, cube root, with and without the use of technology.</td>
<td>• How are functions and their graphs related?</td>
<td>Ivan’s T-shirt Decoration.</td>
</tr>
<tr>
<td>MP 5</td>
<td>• Create models and use arguments to help in solving real life problems.</td>
<td>• How can technology be used to investigate properties of families of functions and their graphs?</td>
<td>Constructing a Box with No Top</td>
</tr>
<tr>
<td>MP 6</td>
<td></td>
<td>• When would it be useful and/or necessary to solve an equation for another variable within that equation?</td>
<td>Construction Site</td>
</tr>
<tr>
<td>MP 7</td>
<td></td>
<td>• How is solving an inequality similar to solving an equation?</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>• How do you represent the solutions to an inequality in interval notation?</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>• Why is it possible to have no solution or infinitely many solutions to an equation or an inequality?</td>
<td></td>
</tr>
</tbody>
</table>
New Jersey Student Learning Standard(s):
F.IF.B.4: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

A.SSE.B.3b: Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.

F.IF.C.7a: Graph linear and quadratic functions and show intercepts, maxima and minima.

Student Learning Objective 2: Analyze functions by determining intervals on which they are increasing, decreasing, or constant and determining their relative minimum and relative maximum values. Identify even and odd functions.

<table>
<thead>
<tr>
<th>MPs</th>
<th>Skills, Strategies &amp; Concepts</th>
<th>Essential Understandings/ Questions</th>
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</tr>
</thead>
</table>
| MP 2 | • Analyze the graph and identify positive and negative values of y.  
• Find the zeros of the functions. Identify increasing and decreasing intervals.  
• Find the maximum and minimum of the functions in a given interval.  
• Identify even and odd functions | • How can you describe a function using its key features?  
• How can algebraic tools like factoring, distributing, and simplifying rational expressions be used to analyze the behavior of functions?  
• How do the characteristics of graphs relate to their corresponding equations?  
• How can algebra help us get information about a graph from an equation?  
• What are common characteristics of polynomials?  
• What is the best method for graphing polynomials?  
• How can technology assist in the understanding of graphing polynomials? | Odd and Even Functions  
Sum of Even and Odd  
NJD Construction Cost Curve |
New Jersey Student Learning Standard(s):
F-IF.C.7: Graph function expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
F-BF.B.3: Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them

Student Learning Objective 3: Recognize graph of common functions, and use vertical, horizontal shifts, reflections, and non-rigid transformations to graph them.

<table>
<thead>
<tr>
<th>MPs</th>
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</tr>
</thead>
<tbody>
<tr>
<td>MP 2</td>
<td>• Analyze the effect of the coefficients on the graph of a function.</td>
<td>• (Given a verbal description) How would you sketch a graph of this function?</td>
<td>Chemco Manufacturing</td>
</tr>
<tr>
<td>MP 4</td>
<td>• Identify horizontal and vertical shifts.</td>
<td>• How do you write equations and draw graphs for the simple transformations of functions?</td>
<td></td>
</tr>
<tr>
<td>MP 5</td>
<td>• Identify reflections and non-rigid transformations to the graph.</td>
<td>• How can coefficients and constants be used to translate, stretch, and rotate the graphs of functions?</td>
<td></td>
</tr>
<tr>
<td>MP 6</td>
<td></td>
<td></td>
<td></td>
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New Jersey Student Learning Standard(s):
F.BF.A.1b: Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.

F.BF.A.1c: Compose functions. For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.

F.BF.B.5: Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

F.BF.B.4: Find inverse functions.

F.BF.B.4b: (+) Verify by composition that one function is the inverse of another.

F.BF.B.4c: (+) Read values of an inverse function from a graph or a table, given that the function has an inverse.

Student Learning Objective 4: Add, subtract, multiply, and divide functions. Find composition of functions. Determine if graphs are one-to-one and find the inverse of functions.

<table>
<thead>
<tr>
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<th>Tasks/Activities</th>
</tr>
</thead>
<tbody>
<tr>
<td>MP 1</td>
<td>• Find arithmetic combinations and compositions of functions.</td>
<td>• What is a composite function?</td>
<td>Speed</td>
</tr>
<tr>
<td>MP 3</td>
<td>• Find inverse functions.</td>
<td>• How do we create composition of two or more functions?</td>
<td>Myoglobin and Hemoglobin</td>
</tr>
<tr>
<td>MP 5</td>
<td></td>
<td>• What do we know about the domain of composite functions?</td>
<td>Bacteria and Temperature</td>
</tr>
<tr>
<td>MP 6</td>
<td></td>
<td>• How do we decompose a function?</td>
<td>Volume of Balloons</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• How can we verify by composition that one function is the inverse of another?</td>
<td>How Cold Is Oslo?</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• How can you find the inverse of a relation or function?</td>
<td>Insurance Sales</td>
</tr>
<tr>
<td></td>
<td></td>
<td>• How can you determine whether the inverse of a function is a function?</td>
<td></td>
</tr>
</tbody>
</table>
**New Jersey Student Learning Standard(s):**

**S.ID.B.6a:** Fit a function to the data; use function fitted to data to solve problems in the context of the data. Use given function or choose a function suggested by the context. *Emphasize linear, quadratic, and exponential models.*

**S.ID.B.6b:** Informally assess the fit of a function by plotting and analyzing residuals.

**S.ID.B.6c:** Fit a linear function for a scatter plot that suggests a linear association.

**Student Learning Objective 5:** Construct scatter plots and interpret data from linear models. Use scatter plots and a graphing utility to find model for data.

<table>
<thead>
<tr>
<th>MPs</th>
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<th>Essential Understandings/ Questions</th>
<th>Tasks/Activities</th>
</tr>
</thead>
<tbody>
<tr>
<td>MP 1</td>
<td>• Use scatter plots to find models for data.</td>
<td>• How can I use a scatter plot to draw informal inference about the correlation between two variables?</td>
<td>World Population</td>
</tr>
<tr>
<td></td>
<td>• Use technology to create lines of best fit and plot them</td>
<td>• How do I determine an equation for the line of best fit using a graphing calculator?</td>
<td>Americans spending</td>
</tr>
<tr>
<td>MP 3</td>
<td></td>
<td></td>
<td>Light Intensity</td>
</tr>
<tr>
<td>MP 6</td>
<td></td>
<td></td>
<td>Motion Detector</td>
</tr>
</tbody>
</table>
New Jersey Student Learning Standard(s):
A.REI.B.4a: Use the method of completing the square to transform any quadratic equation in $x$ into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.

F.IF.C.7a: Graph linear and quadratic functions and show intercepts, maxima and minima.

**Student Learning Objective 6:** Write quadratic functions in standard and vertex form and use results to sketch graphs of functions.

<table>
<thead>
<tr>
<th>MPs</th>
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<th>Tasks/Activities</th>
</tr>
</thead>
<tbody>
<tr>
<td>MP 1</td>
<td>• Find the coordinate of the vertex.</td>
<td>• What are the advantages of a quadratic function in vertex form? In standard form?</td>
<td>Fireworks</td>
</tr>
<tr>
<td>MP 3</td>
<td>• Write the quadratic functions in both forms.</td>
<td>• How is any quadratic function related to the parent quadratic function?</td>
<td>Effectively Explain and Correct</td>
</tr>
<tr>
<td>MP 5</td>
<td></td>
<td>• How are the real solutions of a quadratic equation related to the graph of the related quadratic function?</td>
<td></td>
</tr>
<tr>
<td>MP 7</td>
<td></td>
<td>• How can a quadratic function be converted from standard to vertex form, and what is the significance of each?</td>
<td></td>
</tr>
</tbody>
</table>
New Jersey Student Learning Standard(s):

A.SSE.B.3a: Factor a quadratic expression to find real the zeros of the function it defines.

A.APR.B.3: Identity zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

F.IF.C.7: Graph function expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

F.IF.C.8a: Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

N.CN.C.7: Solve quadratic equations with real coefficients that have complex solutions.

N.CN.C.9: Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

A.REI.B.4a: Use the method of completing the square to transform any quadratic equation in $x$ into an equation of the form $(x – p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.

F.IF.C.7a: Graph linear and quadratic functions and show intercepts, maxima and minima.

A.REI.B.4b: Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers $a$ and $b$.

**Student Learning Objective 7:** Analyze polynomial functions by finding all zeros including complex ones using the Fundamental Theorem of Algebra, factoring, and graphing them, both manually and using technology.
| MP 1 | • Find all the zeros of the polynomial function including complex zeros. |
| MP 2 | • Find the conjugate of a complex zero. |
| MP 3 | • Factor polynomial functions. |
| MP 5 | • Use Descartes rule of signs and the leading coefficient test to determine the number of zeros. |
| MP 6 | • Identify intervals where the function is increasing or decreasing. |

- **Polynomial Farm**
- **Inscribed Rectangle**
- **Revenue Calculation**

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**New Jersey Student Learning Standard(s):**

A.APR.D.6: Rewrite simple rational expressions in different forms; write \( \frac{a(x)}{b(x)} \) in the form \( q(x) + \frac{r(x)}{b(x)} \), where \( a(x), b(x), q(x), \) and \( r(x) \) are polynomials with the degree of \( r(x) \) less than the degree of \( b(x) \), using inspection, long division, or, for the more complicated examples, a computer algebra system.

**Student Learning Objective 8:** Use long division and synthetic division to divide polynomials.

<table>
<thead>
<tr>
<th>MPs</th>
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<th>Essential Understandings/ Questions</th>
<th>Tasks/Activities</th>
</tr>
</thead>
</table>
| MP 2 | • Divide polynomials using long and synthetic divisions.  
• Remainder theorem | • How do you divide a polynomial by another polynomial and use polynomial division to find the rational and real zeros of polynomials?  
• Why is synthetic division used instead of polynomial long division?  
• How can long division of polynomials be used to find intercepts, asymptotes, and the general behavior of rational functions? | Long Division |
New Jersey Student Learning Standard(s):

F.IF.C.7: Graph function expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

F.BF.A.1: Write a function that describes a relationship between two quantities.

S.ID.B.6a: Fit a function to the data; use function fitted to data to solve problems in the context of the data. Use given function or choose a function suggested by the context. *Emphasize linear, quadratic, and exponential models.*

A.SSE.B.3b: Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.

F.IF.B.4: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

F.IF.C.7a: Graph linear and quadratic functions and show intercepts, maxima and minima.

Student Learning Objective 9: Analyze rational functions by finding all horizontal, vertical and slanted asymptotes and graphing them, both manually and using technology. Use polynomial and rational functions to model and solve real-life problems including ones with minimum and maximum values.

<table>
<thead>
<tr>
<th>MPs</th>
<th>Skills, Strategies &amp; Concepts</th>
<th>Essential Understandings/Questions</th>
<th>Tasks/Activities</th>
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<tbody>
<tr>
<td>MP 2</td>
<td>Find horizontal and vertical asymptotes.</td>
<td>How can I identify the characteristics of a rational function?</td>
<td>Concentrations</td>
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<td>MP 6</td>
<td>Find slanted asymptotes.</td>
<td>How are rational functions related to each other and to inverse functions?</td>
<td>Mario Car Racing</td>
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<td>Analyze the function as ( x ) approaches the vertical asymptotes.</td>
<td>How do rational functions model real-world problems and their solutions?</td>
<td>Minimize The Metal in a Can</td>
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<tr>
<td></td>
<td>Create models and use arguments to help in solving real life problems.</td>
<td>How do polynomial functions model real-world problems and their solutions?</td>
<td>Boyle’s Law</td>
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<td></td>
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<td>Modeling real-world phenomena using algebraic functions, which can be simplified and analyzed with or without graphing technology, is a major application of higher algebra and calculus.</td>
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<tr>
<td>Project 1</td>
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<td><strong>Transformations of Functions</strong></td>
<td><strong>Stomp Rockets! Projectile motion and Quadratic Equations</strong></td>
<td><strong>Functions, Functions Everywhere!</strong></td>
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<tr>
<td><strong>Essential Question:</strong></td>
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<td><strong>Essential Question:</strong></td>
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<tr>
<td>• How can you describe a function using its key features?</td>
<td>• How does the interpretation of a quadratic function facilitate informed decision making in real life context?</td>
<td>• What are the major families of functions?</td>
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<tr>
<td>• How do you write equations and draw graphs for the simple transformation of functions?</td>
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<td>• What do they look like algebraically, graphically, numerically and verbally?</td>
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<tr>
<td><strong>Skills:</strong></td>
<td><strong>Skills:</strong></td>
<td><strong>Skills:</strong></td>
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<tr>
<td>• Recognize graphs of parent functions.</td>
<td>• Analyze data to make informed assertions.</td>
<td>• Make decisions based on analysis and support the decisions made.</td>
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<tr>
<td>• Follow directions and steps precisely to accurately construct the graphs.</td>
<td>• Analyze and interpret a quadratic function.</td>
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</tbody>
</table>

**Honors Project (must complete all)**
## Unit 1 Vocabulary

- algebraic function
- axis of symmetry
- bell-shaped curve
- Change-Of-Base Formula
- common logarithmic function
- common logarithmic model
- complex conjugates
- complex numbers
- composition
- condense
- constant
- constant function
- continuous compounding
- continuous function
- correlation coefficient
- cubic function
- dependent variable
- Descartes’ Rule of Signs
- division algorithm
- domain
- even function
- expand
- exponential decay model
- exponential function
- exponential growth modes
- extrema
- factor
- function
- Fundamental Theorem Of Algebra
- Gaussian model
- general form
- greatest integer function
- horizontal asymptote
- horizontal Line Test
- horizontal shift
- horizontal shrink
- horizontal stretch
- imaginary number
- imaginary unit $i$
- independent variable
- Intermediate Value Theorem
- inverse function
- irreducible
- Leading Coefficient Test
- Linear Factorization Theorem
- linear function
- linear regression equation
- negative correlation
- non-rigid transformations
- odd function
- one-to-one function
- parabola
- parallel line
- perpendicular lines
- piecewise defined function
- point-slope form
- polynomial function
- positive correlation
- power function
- prime
- principal square root
- quadratic function
- range
- rational function
- rational numbers
- Rational Zero Test
- real number
- reflection
- relation
- relative maximum
- relative minimum
- Remainder Theorem
- repeated zero
- rigid transformations
- scatter plot
- sigmoidal curve
- slant (or oblique) asymptote
- Slope
- slope-intercept form
- solution
- standard form
- standard normal distribution
- synthetic division
- transcendental function
- transformations
- upper bound
- variation in sign
- vertex
- vertical asymptote
- Vertical Line Test
- vertical shift
- vertical shrink
- vertical stretch
- x-intercepts
- zero
## References & Suggested Instructional Websites

- [http://www.mathwords.com/index_adv_alg_precal.htm](http://www.mathwords.com/index_adv_alg_precal.htm)
- [http://www.hershey.k12.pa.us/Page/3608](http://www.hershey.k12.pa.us/Page/3608)
- [https://sites.google.com/a/evergreenps.org/ms-griffin-s-math-classes/updates](https://sites.google.com/a/evergreenps.org/ms-griffin-s-math-classes/updates)
- [https://sites.google.com/site/dgrahamcalculus/trigpre-calculus/trig-pre-calculus-worksheets](https://sites.google.com/site/dgrahamcalculus/trigpre-calculus/trig-pre-calculus-worksheets)
- [https://www.ixl.com/math/precalculus](https://www.ixl.com/math/precalculus)
- [https://www.illustrativemathematics.org/](https://www.illustrativemathematics.org/)
- The Data and Story Library (DASL). [http://lib.stat.cmu.edu/DASL/](http://lib.stat.cmu.edu/DASL/)
Field Trip Ideas

**SIX FLAGS GREAT ADVENTURE:** This educational event includes workbooks and special science and math related shows throughout the day. Your students will leave with a better understanding of real world applications of the material they have learned in the classroom. Each student will have the opportunity to experience different rides and attractions linking mathematical and scientific concepts to what they are experiencing.

[www.sixflags.com](http://www.sixflags.com)

**MUSEUM of MATHEMATICS:** Mathematics illuminates the patterns that abound in our world. The National Museum of Mathematics strives to enhance public understanding and perception of mathematics. Its dynamic exhibits and programs stimulate inquiry, spark curiosity, and reveal the wonders of mathematics. The Museum’s activities lead a broad and diverse audience to understand the evolving, creative, human, and aesthetic nature of mathematics.

[www.momath.org](http://www.momath.org)

**LIBERTY SCIENCE CENTER:** An interactive science museum and learning center located in Liberty State Park. The center, which first opened in 1993 as New Jersey's first major state science museum, has science exhibits, the largest IMAX Dome theater in the United States, numerous educational resources, and the original *Hoberman sphere.*

[http://lsc.org/plan-your-visit/](http://lsc.org/plan-your-visit/)