MATHEMATICS

Pre-Calculus: Unit 4
Analytic Geometry and Statistics
Course Philosophy/Description

The high school Pre-Calculus course covers mathematical topics ranging from Basics of Functions to Limits of Functions. It provides opportunities to the students to expand their knowledge base and understanding of mathematics in general. The overarching goal of the course is to build a solid foundation for the students who choose Mathematics, Engineering, Sciences, or Business as their college major and/or career options. The major topics in the course such as, Polynomials, Exponents, Trigonometry, Logarithms, Complex numbers, Series/sequences, and Limits help generate students’ inquiries about the mathematical nature, complexities, and applications of these topics in real-life situations. Students not only acquire new knowledge, but also deepen their topical and overall understanding of the content for future transfer to new situations or other disciplines.

Assessment results from this course may be used for the purpose of placements into Calculus/AP Calculus, Statistics, Physics/AP Physics, or other higher level courses.
**ESL Framework**

This ESL framework was designed to be used by bilingual, dual language, ESL and general education teachers. Bilingual and dual language programs use the home language and a second language for instruction. ESL teachers and general education or bilingual teachers may use this document to collaborate on unit and lesson planning to decide who will address certain components of the SLO and language objective. ESL teachers may use the appropriate leveled language objective to build lessons for ELLs which reflects what is covered in the general education program. In this way, whether it is a pull-out or push-in model, all teachers are working on the same Student Learning Objective connected to the Common Core standard. The design of language objectives are based on the alignment of the World-Class Instructional Design Assessment (WIDA) Consortium’s English Language Development (ELD) standards with the Common Core State Standards (CCSS). WIDA’s ELD standards advance academic language development across content areas ultimately leading to academic achievement for English learners. As English learners are progressing through the six developmental linguistic stages, this framework will assist all teachers who work with English learners to appropriately identify the language needed to meet the requirements of the content standard. At the same time, the language objectives recognize the cognitive demand required to complete educational tasks.

Even though listening and reading (receptive) skills differ from speaking and writing (expressive) skills across proficiency levels the cognitive function should not be diminished. For example, an Entering Level One student only has the linguistic ability to respond in single words in English with significant support from their home language. However, they could complete a Venn diagram with single words which demonstrates that they understand how the elements compare and contrast with each other or they could respond with the support of their native language with assistance from a teacher, para-professional, peer or a technology program.

[http://www.state.nj.us/education/modelcurriculum/ela/ELLOverview.pdf](http://www.state.nj.us/education/modelcurriculum/ela/ELLOverview.pdf)
# Pacing Chart – Unit 4

<table>
<thead>
<tr>
<th>#</th>
<th>Student Learning Objective</th>
<th>NJSLS</th>
<th>Pre-Calculus with Limits Correlation</th>
</tr>
</thead>
</table>
| 1  | • Use the Binomial Theorem to calculate binomial coefficients and write binomial expansions.  
    • Use Pascal’s Triangle to calculate binomial coefficients.  
    • Use counting principles to solve more complicated counting problems. | S.CP.A.2  
    S.CP.B.7  
    S.CP.B.9 | 9.5, 9.6, 9.7 |
| 2  | • Evaluates parametric equations for given values of parameter.  
    • Graph curves that are represented by sets of parametric equations.  
    • Represent complex numbers in rectangular and polar form | N.CN.B.4  
    F.IF.C.7  
    N.VM.A.3 | 10.6, 10.7, 10.8 |
| 3  | • Use the definition of a limit to estimate limits.  
    • Determine whether limits of functions exist.  
    • Use properties of limits to evaluate limits.  
    • Evaluate one-sided limits. | F.IF.A.3  
    F.IF.B.4 | 12.1, 12.2 |
| 4  | • Approximate limits of functions graphically and numerically.  
    • Use a tangent line to approximate the slope of a graph at a point.  
    • Use the limit definition of slope to find exact slopes of graphs. | F.LE.A.1b | 12.3, 12.4 |
Research about Teaching and Learning Mathematics

Structure teaching of mathematical concepts and skills around problems to be solved (Checkly, 1997; Wood & Sellars, 1996; Wood & Sellars, 1997)
Encourage students to work cooperatively with others (Johnson & Johnson, 1975; Davidson, 1990)
Use group problem-solving to stimulate students to apply their mathematical thinking skills (Artzt & Armour-Thomas, 1992)
Students interact in ways that support and challenge one another’s strategic thinking (Artzt, Armour-Thomas, & Curcio, 2008)
Activities structured in ways allowing students to explore, explain, extend, and evaluate their progress (National Research Council, 1999)
There are three critical components to effective mathematics instruction (Shellard & Moyer, 2002):

- Teaching for conceptual understanding
- Developing children’s procedural literacy
- Promoting strategic competence through meaningful problem-solving investigations

Teachers should be:

- Demonstrating acceptance and recognition of students’ divergent ideas.
- Challenging students to think deeply about the problems they are solving, extending thinking beyond the solutions and algorithms required to solve the problem
- Influencing learning by asking challenging and interesting questions to accelerate students’ innate inquisitiveness and foster them to examine concepts further.
- Projecting a positive attitude about mathematics and about students’ ability to “do” mathematics

Students should be:

- Actively engaging in “doing” mathematics
- Solving challenging problems
- Investigating meaningful real-world problems
- Making interdisciplinary connections
- Developing an understanding of mathematical knowledge required to “do” mathematics and connect the language of mathematical ideas with numerical representations
- Sharing mathematical ideas, discussing mathematics with one another, refining and critiquing each other’s ideas and understandings
- Communicating in pairs, small group, or whole group presentations
- Using multiple representations to communicate mathematical ideas
- Using connections between pictures, oral language, written symbols, manipulative models, and real-world situations
- Using technological resources and other 21st century skills to support and enhance mathematical understanding
Mathematics is not a stagnate field of textbook problems; rather, it is a dynamic way of constructing meaning about the world around us, generating knowledge and understanding about the real world every day. Students should be metaphorically rolling up their sleeves and “doing mathematics” themselves, not watching others do mathematics for them or in front of them. (Protheroe, 2007)

**Balanced Mathematics Instructional Model**

Balanced math consists of three different learning opportunities; guided math, shared math, and independent math. Ensuring a balance of all three approaches will build conceptual understanding, problem solving, computational fluency, and procedural fluency. Building a balanced mathematical understanding is the focal point of developing mathematical proficiency. Students should frequently work on rigorous tasks, talk about the math, explain their thinking, justify their answer or process, build models with graphs or charts or manipulatives, and use technology.

When balanced math is used in the classroom it provides students opportunities to:

- solve problems
- make connections between math concepts and real-life situations
- communicate mathematical ideas (orally, visually and in writing)
- choose appropriate materials to solve problems
- reflect and monitor their own understanding of the math concepts
- practice strategies to build procedural and conceptual confidence

Teacher builds conceptual understanding by modeling through demonstration, explicit instruction, and think alouds, as well as guiding students as they practice math strategies and apply problem solving strategies. (whole group or small group instruction)

Teacher and students practice mathematics processes together through interactive activities, problem solving, and discussion. (whole group or small group instruction)

Students practice math strategies independently to build procedural and computational fluency. Teacher assesses learning and reteaches as necessary. (whole group instruction, small group instruction, or centers)
<table>
<thead>
<tr>
<th>Effective Pedagogical Routines/Instructional Strategies</th>
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<tr>
<td><strong>Collaborative Problem Solving</strong></td>
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<tr>
<td>Connect Previous Knowledge to New Learning</td>
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<tr>
<td>Making Thinking Visible</td>
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<tr>
<td><strong>Develop and Demonstrate Mathematical Practices</strong></td>
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<tr>
<td>Inquiry-Oriented and Exploratory Approach</td>
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<tr>
<td>Multiple Solution Paths and Strategies</td>
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<tr>
<td>Use of Multiple Representations</td>
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<tr>
<td>Explain the Rationale of your Math Work</td>
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<tr>
<td>Quick Writes</td>
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<tr>
<td>Pair/Trio Sharing</td>
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<tr>
<td>Turn and Talk</td>
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<tr>
<td>Charting</td>
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<tr>
<td>Gallery Walks</td>
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<tr>
<td>Small Group and Whole Class Discussions</td>
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<tr>
<td>Student Modeling</td>
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<tr>
<td><strong>Analyze Student Work</strong></td>
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<tr>
<td>Identify Student’s Mathematical Understanding</td>
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<tr>
<td>Identify Student’s Mathematical Misunderstandings</td>
</tr>
<tr>
<td>Interviews</td>
</tr>
<tr>
<td>Role Playing</td>
</tr>
<tr>
<td><strong>Anticipate Likely and Possible Student Responses</strong></td>
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<tr>
<td>Diagrams, Charts, Tables, and Graphs</td>
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<tr>
<td>Multiple Response Strategies</td>
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<tr>
<td><strong>Collect Different Student Approaches</strong></td>
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<tr>
<td>Asking Assessing and Advancing Questions</td>
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<tr>
<td>Revoicing</td>
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<tr>
<td>Marking</td>
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<tr>
<td>Recapping</td>
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<tr>
<td>Challenging</td>
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<tr>
<td><strong>Pressing for Accuracy and Reasoning</strong></td>
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<tr>
<td>Maintain the Cognitive Demand</td>
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</tbody>
</table>
# Educational Technology Standards

## Technology Operations and Concepts
- Construct a spreadsheet workbook with multiple worksheets, rename tabs to reflect the data on the worksheet, and use mathematical or logical functions, charts and data from all worksheets to convey the results.

**Example:** Using technology, students are able to generate plots, regression functions, and correlation coefficients, and to simulate many possible outcomes in statistics and probability in a short amount of time.

## Digital Citizenship
- Analyze the capabilities and limitations of current and emerging technology resources and assess their potential to address personal, social, lifelong learning and career needs.

**Example:** Students will be able to use critical thinking skills to plan and conduct, research, manage projects, solve problems, and make informed decisions using appropriate digital tools and resource.

## Research and Information Literacy
- Produce a position statement about a real world problem by developing a systematic plan of investigation with peers and experts synthesizing information from multiple sources.

**Example:** Students will be able to produce a position statement about the real world problem by developing systematic plan of investigation with peers and experts synthesizing information from multiple sources.

## Critical Thinking, Problem Solving, Decision Making
- Evaluate the strengths and limitations of emerging technologies and their impact on educational, career, personal and or social needs.

**Example:** A problem-solving approach will allow students to construct their own ideas about mathematics and to take responsibility for their own learning.
Career Ready Practices describe the career-ready skills that all educators in all content areas should seek to develop in their students. They are practices that have been linked to increase college, career, and life success. Career Ready Practices should be taught and reinforced in all career exploration and preparation programs with increasingly higher levels of complexity and expectation as a student advances through a program of study.

- **CRP2. Apply appropriate academic and technical skills.**
  Career-ready individuals readily access and use the knowledge and skills acquired through experience and education to be more productive. They make connections between abstract concepts with real-world applications, and they make correct insights about when it is appropriate to apply the use of an academic skill in a workplace situation.

  **Example:** Students will use technology to acquire, manipulate, analyze and report data, display and communicate STEM information and apply science and mathematical concepts to the development of plans, processes and projects that address real world problems.

- **CRP4 – Communicate clearly and effectively and with reason**
  Career-ready individuals communicate thoughts, ideas, and action plans with clarity, whether using written, verbal, and/or visual methods. They communicate in the workplace with clarity and purpose to make maximum use of their own and others’ time. They are excellent writers; they master conventions, word choice, and organization, and use effective tone and presentation skills to articulate ideas. They are skilled at interacting with others; they are active listeners and speak clearly and with purpose. Career-ready individuals think about the audience for their communication and prepare accordingly to ensure the desired outcome.

  **Example:** Student will apply critical thinking skills to review information, explain statistical analysis, and to translate, interpret and summarize research and statistical data.

- **CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.**
  Career-ready individuals readily recognize problems in the workplace, understand the nature of the problem, and devise effective plans to solve the problem. They are aware of problems when they occur and take action quickly to address the problem; they thoughtfully investigate the root cause of the problem prior to introducing solutions. They carefully consider the options to solve the problem. Once a solution is agreed upon, they follow through to ensure the problem is solved, whether through their own actions or the actions of others.

  **Example:** Throughout this unit, students work to improve their critical-thinking skills and problem-solving techniques.

- **CRP11. Use technology to enhance productivity.**
Career Ready Practices

Career-ready individuals find and maximize the productive value of existing and new technology to accomplish workplace tasks and solve workplace problems. They are flexible and adaptive in acquiring new technology. They are proficient with ubiquitous technology applications. They understand the inherent risks-personal and organizational-of technology applications, and they take actions to prevent or mitigate these risks.

Example: Students will use digital tools such as TI-83/84/89, Interactive Whiteboard, etc. to access, manage, evaluate, and synthesize information in order to solve problems individually and collaboratively to construct and communicate knowledge.
WIDA Proficiency Levels

At the given level of English language proficiency, English language learners will process, understand, produce or use

<table>
<thead>
<tr>
<th>Level</th>
<th>Language Proficiency</th>
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</thead>
</table>
| 6- Reaching | Specialized or technical language reflective of the content areas at grade level  
• A variety of sentence lengths of varying linguistic complexity in extended oral or written discourse as required by the specified grade level  
• Oral or written communication in English comparable to proficient English peers |
| 5- Bridging | Specialized or technical language of the content areas  
• A variety of sentence lengths of varying linguistic complexity in extended oral or written discourse, including stories, essays or reports  
• Oral or written language approaching comparability to that of proficient English peers when presented with grade level material. |
| 4- Expanding | Specific and some technical language of the content areas  
• A variety of sentence lengths of varying linguistic complexity in oral discourse or multiple, related sentences or paragraphs  
• Oral or written language with minimal phonological, syntactic or semantic errors that may impede the communication, but retain much of its meaning, when presented with oral or written connected discourse, with sensory, graphic or interactive support |
| 3- Developing | General and some specific language of the content areas  
• Expanded sentences in oral interaction or written paragraphs  
• Oral or written language with phonological, syntactic or semantic errors that may impede the communication, but retain much of its meaning, when presented with oral or written, narrative or expository descriptions with sensory, graphic or interactive support |
| 2- Beginning | General language related to the content area  
• Phrases or short sentences  
• Oral or written language with phonological, syntactic, or semantic errors that often impede the communication when presented with one to multiple-step commands, directions, or a series of statements with sensory, graphic or interactive support |
| 1- Entering | Pictorial or graphic representation of the language of the content areas  
• Words, phrases or chunks of language when presented with one-step commands directions, WH-, choice or yes/no questions, or statements with sensory, graphic or interactive support |
# Language Development Supports for English Language Learners To Increase Comprehension and Communication Skills

## Environment
- Welcoming and stress-free
- Respectful of linguistic and cultural diversity
- Honors students' background knowledge
- Sets clear and high expectations
- Includes routines and norms
- Is thinking-focused vs. answer-seeking
- Offers multiple modalities to engage in content learning and to demonstrate understanding
- Includes explicit instruction of specific language targets
- Provides participation techniques to include all learners
- Integrates learning centers and games in a meaningful way
- Provides opportunities to practice and refine receptive and productive skills in English as a new language
- Integrates meaning and purposeful tasks/activities that:
  - Are accessible by all students through multiple entry points
  - Are relevant to students' lives and cultural experiences
  - Build on prior mathematical learning
  - Demonstrate high cognitive demand
  - Offer multiple strategies for solutions
  - Allow for a language learning experience in addition to content

## Sensory Supports
- Real-life objects (realia) or concrete objects
- Physical models
- Manipulatives
- Pictures & photographs
- Visual representations or models such as diagrams or drawings
- Videos & films
- Newspapers or magazines
- Gestures
- Physical movements
- Music & songs

## Graphic Supports
- Graphs
- Charts
- Timelines
- Number lines
- Graphic organizers
- Graphing paper

## Interactive Supports
- In a whole group
- In a small group
- With a partner such as Turn-and-Talk
- In pairs as a group (first, two pairs work independently, then they form a group of four)
- In triads
- Cooperative learning structures such as Think-Pair-Share
- Interactive websites or software
- With a mentor or coach

## Verbal and Textual Supports
- Labeling
- Students' native language
- Modeling
- Repetitions
- Paraphrasing
- Summarizing
- Guiding questions
- Clarifying questions
- Probing questions
- Leveled questions such as What? Where? How? Why?
- Questioning prompts & cues
- Word Banks
- Sentence starters
- Sentence frames
- Discussion frames
- Talk moves, including Wait Time

# Building Equity in Your Teaching Practice

How do the essential questions highlight the connection between the big ideas of the unit and equity in your teaching practice?

<table>
<thead>
<tr>
<th>CONTENT INTEGRATION</th>
<th>KNOWLEDGE CONSTRUCTION</th>
<th>PREJUDICE REDUCTION</th>
<th>EQUITABLE PEDAGOGY</th>
<th>EMPOWERING SCHOOL CULTURE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Teachers use examples and content from a variety of cultures &amp; groups.</td>
<td>Teachers help students understand how knowledge is created and influenced by cultural assumptions, perspectives &amp; biases.</td>
<td>Teachers implement lessons and activities to assert positive images of ethnic groups &amp; improve intergroup relations.</td>
<td>Teachers modify techniques and methods to facilitate the academic achievement of students from diverse backgrounds.</td>
<td>Using the other four dimensions to create a safe and healthy educational environment for all.</td>
</tr>
</tbody>
</table>

This unit / lesson is connected to other topics explored with students.

This unit / lesson provides context to the history of privilege and oppression.

This unit / lesson addresses power relationships.

This unit / lesson helps students to develop research and critical thinking skills.

This curriculum creates windows and mirrors* for students.

This unit / lesson helps students question and unpack biases & stereotypes.

This unit / lesson helps students examine, research and question information and sources.

This unit / lesson challenges dominant perspectives.

This instruction has been modified to meet the needs of each student.

Students feel respected and their cultural identities are valued.

The curriculum encourages discussion and understanding about the groups of people being represented.

Opportunities are provided for student to reflect on their learning and provide feedback.

There are opportunities for students to connect with the community.

My classroom is welcoming and supportive for all students?

I am aware of and sensitive to the needs of my students and their families.

There are effective parent communication systems established. Parents can talk to me about issues as they arise in my classroom.

Culturally Relevant Pedagogy Examples

- **Integrate Relevant Word Problems**: Contextualize equations using word problems that reference student interests and cultures.
  
  **Example**: When learning about probability rules, problems that relate to student interests such as music, sports and art enable the students to understand and relate to the concept in a more meaningful way.

- **Everyone has a Voice**: Create a classroom environment where students know that their contributions are expected and valued.
  
  **Example**: Norms for sharing are established that communicate a growth mindset for mathematics. All students are capable of expressing mathematical thinking and contributing to the classroom community. Students learn new ways of looking at problem solving by working with and listening to each other.

- **Run Problem Based Learning Scenarios**: Encourage mathematical discourse among students by presenting problems that are relevant to them, the school and/or the community.
  
  **Example**: Using a Place Based Education (PBE) model, students explore math concepts such as systems of equations while determining ways to address problems that are pertinent to their neighborhood, school or culture.

- **Encourage Student Leadership**: Create an avenue for students to propose problem solving strategies and potential projects.
  
  **Example**: Students can learn to find the limits of functions by creating problems together and deciding if the problems fit the necessary criteria. This experience will allow students to discuss and explore their current level of understanding by applying the concepts to relevant real-life experiences.

- **Present New Concepts Using Student Vocabulary**: Use student diction to capture attention and build understanding before using academic terms.
  
  **Example**: Teach math vocabulary in various modalities for students to remember. Use multi-modal activities, analogies, realia, visual cues, graphic representations, gestures, pictures and cognates. Directly explain and model the idea of vocabulary words having multiple meanings. Students can create the Word Wall with their definitions and examples to foster ownership.
### Differentiated Instruction

**Accommodate Based on Students Individual Needs: Strategies**

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<thead>
<tr>
<th>Time/General</th>
<th>Processing</th>
<th>Comprehension</th>
<th>Recall</th>
</tr>
</thead>
<tbody>
<tr>
<td>Extra time for assigned tasks</td>
<td>Extra Response time</td>
<td>Precise processes for balanced math instruction model</td>
<td>Teacher-made checklist</td>
</tr>
<tr>
<td>Adjust length of assignment</td>
<td>Have students verbalize steps</td>
<td>Short manageable tasks</td>
<td>Use visual graphic organizers</td>
</tr>
<tr>
<td>Timeline with due dates for reports and projects</td>
<td>Repeat, clarify or reword directions</td>
<td>Brief and concrete directions</td>
<td>Reference resources to promote independence</td>
</tr>
<tr>
<td>Communication system between home and school</td>
<td>Mini-breaks between tasks</td>
<td>Provide immediate feedback</td>
<td>Visual and verbal reminders</td>
</tr>
<tr>
<td>Provide lecture notes/outline</td>
<td>Provide a warning for transitions</td>
<td>Small group instruction</td>
<td>Graphic organizers</td>
</tr>
<tr>
<td></td>
<td>Partnering</td>
<td>Emphasize multi-sensory learning</td>
<td></td>
</tr>
</tbody>
</table>

**Assistive Technology**
- Computer/whiteboard
- Tape recorder
- Video Tape

**Tests/Quizzes/Grading**
- Extended time
- Study guides
- Shortened tests
- Read directions aloud

**Behavior/Attention**
- Consistent daily structured routine
- Simple and clear classroom rules
- Frequent feedback

**Organization**
- Individual daily planner
- Display a written agenda
- Note-taking assistance
- Color code materials
## Differentiated Instruction

### Accommodate Based on Content Needs: Strategies

- Anchor charts to model strategies
- Review Algebra concepts to ensure students have the information needed to progress in understanding
- Pre-teach pertinent vocabulary
- Provide reference sheets that list formulas, step-by-step procedures, theorems, and modeling of strategies
- Word wall with visual representations of mathematical terms
- Teacher modeling of thinking processes involved in solving, graphing, and writing equations
- Introduce concepts embedded in real-life context to help students relate to the mathematics involved
- Record formulas, processes, and mathematical rules in reference notebooks
- Graphing calculator to assist with computations and graphing of trigonometric functions
- Utilize technology through interactive sites to represent nonlinear data
- Graphic organizers to help students interpret the meaning of terms in an expression or equation in context
- Translation dictionary
- Sentence stems to provide additional language support for ELL students.
# Interdisciplinary Connections

*Model interdisciplinary thinking to expose students to other disciplines.*

## Social Studies Connection:
**Name of Task:** Security System at Quail Creek NJSLS: 9.1.12.A.4

Ten homes in the Quail Creek area were identified by a surveyor; you were told that 4 of the homes have security system.

1. If you are to pick 3 homes randomly:
   a. What is the probability that all 3 homes have security system?
   b. What is the probability that only 1 has a security system?

2. If 40% of the homes constructed in the Quail creek area include a security system. Three homes are selected at random:
   a. What is the probability all three of the selected homes have a security system?
   b. What is the probability none of the selected homes have a security system?
   c. What is the probability that at least one has a security system?

3. Create probability distribution tables for part 1 and part 2

## Science Connection:
**Name of Task:** Rabbits Population NJSLS: HS-LS1-2; HS-LS1-4

The population of rabbits over a 2 year period in a certain county is given below

1. Draw a scatter plot of the data
2. Find a logistic regression model for the data. Find the limit of that model as time approaches infinity
3. What can you conclude about the limit of the rabbit population growth in the county?
4. Provide a reasonable explanation for the population growth limit.

**Name of Task:** Rock Toss NJSLS: HS-LS1-2; HS-LS1-4

A rock is thrown straight up from level ground. The velocity of the rock at any time $t$ (sec) is $v(t) = 48 - 32t$ ft/sec

1. Graph the velocity function.
2. At what time does the rock reach its maximum height?
3. Find how far the rock has traveled at its maximum height.
4. Graph the pathway followed by the rock
5. How far away from the launching point would the rock land?

*Tasks can be found within the additional task folders*
# Enrichment

## What is the purpose of Enrichment?

- The purpose of enrichment is to provide extended learning opportunities and challenges to students who have already mastered, or can quickly master, the basic curriculum. Enrichment gives the student more time to study concepts with greater depth, breadth, and complexity.
- Enrichment also provides opportunities for students to pursue learning in their own areas of interest and strengths.
- Enrichment keeps advanced students engaged and supports their accelerated academic needs.
- Enrichment provides the most appropriate answer to the question, “What do you do when the student already knows it?”

## Enrichment is…

- Planned and purposeful
- *Different*, or differentiated, work – not just *more* work
- Responsive to students’ needs and situations
- A promotion of high-level thinking skills and making connections within content
- The ability to apply different or multiple strategies to the content
- The ability to synthesize concepts and make real world and cross-curricular connections
- Elevated contextual complexity
- Sometimes independent activities, sometimes direct instruction
- Inquiry based or open-ended assignments and projects
- Using supplementary materials in addition to the normal range of resources
- Choices for students
- Tiered/Multi-level activities with flexible groups (may change daily or weekly)

## Enrichment is not…

- Just for gifted students (some gifted students may need intervention in some areas just as some other students may need frequent enrichment)
- Worksheets that are more of the same (busywork)
- Random assignments, games, or puzzles not connected to the content areas or areas of student interest
- Extra homework
- A package that is the same for everyone
- Thinking skills taught in isolation
- Unstructured free time
Assessments

Required District/State Assessments
SGO Assessments

Suggested Formative/Summative Classroom Assessments
Describe Learning Vertically
Identify Key Building Blocks
Make Connections (between and among key building blocks)
Short/Extended Constructed Response Items
Multiple-Choice Items (where multiple answer choices may be correct)
Drag and Drop Items
Use of Equation Editor
Quizzes
Journal Entries/Reflections/Quick-Writes
Accountable talk
Projects
Portfolio
Observation
Graphic Organizers/Concept Mapping
Presentations
Role Playing
Teacher-Student and Student-Student Conferencing
Homework
### New Jersey Student Learning Standards

**S.CP.A.2**: Understand that two events $A$ and $B$ are independent if the probability of $A$ and $B$ occurring together is the product of their probabilities, and use this characterization to determine if they are independent.

**S.CP.B.7**: Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, and interpret the answer in terms of the model.

**S.CP.B.9**: Use permutations and combinations to compute probabilities of compound events and solve problems.

**N.CN.B.4**: Represent complex numbers on the complex plain and in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.

**N.VM.A3**: Solve problems involving velocity and other quantities that can be represented by vectors.

**N.VM.B.4**: Add and Subtract vectors.

**N.VM.B.5**: Multiply a vector by a scalar.

**F.IF.A.3**: Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.

**F.IF.B.4**: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing the features. Give a verbal description of the relationship. *Key features include: intercepts, intervals where the function is increasing, decreasing, positive or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*

**F.IF.C.7**: Graph function expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

**F.LE.A.1b**: Recognize situation in which one quantity changes at a constant rate per unit interval relative to another.
<table>
<thead>
<tr>
<th>Mathematical Practices</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Make sense of problems and persevere in solving them.</td>
</tr>
<tr>
<td>2. Reason abstractly and quantitatively.</td>
</tr>
<tr>
<td>3. Construct viable arguments and critique the reasoning of others.</td>
</tr>
<tr>
<td>4. Model with mathematics.</td>
</tr>
<tr>
<td>5. Use appropriate tools strategically.</td>
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<tr>
<td>6. Attend to precision.</td>
</tr>
<tr>
<td>7. Look for and make use of structure.</td>
</tr>
<tr>
<td>8. Look for and express regularity in repeated reasoning.</td>
</tr>
</tbody>
</table>
Course: Pre-Calculus  |  Unit: 4  |  Topic: Analytic Geometry and Statistics

NJSLS:


Unit Focus:

- Use the counting principles and the probability rules to analyze the Binomial Theorem and the binomial probability distribution.
- Introduce parametric and polar forms for writing and graphing equations.
- Find the limits of functions either by using its definition, by approximating it graphically and numerically or by evaluating one-sided ones.
- Use a tangent line to approximate the slope of a graph at a point.
- Find the tangent lines of a function.

New Jersey Student Learning Standard(s):

S.CP.A.2: Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.

S.CP.B.7: Apply the Addition Rule, \( P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) \), and interpret the answer in terms of the model.

S.CP.B.9: Use permutations and combinations to compute probabilities of compound events and solve problems.

Student Learning Objective 1: Use the counting principles and the probability rules to analyze the Binomial Yhoreum and the binomial probability distribution.

<table>
<thead>
<tr>
<th>MPs</th>
<th>Skills, Strategies &amp; Concepts</th>
<th>Essential Understandings/ Questions</th>
<th>Tasks/Activities</th>
</tr>
</thead>
<tbody>
<tr>
<td>MP 2</td>
<td>Calculate basic combinations, permutations, and factorials.</td>
<td>How can the counting principle be used to find sample spaces and probabilities?</td>
<td>Straight Poker</td>
</tr>
<tr>
<td>MP 5</td>
<td>Calculate binomial probability distributions.</td>
<td>What is the difference between permutations and combinations?</td>
<td>Security System at Quail Creek</td>
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<td></td>
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<td>Is It Fair?</td>
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</tbody>
</table>
- Use the Binomial Theorem to expand binomials.
- Analyzing Pascal’s Triangle for its values and symmetries.

- What methods can be used to find binomial coefficients?
- How do you use binomial coefficients to write binomial expansions?
- The Binomial Theorem can be used to expand polynomials and to determine the probability of an event.
- The Binomial Theorem can be applied easily using tools like Pascal’s Triangle.

New Jersey Student Learning Standard(s):

**N.CN.B.4**: Represent complex numbers on the complex plain and in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.

**F.IF.C.7**: Graph function expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

**N.VM.A3**: Solve problems involving velocity and other quantities that can be represented by vectors.

**Student Learning Objective 2**: Evaluates parametric equations for given values of parameter. Graph curves that are represented by sets of parametric equations. Represent complex numbers in rectangular and polar form.

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<tr>
<td>MP 2</td>
<td>Use vector operations to find direction, angles and magnitude.</td>
<td>Functions and relations can be represented using vectors, parametric equations, and polar coordinates.</td>
<td>Amusement Park</td>
</tr>
<tr>
<td>MP 5</td>
<td>Calculate velocity and find a force.</td>
<td>Parametric equations can be used to simulate motion.</td>
<td>Planetary Orbits</td>
</tr>
<tr>
<td></td>
<td>Graph curves represented by parametric equations.</td>
<td>Polar equations can be converted to rectangular form and rectangular</td>
<td></td>
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</tbody>
</table>
Plot points in the polar coordinate plane. Equations can be converted to polar form.

New Jersey Student Learning Standard(s):

F.IF.A.3: Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.

F.IF.B.4: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing the features. Give a verbal description of the relationship. Key features include: intercepts, intervals where the function is increasing, decreasing, positive or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

Student Learning Objective 3: Use the definition of a limit to estimate limits. Determine whether limits of functions exist. Use properties of limits to evaluate limits. Evaluate one-sided limits.

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<tr>
<td>MP 2</td>
<td>• Find limit at infinity.</td>
<td>• What is the limit of a function and how can a limit be used to determine the continuity of a function?</td>
<td>Rock Toss</td>
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<tr>
<td>MP 5</td>
<td>• Computing integral.</td>
<td></td>
<td>Rabbit Population</td>
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<td></td>
<td>• Find left and right limit at a point.</td>
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</tbody>
</table>
New Jersey Student Learning Standard(s):  
F.LE.A.1b: Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

Student Learning Objective 4: Approximate limits of functions graphically and numerically. Use a tangent line to approximate the slope of a graph at a point. Use the limit definition of slope to find exact slopes of graphs.

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| MP 2 MP 5 | • Define a limit.  
• Find the value of a limit graphically and as a table, including limits at infinity.  
• Find the limit algebraically.  
• Find one sided limits graphically and algebraically.  
• Determine the x-values at which a function is continuous/discontinuous. | • What characteristics does a function need to have for a limit to exist?  
• What does it mean to have a limit equal infinity?  
• Why does the slope of a secant line change as x approaches a point?  
• What is rate of change and how can functions and graphs help model it?  
• Is a function continuous at a particular value of x?  
• Is a function considered a continuous function?  
• The average rate of change between two points on a function can be written as a function. | Free Fall on Another Planet |
**Unit 4 Vocabulary**

- derivative
- direct substitution
- directed angle
- directed distance
- dividing out technique
- indeterminate form
- limacon
- limit
- one-sided limit
- orientation of a curve
- parameter
- parametric equations

- parametric mode
- plane curve
- polar axis
- polar coordinate mode
- polar coordinate system
- polar coordinates
- pole
- rationalizing technique
- rectangular for
- secant line
- slope of a graph
- tangent line
<table>
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<tr>
<th>References &amp; Suggested Instructional Websites</th>
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<tbody>
<tr>
<td>• <a href="https://www.ixl.com/standards/new-jersey/math/high-school">https://www.ixl.com/standards/new-jersey/math/high-school</a></td>
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<tr>
<td>• <a href="http://www.mathwords.com/index_adv_alg_precal.htm">http://www.mathwords.com/index_adv_alg_precal.htm</a></td>
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<td>• <a href="https://sites.google.com/site/dgrahamcalculus/trigpre-calculus/trig-pre-calculus-worksheets">https://sites.google.com/site/dgrahamcalculus/trigpre-calculus/trig-pre-calculus-worksheets</a></td>
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<td>• <a href="https://www.illustrativemathematics.org/">https://www.illustrativemathematics.org/</a></td>
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<td>• <a href="https://www.ck12.org/trigonometry/Rectangular-to-Polar-Form-for-Equations/">https://www.ck12.org/trigonometry/Rectangular-to-Polar-Form-for-Equations/</a></td>
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<td>• Statistics Education Web (STEW). <a href="http://www.amstat.org/education/STEW/">http://www.amstat.org/education/STEW/</a></td>
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<td>• The Data and Story Library (DASL). <a href="http://lib.stat.cmu.edu/DASL/">http://lib.stat.cmu.edu/DASL/</a></td>
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## Field Trip Ideas

**SIX FLAGS GREAT ADVENTURE:** This educational event includes workbooks and special science and math related shows throughout the day. Your students will leave with a better understanding of real world applications of the material they have learned in the classroom. Each student will have the opportunity to experience different rides and attractions linking mathematical and scientific concepts to what they are experiencing.

[www.sixflags.com](http://www.sixflags.com)

**MUSEUM of MATHEMATICS:** Mathematics illuminates the patterns that abound in our world. The National Museum of Mathematics strives to enhance public understanding and perception of mathematics. Its dynamic exhibits and programs stimulate inquiry, spark curiosity, and reveal the wonders of mathematics. The Museum’s activities lead a broad and diverse audience to understand the evolving, creative, human, and aesthetic nature of mathematics.

[www.momath.org](http://www.momath.org)

**LIBERTY SCIENCE CENTER:** An interactive science museum and learning center located in Liberty State Park. The center, which first opened in 1993 as New Jersey's first major state science museum, has science exhibits, the largest IMAX Dome theater in the United States, numerous educational resources, and the original *Hoberman sphere*.

[http://lsc.org/plan-your-visit/](http://lsc.org/plan-your-visit/)