Calculus
Unit 3: Derivatives & Application of Derivatives
Course Philosophy/Description

Calculus is a college prep course that introduces students to the four major concepts in calculus: The Limit, The Derivative, The Definite Integral and The Indefinite Integral. This course will prepare students for further study in all branches of higher mathematics, science and related fields. By the end of the course students will have learned Limits and Continuity; the Fundamental Theorems of Calculus; Definition of the Derivative of a Function and Techniques of Differentiation; Applications of the Derivative to maximize or minimize a function; the Chain Rule, Mean Value Theorem; Rate of Change problems; Curve Sketching; Definite and Indefinite integration of algebraic, trigonometric, and transcendental functions. The numerical and graphical procedures covered in this course can be applied to any kind of functions they have encountered in their previous courses. The use of technology reinforces these approaches to confirm and interpret the results. Calculus is a transition course linking the mathematical and algebraic procedures taught in previous classes with the higher-level skills required in post-secondary technical programs.

**PREREQUISITES:**

Before studying Calculus, all students must successfully complete coursework for Algebra I, Geometry, Algebra II, and Pre-Calculus. Students must be familiar with the properties, the algebra, the graphs and the language of functions.
This ESL framework was designed to be used by bilingual, dual language, ESL and general education teachers. Bilingual and dual language programs use the home language and a second language for instruction. ESL teachers and general education or bilingual teachers may use this document to collaborate on unit and lesson planning to decide who will address certain components of the SLO and language objective. ESL teachers may use the appropriate leveled language objective to build lessons for ELLs which reflects what is covered in the general education program. In this way, whether it is a pull-out or push-in model, all teachers are working on the same Student Learning Objective connected to the Common Core standard. The design of language objectives are based on the alignment of the World-Class Instructional Design Assessment (WIDA) Consortium’s English Language Development (ELD) standards with the Common Core State Standards (NJSLS). WIDA’s ELD standards advance academic language development across content areas ultimately leading to academic achievement for English learners. As English learners are progressing through the six developmental linguistic stages, this framework will assist all teachers who work with English learners to appropriately identify the language needed to meet the requirements of the content standard. At the same time, the language objectives recognize the cognitive demand required to complete educational tasks. Even though listening and reading (receptive) skills differ from speaking and writing (expressive) skills across proficiency levels the cognitive function should not be diminished. For example, an Entering Level One student only has the linguistic ability to respond in single words in English with significant support from their home language. However, they could complete a Venn diagram with single words which demonstrates that they understand how the elements compare and contrast with each other or they could respond with the support of their home language (L1) with assistance from a teacher, para-professional, peer or a technology program.

http://www.state.nj.us/education/modelcurriculum/ela/ELLOverview.pdf
<table>
<thead>
<tr>
<th>#</th>
<th>Student Learning Objective</th>
<th>NJSLS</th>
<th>Calculus Correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>• Calculate slopes and derivatives using the definition of derivative.</td>
<td>F-IF.A.1-3, F-IF.B.4 APR.D.6-7 ASSE.A.1a,b F-FT.A.1,3; B.5,6</td>
<td>2.1, 2.2, 2.3, 3.6</td>
</tr>
<tr>
<td></td>
<td>• Graph $f$ from the graph of $f'$, graph $f''$ from the graph of $f$.</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Find where a function is not differentiable and distinguish between corners, cups, discontinuities, and vertical tangents.</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Approximate derivatives numerically and graphically.</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Use rules of differentiation to calculate derivatives, including second and higher order derivatives.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>• Use derivatives to analyze straight-line motion and solve other problems involving rates of change.</td>
<td>F-IF.A.1-3, F-IF.B.4 APR.D.6-7 ASSE.A.1a,b F-FT.A.1,3; B.5,6; C.8</td>
<td>2.4</td>
</tr>
<tr>
<td></td>
<td>• Use the rules for differentiating the six basic trigonometric functions.</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Differentiate composite functions using the Chain Rule.</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Find the slope of parameterized curves.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>• Find the derivatives using implicit differentiation.</td>
<td>F-IF.A.1-3, F-IF.B.4 APR.D.6-7 ASSE.A.1a,b F-FT.A.1,3; B.5,6; C.8</td>
<td>2.5, 2.6</td>
</tr>
<tr>
<td></td>
<td>• Find derivatives using the Power Rule for Rational Powers of $x$.</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Calculate derivatives of functions involving the inverse trigonometric functions.</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Calculate derivatives of exponential and logarithmic functions.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>• Determine the local or global extreme values of a function.</td>
<td>F-IF.A.1-3, F-IF.B.4 APR.D.6-7 ASSE.A.1a,b F-FT.A.1,3; B.5,6; C.8</td>
<td>3.1, 3.2, 3.3, 3.4, 3.7</td>
</tr>
<tr>
<td></td>
<td>• Apply the Mean Value Theorem and find the intervals on which a function is increasing or decreasing.</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Use the First and Second Derivative Test to determine local extreme values of a function.</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Determine the concavity of a function and locate the points of inflection by analyzing the second derivative.</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Solve application problems involving finding minimum or maximum values of functions.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Research about Teaching and Learning Mathematics**
Structure teaching of mathematical concepts and skills around problems to be solved (Checkly, 1997; Wood & Sellars, 1996; Wood & Sellars, 1997)

Encourage students to work cooperatively with others (Johnson & Johnson, 1975; Davidson, 1990)

Use group problem-solving to stimulate students to apply their mathematical thinking skills (Artzt & Armour-Thompson, 1992)

Students interact in ways that support and challenge one another’s strategic thinking (Artzt, Armour-Thomas, & Curcio, 2008)

Activities structured in ways allowing students to explore, explain, extend, and evaluate their progress (National Research Council, 1999)

There are three critical components to effective mathematics instruction (Shellard & Moyer, 2002):

- Teaching for conceptual understanding
- Developing children’s procedural literacy
- Promoting strategic competence through meaningful problem-solving investigations

Teachers should be:

- Demonstrating acceptance and recognition of students’ divergent ideas
- Challenging students to think deeply about the problems they are solving, extending thinking beyond the solutions and algorithms required to solve the problem
- Influencing learning by asking challenging and interesting questions to accelerate students’ innate inquisitiveness and foster them to examine concepts further
- Projecting a positive attitude about mathematics and about students’ ability to “do” mathematics

Students should be:

- Actively engaging in “doing” mathematics
- Solving challenging problems
- Investigating meaningful real-world problems
- Making interdisciplinary connections
- Developing an understanding of mathematical knowledge required to “do” mathematics and connect the language of mathematical ideas with numerical representations
- Sharing mathematical ideas, discussing mathematics with one another, refining and critiquing each other’s ideas and understandings
- Communicating in pairs, small group, or whole group presentations
- Using multiple representations to communicate mathematical ideas
- Using connections between pictures, oral language, written symbols, manipulative models, and real-world situations
- Using technological resources and other 21st century skills to support and enhance mathematical understanding
Mathematics is not a stagnate field of textbook problems; rather, it is a dynamic way of constructing meaning about the world around us, generating knowledge and understanding about the real world every day. Students should be metaphorically rolling up their sleeves and “doing mathematics” themselves, not watching others do mathematics for them or in front of them. (Protheroe, 2007)

**Balanced Mathematics Instructional Model**

Balanced math consists of three different learning opportunities; guided math, shared math, and independent math. Ensuring a balance of all three approaches will build conceptual understanding, problem solving, computational fluency, and procedural fluency. Building conceptual understanding is the focal point of developing mathematical proficiency. Students should frequently work on rigorous tasks, talk about the math, explain their thinking, justify their answer or process, build models with graphs or charts or manipulatives, and use technology.

When balanced math is used in the classroom it provides students opportunities to:

- solve problems
- make connections between math concepts and real-life situations
- communicate mathematical ideas (orally, visually and in writing)
- choose appropriate materials to solve problems
- reflect and monitor their own understanding of the math concepts
- practice strategies to build procedural and conceptual confidence

Teacher builds conceptual understanding by modeling through demonstration, explicit instruction, and think alouds, as well as guiding students as they practice math strategies and apply problem solving strategies. (whole group or small group instruction)

Teacher and students practice mathematics processes together through interactive activities, problem solving, and discussion. (whole group or small group instruction)

Students practice math strategies independently to build procedural and computational fluency. Teacher assesses learning and reteaches as necessary. (whole group instruction, small group instruction, or centers)
<table>
<thead>
<tr>
<th>Effective Pedagogical Routines/Instructional Strategies</th>
</tr>
</thead>
<tbody>
<tr>
<td>Collaborative Problem Solving</td>
</tr>
<tr>
<td>Connect Previous Knowledge to New Learning</td>
</tr>
<tr>
<td>Making Thinking Visible</td>
</tr>
<tr>
<td>Develop and Demonstrate Mathematical Practices</td>
</tr>
<tr>
<td>Inquiry-Oriented and Exploratory Approach</td>
</tr>
<tr>
<td>Multiple Solution Paths and Strategies</td>
</tr>
<tr>
<td>Use of Multiple Representations</td>
</tr>
<tr>
<td>Explain the Rationale of your Math Work</td>
</tr>
<tr>
<td>Quick Writes</td>
</tr>
<tr>
<td>Pair/Trio Sharing</td>
</tr>
<tr>
<td>Turn and Talk</td>
</tr>
<tr>
<td>Charting</td>
</tr>
<tr>
<td>Gallery Walks</td>
</tr>
<tr>
<td>Small Group and Whole Class Discussions</td>
</tr>
<tr>
<td>Student Modeling</td>
</tr>
<tr>
<td>Analyze Student Work</td>
</tr>
<tr>
<td>Identify Student’s Mathematical Understanding</td>
</tr>
<tr>
<td>Identify Student’s Mathematical Misunderstandings</td>
</tr>
<tr>
<td>Interviews</td>
</tr>
<tr>
<td>Role Playing</td>
</tr>
<tr>
<td>Diagrams, Charts, Tables, and Graphs</td>
</tr>
<tr>
<td>Anticipate Likely and Possible Student Responses</td>
</tr>
<tr>
<td>Collect Different Student Approaches</td>
</tr>
<tr>
<td>Multiple Response Strategies</td>
</tr>
<tr>
<td>Asking Assessing and Advancing Questions</td>
</tr>
<tr>
<td>Revoicing</td>
</tr>
<tr>
<td>Marking</td>
</tr>
<tr>
<td>Recapping</td>
</tr>
<tr>
<td>Challenging</td>
</tr>
<tr>
<td>Pressing for Accuracy and Reasoning</td>
</tr>
<tr>
<td>Maintain the Cognitive Demand</td>
</tr>
</tbody>
</table>
## Educational Technology Standards


<table>
<thead>
<tr>
<th>➢ Technology Operations and Concepts</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Construct a spreadsheet workbook with multiple worksheets, rename tabs to reflect the data on the worksheet, and use mathematical or logical functions, charts and data from all worksheets to convey the results.</td>
</tr>
<tr>
<td><strong>Example:</strong> Students will use appropriate digital tools to and software to problem solve, check for accuracy and support answers.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>➢ Digital Citizenship</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Analyze the capabilities and limitations of current and emerging technology resources and assess their potential to address personal, social, lifelong learning and career needs.</td>
</tr>
<tr>
<td><strong>Example:</strong> Students will be able to use critical thinking skills to plan, conduct, research, manage projects, solve problems, and make informed decisions using appropriate digital tools and resources.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>➢ Research and Information Literacy</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Produce a position statement about a real world problem by developing a systematic plan of investigation with peers and experts synthesizing information from multiple sources.</td>
</tr>
<tr>
<td><strong>Example:</strong> Students will produce a position statement about a real world problem by developing a systematic plan of investigation with peers and experts synthesizing information from multiple sources.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>➢ Design: Critical Thinking, Problem Solving, and Decision Making</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Evaluate the strengths and limitations of emerging technologies and their impact on educational, career, personal and or social needs.</td>
</tr>
<tr>
<td><strong>Example:</strong> A problem-solving approach will allow students to construct their own idea about mathematics and to take responsibility for their own learning.</td>
</tr>
</tbody>
</table>
Career Ready Practices

Career Ready Practices describe the career-ready skills that all educators in all content areas should seek to develop in their students. They are practices that have been linked to increase college, career, and life success. Career Ready Practices should be taught and reinforced in all career exploration and preparation programs with increasingly higher levels of complexity and expectation as a student advances through a program of study.

- **CRP2. Apply appropriate academic and technical skills.**
  
  Career-ready individuals readily access and use the knowledge and skills acquired through experience and education to be more productive. They make connections between abstract concepts with real-world applications, and they make correct insights about when it is appropriate to apply the use of an academic skill in a workplace situation.

  **Example:** Students are able to apply concepts in the classroom to solve real-life application problems and analyze their solutions for accuracy.

- **CRP4. Communicate clearly and effectively and with reason.**

  Career-ready individuals communicate thoughts, ideas, and action plans with clarity, whether using written, verbal, and/or visual methods. They communicate in the workplace with clarity and purpose to make maximum use of their own and others’ time. They are excellent writers; they master conventions, word choice, and organization, and use effective tone and presentation skills to articulate ideas. They are skilled at interacting with others; they are active listeners and speak clearly and with purpose. Career-ready individuals think about the audience for their communication and prepare accordingly to ensure the desired outcome.

  **Example:** Students are able to listen, communicate and have constructed arguments to justify their approach and conclusion to a problem.

- **CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.**

  Career-ready individuals readily recognize problems in the workplace, understand the nature of the problem, and devise effective plans to solve the problem. They are aware of problems when they occur and take action quickly to address the problem; they thoughtfully investigate the root cause of the problem prior to introducing solutions. They carefully consider the options to solve the problem. Once a solution is agreed upon, they follow through to ensure the problem is solved, whether through their own actions or the actions of others.

  **Example:** Students are able to utilize concepts learnt in the classroom to analyze problems, create a plan to solve the problem and check their solution for correctness.

- **CRP11. Use technology to enhance productivity.**

  Career-ready individuals find and maximize the productive value of existing and new technology to accomplish workplace tasks and solve workplace problems. They are flexible and adaptive in acquiring new technology. They are proficient with ubiquitous technology applications. They understand the inherent risks—personal and organizational—of technology applications, and they take actions to prevent or mitigate these risks.

  **Example:** Students are able to use a variety of technology such as the TI-84 and online resources to effectively help them in problem solving.
### WIDA Proficiency Levels

At the given level of English language proficiency, English language learners will process, understand, produce or use

<table>
<thead>
<tr>
<th>Level</th>
<th>Description</th>
</tr>
</thead>
</table>
| **6- Reaching** | - Specialized or technical language reflective of the content areas at grade level  
- A variety of sentence lengths of varying linguistic complexity in extended oral or written discourse as required by the specified grade level  
- Oral or written communication in English comparable to proficient English peers |
| **5- Bridging** | - Specialized or technical language of the content areas  
- A variety of sentence lengths of varying linguistic complexity in extended oral or written discourse, including stories, essays or reports  
- Oral or written language approaching comparability to that of proficient English peers when presented with grade level material. |
| **4- Expanding** | - Specific and some technical language of the content areas  
- A variety of sentence lengths of varying linguistic complexity in oral discourse or multiple, related sentences or paragraphs  
- Oral or written language with minimal phonological, syntactic or semantic errors that may impede the communication, but retain much of its meaning, when presented with oral or written connected discourse, with sensory, graphic or interactive support |
| **3- Developing** | - General and some specific language of the content areas  
- Expanded sentences in oral interaction or written paragraphs  
- Oral or written language with phonological, syntactic or semantic errors that may impede the communication, but retain much of its meaning, when presented with oral or written, narrative or expository descriptions with sensory, graphic or interactive support |
| **2- Beginning** | - General language related to the content area  
- Phrases or short sentences  
- Oral or written language with phonological, syntactic, or semantic errors that often impede the communication when presented with one to multiple-step commands, directions, or a series of statements with sensory, graphic or interactive support |
| **1- Entering** | - Pictorial or graphic representation of the language of the content areas  
- Words, phrases or chunks of language when presented with one-step commands directions, WH-, choice or yes/no questions, or statements with sensory, graphic or interactive support |
# Language Development Supports for English Language Learners to Increase Comprehension and Communication Skills

## Environment

- Welcoming and stress-free
- Respectful of linguistic and cultural diversity
- Honors students’ background knowledge
- Sets clear and high expectations
- Includes routines and norms
- Is thinking-focused vs. answer-seeking
- Offers multiple modalities to engage in content learning and to demonstrate understanding
- Includes explicit instruction of specific language targets
- Provides participation techniques to include all learners
- Integrates learning centers and games in a meaningful way
- Provides opportunities to practice and refine receptive and productive skills in English as a new language
- Integrates meaning and purposeful tasks/activities that:
  - Are accessible by all students through multiple entry points
  - Are relevant to students’ lives and cultural experiences
  - Build on prior mathematical learning
  - Demonstrate high cognitive demand
  - Offer multiple strategies for solutions
  - Allow for a language learning experience in addition to content

## Sensory Supports*

- Real-life objects (realia) or concrete objects
- Physical models
- Manipulatives
- Pictures & photographs
- Visual representations or models such as diagrams or drawings
- Videos & films
- Newspapers or magazines
- Gestures
- Physical movements
- Music & songs

## Graphic Supports*

- Graphs
- Charts
- Timelines
- Number lines
- Graphic organizers
- Graphing paper

## Interactive Supports*

- In a whole group
- In a small group
- With a partner such as Turn-and-Talk
- In pairs as a group (first, two pairs work independently, then they form a group of four)
- In triads
- Cooperative learning structures such as Think-Pair-Share
- Interactive websites or software
- With a mentor or coach

## Verbal and Textual Supports

- Labeling
- Students’ native language
- Modeling
- Repetitions
- Paraphrasing
- Summarizing
- Guiding questions
- Clarifying questions
- Probing questions
- Leveled questions such as What? Where? How? Why?
- Questioning prompts & cues
- Word Banks
- Sentence starters
- Sentence frames
- Discussion frames
- Talk moves, including Wait Time

BUILDING EQUITY IN YOUR TEACHING PRACTICE

How do the essential questions highlight the connection between the big ideas of the unit and equity in your teaching practice?

**CONTENT INTEGRATION**
Teachers use examples and content from a variety of cultures & groups.

This unit / lesson is connected to other topics explored with students.

There are multiple viewpoints reflected in the content of this unit / lesson.

The materials and resources are reflective of the diverse identities and experiences of students.

The content affirms students, as well as exposes them to experiences other than their own.

**KNOWLEDGE CONSTRUCTION**
Teachers help students understand how knowledge is created and influenced by cultural assumptions, perspectives & biases.

This unit / lesson provides context to the history of privilege and oppression.

This unit / lesson addresses power relationships.

This unit / lesson help students to develop research and critical thinking skills.

This curriculum creates windows and mirrors* for students.

**PREJUDICE REDUCTION**
Teachers implement lessons and activities to assert positive images of ethnic groups & improve intergroup relations.

This unit / lesson help students question and unpack biases & stereotypes.

This unit / lesson help students examine, research and question information and sources.

The curriculum encourage discussion and understanding about the groups of people being represented.

This unit / lesson challenges dominant perspectives.

**EQUITABLE PEDAGOGY**
Teachers modify techniques and methods to facilitate the academic achievement of students from diverse backgrounds.

The instruction has been modified to meet the needs of each student.

Students feel respected and their cultural identities are valued.

Additional supports have been provided for students to become successful and independent learners.

Opportunities are provided for student to reflect on their learning and provide feedback.

**EMPOWERING SCHOOL CULTURE**
Using the other four dimensions to create a safe and healthy educational environment for all.

There are opportunities for students to connect with the community.

My classroom is welcoming and supportive for all students?

I am aware of and sensitive to the needs of my students and their families.

There are effective parent communication systems established. Parents can talk to me about issues as they arise in my classroom.

---

## Culturally Relevant Pedagogy Examples

- **Call on Each Student**: Encourage each student to share his or her thoughts through call-and-response, keeping the class’s attention in the process.
  
  **Example**: Foster confidence. Make the assessment process less intimidating by offering different ways to demonstrate skills and understanding. For example, avoid handing out quizzes that are purely multiple choice or fill-in-the-blank. Among other question types, mix in problems that involve explaining the step necessary to get to the answer. After, give students time to assess their own progress and performance, helping them focus on growth.

- **Integrate Relevant Word Problems**: Contextualize equations using word problems that reference student interests and cultures.
  
  **Example**: When learning about derivatives grab students’ attention by including optimization problems that are relevant to their interest.

- **Present New Concepts Using Student Vocabulary**: Use student diction to capture attention and build understanding before using academic terms.
  
  **Example**: Teach math vocabulary in various modalities for students to remember. Use multi-modal activities, analogies, realia, visual cues, graphic representations, gestures, pictures and cognates. Directly explain and model the idea of vocabulary words having multiple meanings. Students can create the Word Wall with their definitions and examples to foster ownership.

- **Gamify Lessons**: Appeal to gaming culture by, for example, writing instruction manuals for projects and offering rewards such as badges.
  
  **Example**: Appeal to students’ personal interest to keep them engaged. Create a game to check students understanding of applying the rules of derivatives.
  
  - [https://kahoot.com/](https://kahoot.com/)
  - [http://www.classtools.net/](http://www.classtools.net/)
# Differentiated Instruction

**Accommodate Based on Students Individual Needs: Strategies**

<table>
<thead>
<tr>
<th>Time/General</th>
<th>Processing</th>
<th>Comprehension</th>
<th>Recall</th>
</tr>
</thead>
<tbody>
<tr>
<td>Extra time for assigned tasks</td>
<td>Extra Response time</td>
<td>Precise processes for balanced math instructional model</td>
<td>Teacher-made checklist</td>
</tr>
<tr>
<td>Adjust length of assignment</td>
<td>Have students verbalize steps</td>
<td>Short manageable tasks</td>
<td>Use visual graphic organizers</td>
</tr>
<tr>
<td>Timeline with due dates for reports and projects</td>
<td>Repeat, clarify or reword directions</td>
<td>Brief and concrete directions</td>
<td>Reference resources to promote independence</td>
</tr>
<tr>
<td>Communication system between home and school</td>
<td>Mini-breaks between tasks</td>
<td>Provide immediate feedback</td>
<td>Visual and verbal reminders</td>
</tr>
<tr>
<td>Provide lecture notes/outline</td>
<td>Provide a warning for transitions</td>
<td>Small group instruction</td>
<td>Graphic organizers</td>
</tr>
<tr>
<td></td>
<td>Reading partners</td>
<td>Emphasize multi-sensory learning</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Assistive Technology</th>
<th>Tests/Quizzes/Grading</th>
<th>Behavior/Attention</th>
<th>Organization</th>
</tr>
</thead>
<tbody>
<tr>
<td>Computer/whiteboard</td>
<td>Extended time</td>
<td>Consistent daily structured routine</td>
<td>Individual daily planner</td>
</tr>
<tr>
<td>Tape recorder</td>
<td>Study guides</td>
<td>Simple and clear classroom rules</td>
<td>Display a written agenda</td>
</tr>
<tr>
<td>Video Tape</td>
<td>Shortened tests</td>
<td>Frequent feedback</td>
<td>Note-taking assistance</td>
</tr>
<tr>
<td></td>
<td>Read directions aloud</td>
<td></td>
<td>Color code materials</td>
</tr>
</tbody>
</table>

- **Behavior/Attention**
  - Consistent daily structured routine
  - Simple and clear classroom rules
  - Frequent feedback

- **Organization**
  - Individual daily planner
  - Display a written agenda
  - Note-taking assistance
  - Color code materials
## Differentiated Instruction

**Accommodate Based on Content Needs: Strategies**

- Anchor charts to model strategies
- Review Algebra concepts to ensure students have the information needed to progress in understanding
- Pre-teach pertinent vocabulary
- Provide reference sheets that list formulas, step-by-step procedures, theorems, and modeling of strategies
- Word wall with visual representations of mathematical terms
- Teacher modeling of thinking processes involved in solving, graphing, and writing equations
- Introduce concepts embedded in real-life context to help students relate to the mathematics involved
- Record formulas, processes, and mathematical rules in reference notebooks
- Graphing calculator to assist with computations and graphing of trigonometric functions
- Utilize technology through interactive sites to represent nonlinear data
- Graphic organizers to help students interpret the meaning of terms in an expression or equation in context
- Translation dictionary
- Sentence stems to provide additional language support for ELL students.
Interdisciplinary Connections

Model interdisciplinary thinking to expose students to other disciplines.

Social Studies Connection: NJSLS 6.1.8.B.1.a, 6.1.8.C.1.b, 6.1.8.D.1.a, 6.1.8.B.2.a

Name of Task:

- Landfill


Science Connection: NJSLS HS-PS2-2, HS-PS2-5, HS-PS4-1

Name of Task:

- Water Temperature

http://apcentral.collegeboard.com/apc/public/repository/ap12_calculus_bc_q1.pdf
## Enrichment

**What is the purpose of Enrichment?**

- The purpose of enrichment is to provide extended learning opportunities and challenges to students who have already mastered, or can quickly master, the basic curriculum. Enrichment gives the student more time to study concepts with greater depth, breadth, and complexity.
- Enrichment also provides opportunities for students to pursue learning in their own areas of interest and strengths.
- Enrichment keeps advanced students engaged and supports their accelerated academic needs.
- Enrichment provides the most appropriate answer to the question, “What do you do when the student already knows it?”

**Enrichment is…**

- Planned and purposeful
- *Different*, or differentiated, work – not just *more* work
- Responsive to students’ needs and situations
- A promotion of high-level thinking skills and making connections within content
- The ability to apply different or multiple strategies to the content
- The ability to synthesize concepts and make real world and cross-curricular connections
- Elevated contextual complexity
- Sometimes independent activities, sometimes direct instruction
- Inquiry based or open ended assignments and projects
- Using supplementary materials in addition to the normal range of resources
- Choices for students
- Tiered/Multi-level activities with flexible groups (may change daily or weekly)

**Enrichment is not…**

- Just for gifted students (some gifted students may need intervention in some areas just as some other students may need frequent enrichment)
- Worksheets that are more of the same (busywork)
- Random assignments, games, or puzzles not connected to the content areas or areas of student interest
- Extra homework
- A package that is the same for everyone
- Thinking skills taught in isolation
- Unstructured free time
Assessments

**Required District/State Assessments**
- SGO Assessments

**Suggested Formative/Summative Classroom Assessments**
- Describe Learning Vertically
- Identify Key Building Blocks
- Make Connections (between and among key building blocks)
- Short/Extended Constructed Response Items
- Multiple-Choice Items (where multiple answer choices may be correct)
- Drag and Drop Items
- Use of Equation Editor
- Quizzes
- Journal Entries/Reflections/Quick-Writes
- Accountable talk
- Projects
- Portfolio
- Observation
- Graphic Organizers/Concept Mapping
- Presentations
- Role Playing
- Teacher-Student and Student-Student Conferencing
- Homework
New Jersey Student Learning Standards (NJSLS)

**F-IF.A.1** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If $f$ is a function and $x$ is an element of its domain, then $f(x)$ denotes the output of $f$ corresponding to the input $x$. The graph of $f$ is the graph of the equation $y = f(x)$.

**F-TF.A.3** Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosines, and tangent for $\pi-x$, $\pi+x$, and $2\pi-x$ in terms of their values for $x$, where $x$ is any real number.

**F-IF.B.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

**F-IF.B.5** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.

**APR.D.6** Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.

**APR.D.7** Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression.

**A-SSE.1a** Interpret parts of an expression, such as terms, factors, and coefficients.

**A-SSE.1b** Interpret complicated expressions by viewing one or more of their parts as a single entity.

**A-SSE.A.2** Use the structure of an expression to identify ways to rewrite it. For example, see $x^2 - y^2$ as $(x+y)(x-y)$, thus recognizing it as a difference of squares that can be factored $(x+y)^2(x^2+y^2)$

**F-TF.A.1** Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
**New Jersey Student Learning Standards (NJSLS)**

<table>
<thead>
<tr>
<th>Standard</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>F-TF.A.3</strong></td>
<td>Use special triangles to determine geometrically the values of sine, cosine, tangent for (\pi/3), (\pi/4) and (\pi/6), and use the unit circle to express the values of sine, cosine, and tangent for (x), (\pi + x), and (2\pi - x) in terms of their values for (x), where (x) is any real number.</td>
</tr>
<tr>
<td><strong>F-TF.B.5</strong></td>
<td>Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.</td>
</tr>
<tr>
<td><strong>F-TF.B.6</strong></td>
<td>Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.</td>
</tr>
<tr>
<td><strong>F-TF.C.8</strong></td>
<td>Prove the Pythagorean identity (\sin^2(\theta) + \cos^2(\theta) = 1) and use it to find (\sin(\theta)), (\cos(\theta)), or (\tan(\theta)) given (\sin(\theta)), (\cos(\theta)), or (\tan(\theta)) and the quadrant of the angle.</td>
</tr>
</tbody>
</table>
Mathematical Practices for Calculus

**MP 1: Make sense of problems and persevere in solving them.**
Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary.

**MP 2: Reason abstractly and quantitatively.**
Mathematically proficient students make sense of quantities and their relationships in problem situations. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

**MP 3: Construct viable arguments and critique the reasoning of others.**
Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose.

**MP 4: Model with mathematics.**
Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later.

**MP 5: Use appropriate tools strategically.**
Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software.
Mathematical Practices for Calculus

MP 6: Attend to precision.  
Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately.

MP 7: Look for and make use of structure.  
Mathematically proficient students look closely to discern a pattern or structure. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects.

MP 8: Look for and express regularity in repeated reasoning.  
Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.
Grade: 11/12 | Unit: 3 (Three) | Topic: Derivatives & Applications of Derivatives

**Unit 1 NJSLs:** F-IF.A.1, F-IF.A.3, F-IF.B.4, APR.D.6-7, ASSE.A.1a, ASSE.A.1b, F-FT.A.1,3; B.5,6;

**Student Learning Objective (SLO 1):**
- Calculate slopes and derivatives using the definition of derivative.
- Graph $f$ from the graph of $f'$, graph $f'$ from the graph of $f$.
- Find where a function is not differentiable and distinguish between corners, cups, discontinuities, and vertical tangents.
- Approximate derivatives numerically and graphically.
- Use rules of differentiation to calculate derivatives, including second and higher order derivatives.

**New Jersey Student Learning Standards (NJSLS):** F-IF.A.1, F-IF.A.3, F-IF.B.4, APR.D.1-7, ASSE.A.1a,b, F-FT.A.1,3; B.5,6

**F-IF.A.1:** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If $f$ is a function and $x$ is an element of its domain, then $f(x)$ denotes the output of $f$ corresponding to the input $x$. The graph of $f$ is the graph of the equation $y = f(x)$.

**F-IF.A.3:** Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosines, and tangent for $\pi-x$, $\pi+x$, and $2\pi-x$ in terms of their values for $x$, where $x$ is any real number.

**F-IF.B.4:** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

**APR.D.6:** Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.

**APR.D.7:** Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression.

**A-SSE.1a:** Interpret parts of an expression, such as terms, factors, and coefficients.

**A-SSE.1b:** Interpret complicated expressions by viewing one or more of their parts as a single entity.
F-TF.A.1: Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.

F-TF.A.3: Use special triangles to determine geometrically the values of sine, cosine, tangent for \(\pi/3\), \(\pi/4\) and \(\pi/6\), and use the unit circle to express the values of sine, cosine, and tangent for \(x\), \(\pi + x\), and \(2\pi - x\) in terms of their values for \(x\), where \(x\) is any real number.

F-TF.B.5: Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.

F-TF.B.6: Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.

<table>
<thead>
<tr>
<th>MPs</th>
<th>Skills, Strategies &amp; Concepts</th>
<th>Essential Understandings/Questions</th>
<th>Tasks/Activities</th>
</tr>
</thead>
<tbody>
<tr>
<td>• MP 1</td>
<td>Students will learn about: • Calculate slopes and derivatives using the definition of the derivative.</td>
<td>• The derivative gives the value of the slope of the tangent line to a curve at a point.</td>
<td>• Derivatives of a Function Part 1</td>
</tr>
<tr>
<td>• MP 2</td>
<td></td>
<td>• Learn to calculate derivatives using the definition.</td>
<td>• Derivatives of a Function Part 2</td>
</tr>
<tr>
<td>• MP 3</td>
<td>• Graph (f) from the graph (f'), graph (f') from the graph of (f), and graph the derivative of a function given numerically with data.</td>
<td>• The derivative of a function is defined as the limit of a difference quotient and can be determined using a variety of strategies.</td>
<td>• Matching Functions with Derivatives</td>
</tr>
<tr>
<td>• MP 4</td>
<td>• Find where a function is not differentiable and distinguish between corners, cups discontinuities, and vertical tangents.</td>
<td>• Graphs of differentiable functions can be approximated by their tangent lines at points where the derivative exists.</td>
<td>• Differentiability</td>
</tr>
<tr>
<td>• MP 8</td>
<td>• Approximate derivatives numerically and graphically.</td>
<td>• Rules of derivatives of functions are analytically more efficiently</td>
<td>• Differentiation Day 1</td>
</tr>
<tr>
<td></td>
<td>• Use rules of differentiation to calculate derivatives, including second and higher order derivatives.</td>
<td></td>
<td>• Differentiation Day 2</td>
</tr>
</tbody>
</table>

* SEE UNIT 3 TASK FOLDER
• Use derivatives to calculate the instantaneous rate of change.

• Common errors—when calculating derivatives using the definition, students often make errors in evaluating and simplifying the numerator of the difference quotient. When \( f(x) \) is a polynomial or rational function, \( h \) is always a factor of the simplified expression.

---

<table>
<thead>
<tr>
<th>Grade: 11/12</th>
<th>Unit: 3 (Three)</th>
<th>Topic: Derivatives &amp; Applications of Derivatives</th>
</tr>
</thead>
</table>

**Student Learning Objective (SLO 2):**

- Use derivatives to analyze straight-line motion and solve other problems involving rates of change.
- Use the rules for differentiating the six basic trigonometric functions.
- Differentiate composite functions using the Chain Rule.
- Find the slope of parameterized curves.

**New Jersey Student Learning Standards (NJSLS):** F-IF.A.1, F-IF.A.3, F-IF.B.4, APR.D.1-7, ASSE.A.1a,b, F-FT.A.1,3; B.5,6; C.8

- **F-IF.A.1:** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If \( f \) is a function and \( x \) is an element of its domain, then \( f(x) \) denotes the output of \( f \) corresponding to the input \( x \). The graph of \( f \) is the graph of the equation \( y = f(x) \).

- **F-IF.A.3:** Use special triangles to determine geometrically the values of sine, cosine, tangent for \( \pi/3, \pi/4 \) and \( \pi/6 \), and use the unit circle to express the values of sine, cosines, and tangent for \( \pi-x, \pi+x, \) and \( 2\pi-x \) in terms of their values for \( x \), where \( x \) is any real number.

- **F-IF.B.4:** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
APR.D.6: Rewrite simple rational expressions in different forms; write \( \frac{a(x)}{b(x)} \) in the form \( q(x) + \frac{r(x)}{b(x)} \), where \( a(x) \), \( b(x) \), \( q(x) \), and \( r(x) \) are polynomials with the degree of \( r(x) \) less than the degree of \( b(x) \), using inspection, long division, or, for the more complicated examples, a computer algebra system.

APR.D.7: Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression.

A-SSE.1a: Interpret parts of an expression, such as terms, factors, and coefficients.

A-SSE.1b: Interpret complicated expressions by viewing one or more of their parts as a single entity.

F-TF.A.1: Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.

F-TF.A.3: Use special triangles to determine geometrically the values of sine, cosine, tangent for \( \frac{\pi}{3} \), \( \frac{\pi}{4} \) and \( \frac{\pi}{6} \), and use the unit circle to express the values of sine, cosine, and tangent for \( x \), \( \pi + x \), and \( 2\pi - x \) in terms of their values for \( x \), where \( x \) is any real number.

F-TF.B.5: Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.

F-TF.B.6: Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.

F-TF.C.8: Prove the Pythagorean identity \( \sin^2(\theta) + \cos^2(\theta) = 1 \) and use it to find \( \sin(\theta) \), \( \cos(\theta) \), or \( \tan(\theta) \) given \( \sin(\theta) \), \( \cos(\theta) \), or \( \tan(\theta) \) and the quadrant of the angle.

<table>
<thead>
<tr>
<th>MPs</th>
<th>Skills, Strategies &amp; Concepts</th>
<th>Essential Understandings/Questions</th>
<th>Tasks/Activities</th>
</tr>
</thead>
<tbody>
<tr>
<td>• MP 1</td>
<td>Students will learn about: • Use derivatives to analyze straight-line motion and solve other problems involving rates of change.</td>
<td>• Derivatives give the rates at which things change in the world. • Acceleration due to gravity can be either a positive or a negative number, depending on</td>
<td>• The Chain Rule 1 &amp; 2</td>
</tr>
<tr>
<td>• MP 2</td>
<td>• Use rules to differentiate the six trigonometric functions.</td>
<td></td>
<td>• Velocity &amp; Other Rates of Change 1 &amp; 2</td>
</tr>
<tr>
<td>• MP 3</td>
<td></td>
<td></td>
<td>• Velocity of a Particle</td>
</tr>
<tr>
<td>• MP 4</td>
<td></td>
<td></td>
<td>• Derivatives of Trigonometry Functions 1 &amp; 2</td>
</tr>
<tr>
<td>• MP 8</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
- Differentiate composite functions using the Chain Rule.
- Find slopes of parameterized curve.
- Common errors 1- students have trouble distinguishing between speed and velocity.
- Common errors 2- Students sometimes forget to or misapply the basic trigonometric identities. You may wish to review the reciprocal, Pythagorean, angle sum, and half-angle identities.
- Common errors 3- In applying the outside-inside rule to differentiate $f(g(x))$, a common mistake is to omit the factor $g'(x)$ in the answer.

- how one defines a coordinate system.
- Velocity is the rate of change of position, and acceleration is the rate of change of velocity.
- The derivatives of sines and cosines play a key role in describing periodic change.
- Chain Rule is the most widely used differentiation rule in mathematics.
- Direct application of the definition of the derivative can be used to find the derivative for selected functions, including polynomial, power, sine, cosine, exponential, and logarithmic functions.
- The Chain Rule provides a way to differentiate composite functions.
- The Chain Rule is the basis for implicit differentiation and find the derivative of inverse functions.

- Mixed Review

* SEE UNIT 3 TASK FOLDER
<table>
<thead>
<tr>
<th>Grade: 11/12</th>
<th>Unit: 3 (Three)</th>
<th>Topic: Derivatives &amp; Applications of Derivatives</th>
</tr>
</thead>
</table>

**Student Learning Objective (SLO 3):**
- Find the derivatives using implicit differentiation.
- Find derivatives using the Power Rule for Rational Powers of $x$.
- Calculate derivatives of functions involving the inverse trigonometric functions.
- Calculate derivatives of exponential and logarithmic functions.

**New Jersey Student Learning Standards (NJSLS):** F-IF.A.1, F-IF.A.3, F-IF.B.4, APR.D.1-7, ASSE.A.1a,b, F-FT.A.1,3; B.5,6; C.8

**F-IF.A.1:** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If $f$ is a function and $x$ is an element of its domain, then $f(x)$ denotes the output of $f$ corresponding to the input $x$. The graph of $f$ is the graph of the equation $y = f(x)$.

**F-IF.A.3:** Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosines, and tangent for $\pi - x$, $\pi + x$, and $2\pi - x$ in terms of their values for $x$, where $x$ is any real number.

**F-IF.B.4:** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

**APR.D.6:** Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.

**APR.D.7:** Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression.

**A-SSE.1a:** Interpret parts of an expression, such as terms, factors, and coefficients.

**A-SSE.1b:** Interpret complicated expressions by viewing one or more of their parts as a single entity.

**F-FT.A.1:** Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
**F-TF.A.3:** Use special triangles to determine geometrically the values of sine, cosine, tangent for \( \pi/3, \pi/4 \) and \( \pi/6 \), and use the unit circle to express the values of sine, cosine, and tangent for \( x, \pi + x, \) and \( 2\pi - x \) in terms of their values for \( x \), where \( x \) is any real number.

**F-TF.B.5:** Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.

**F-TF.B.6:** Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.

**F-TF.C.8:** Prove the Pythagorean identity \( \sin^2(\theta) + \cos^2(\theta) = 1 \) and use it to find \( \sin(\theta), \cos(\theta), \) or \( \tan(\theta) \) given \( \sin(\theta), \cos(\theta), \) or \( \tan(\theta) \) and the quadrant of the angle.

<table>
<thead>
<tr>
<th>MPs</th>
<th>Skills, Strategies &amp; Concepts</th>
<th>Essential Understandings/Questions</th>
<th>Tasks/Activities</th>
</tr>
</thead>
<tbody>
<tr>
<td>• MP 1</td>
<td>Students will learn about:</td>
<td>• Implicit differentiation allows us to find derivatives of functions that are not defined or written explicitly as a function of a single variable.</td>
<td>• Implicit Differentiation 1-2</td>
</tr>
<tr>
<td>• MP 2</td>
<td>• Find derivatives using implicit differentiation.</td>
<td>• Implicit differentiation is a powerful and important technique. It is used in application.</td>
<td>• Implicit-Inverse Trig Practice</td>
</tr>
<tr>
<td>• MP 3</td>
<td>• Find derivatives using Power Rule for Rational Powers of ( x ).</td>
<td>• The relationship between the graph of a function and its inverse allows us to see the relationship between their derivatives.</td>
<td>• Derivatives of Inverse Functions</td>
</tr>
<tr>
<td>• MP 4</td>
<td>• Calculate derivatives of functions involving inverse functions.</td>
<td>• The relationship between exponential and logarithmic functions provides a powerful</td>
<td>• Derivatives of Logs &amp; Exponential Functions</td>
</tr>
<tr>
<td>• MP 8</td>
<td>• Calculate derivatives of exponential and logarithmic functions.</td>
<td></td>
<td>SEE UNIT 3 TASK FOLDER</td>
</tr>
<tr>
<td></td>
<td>• You may wish to begin this lesson with informal discussion about graphs of equations of the form ( F(x, y) = c ). Use equations whose graphs can be displayed on a graphing utility, in either parametric mode or rectangular mode, by graphing the separate branches in the same window.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
• Discuss the relationship between slopes of linear functions and their inverses.

• Help students understand how derivatives of inverse functions are derived.

• A discussion of logarithmic differentiation is a good way to conclude this lesson. Make sure students understand why the derivative of \( y = x^x \) cannot be found directly using the Power Rule.

• Common errors - It is easy to make mistakes in taking derivatives, which require the Product Rule and/or the Chain Rule. Students forget to use the Product Rule when differentiating the term \( xy \).

<table>
<thead>
<tr>
<th>Grade: 11/12</th>
<th>Unit: 3 (Three)</th>
<th>Topic: Derivatives &amp; Applications of Derivatives</th>
</tr>
</thead>
</table>

**Student Learning Objective (SLO 4):**
- Determine the local or global extreme values of a function.
- Apply the Mean Value Theorem and find the intervals on which a function is increasing or decreasing.
- Use the First and Second Derivative Test to determine local extreme values of a function.
- Determine the concavity of a function and locate the points of inflection by analyzing the second derivative.
- Solve application problems involving finding minimum or maximum values of functions.

**New Jersey Student Learning Standards (NJSLS):** F-IF.A.1, F-IF.A.3, F-IF.B.4, APR.D.1-7, ASSE.A.1a,b, F-FT.A.1,3; B.5,6; C.8
F-IF.A.1: Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If \( f \) is a function and \( x \) is an element of its domain, then \( f(x) \) denotes the output of \( f \) corresponding to the input \( x \). The graph of \( f \) is the graph of the equation \( y = f(x) \).

F-IF.A.3: Use special triangles to determine geometrically the values of sine, cosine, tangent for \( \pi/3, \pi/4 \) and \( \pi/6 \), and use the unit circle to express the values of sine, cosines, and tangent for \( \pi-x, \pi+x, \) and \( 2\pi-x \) in terms of their values for \( x \), where \( x \) is any real number.

F-IF.B.4: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

APR.D.6: Rewrite simple rational expressions in different forms; write \( a(x)/b(x) \) in the form \( q(x) + r(x)/b(x) \), where \( a(x), b(x), q(x), \) and \( r(x) \) are polynomials with the degree of \( r(x) \) less than the degree of \( b(x) \), using inspection, long division, or, for the more complicated examples, a computer algebra system.

APR.D.7: Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression.

A-SSE.1a: Interpret parts of an expression, such as terms, factors, and coefficients.

A-SSE.1b: Interpret complicated expressions by viewing one or more of their parts as a single entity.

F-TF.A.1: Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.

F-TF.A.3: Use special triangles to determine geometrically the values of sine, cosine, tangent for \( \pi/3, \pi/4 \) and \( \pi/6 \), and use the unit circle to express the values of sine, cosines, and tangent for \( x, \pi+x, \) and \( 2\pi-x \) in terms of their values for \( x \), where \( x \) is any real number.

F-TF.B.5: Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.

F-TF.B.6: Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.

F-TF.C.8: Prove the Pythagorean identity \( \sin^2(\theta) + \cos^2(\theta) = 1 \) and use it to find \( \sin(\theta), \cos(\theta), \) or \( \tan(\theta) \) given \( \sin(\theta), \cos(\theta), \) or \( \tan(\theta) \) and the quadrant of the angle.
<table>
<thead>
<tr>
<th>MPs</th>
<th>Skills, Strategies &amp; Concepts</th>
<th>Essential Understandings/Questions</th>
<th>Tasks/Activities</th>
</tr>
</thead>
<tbody>
<tr>
<td>MP 1</td>
<td>Students will learn about:</td>
<td>• Finding maximum and minimum values of functions, called optimization, is an important issue in real-world problems.</td>
<td>• Extreme Values of Functions 1 &amp; 2</td>
</tr>
<tr>
<td>MP 2</td>
<td>Determine the local and global extreme values of a function.</td>
<td>• The Mean Value Theorem is an important theoretical to connect the average and instantaneous rates of change.</td>
<td>• Mean Value Theorem 1-3</td>
</tr>
<tr>
<td>MP 3</td>
<td>Apply the Mean Value Theorem and find the intervals on which a function is increasing or decreasing.</td>
<td>• Differential calculus is a powerful problem-solving tool precisely because of its usefulness for analyzing functions.</td>
<td>• Connecting Graphs 1 &amp; 2</td>
</tr>
<tr>
<td>MP 4</td>
<td>Use the First and Second Derivative Test to determine the local extreme values of a function.</td>
<td>• Historically, optimization problems were among the earliest applications of what we now call differential calculus.</td>
<td>• Optimization 1-4</td>
</tr>
<tr>
<td>MP 8</td>
<td>Determine the concavity of a function and locate the points of inflection by analyzing the second derivatives.</td>
<td>• It’s a Match Up</td>
<td>• Application of Derivatives</td>
</tr>
<tr>
<td></td>
<td>Graph $f$ using information about $f'$.</td>
<td>* SEE UNIT 3 TASK FOLDER</td>
<td></td>
</tr>
<tr>
<td></td>
<td>One way to begin the lesson is to sketch an arbitrary function and discuss the local and global (absolute) minima and maxima of the function.</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>You can motivate the First Derivative Test presenting several graphs and having students discuss the derivatives and how they relate to the extreme values of the function.</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Common errors 1- Students will neglect to find the points where the derivative is undefined.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
- Common error 2- Students have trouble with the use of closed intervals in describing where a function is increasing or decreasing.
<table>
<thead>
<tr>
<th>Vocabulary</th>
</tr>
</thead>
</table>
| • Area below the curve  
• Area between the curve  
• Area under the curve  
• Asymptote  
• Average rate of change  
• Axis of rotation  
• Bounds of integration  
• Chain rule  
• Concave up  
• Concave down  
• Continuous function  
• Continuous differentiable function  
• Critical number  
• Critical point  
• Critical value  
• Curve sketching  
• Cylindrical Shell Method  
• Decreasing function  
• Definite Integral  
• Definite integral rules  
• Derivative  
• Derivative of a Power Series  
• Derivative Rules  
• Discontinuity  
• Disk method  
• Divergent Sequence and Series  
• Explicit Differentiation  
• Extreme Value Theorem  
• Factorial  

• First Derivative Test  
• Fundamental Theorem of Calculus  
• Higher Derivative  
• Implicit Differentiation  
• Infinite Limit  
• Inflection Point  
• Instantaneous Velocity  
• Integral  
• Integration by parts  
• Integration by substitution  
• Intermediate Value Theorem  
• L’Hopital’s Rule  
• Limit  
• Right Limit/ Left Limit  
• Local maximum/ Local minimum  
• Mean Value Theorem  
• One-sided limit  
• Product Rule  
• Projectile Motion  
• Quotient Rule  
• Relative maximum/ Relative minimum  
• Riemann Sum  
• Rolle’s Theorem  
• Sandwich Theorem  
• U-Substitution  
• Volume by parallel cross section |
References & Suggested Instructional Websites

- [http://www.mathwords.com/index_calculus.htm](http://www.mathwords.com/index_calculus.htm)
- [http://www.hershey.k12.pa.us/Page/3607](http://www.hershey.k12.pa.us/Page/3607)
- [https://sites.google.com/a/evergreenps.org/ms-griffin-s-math-classes/calculus-notes-worksheets-and-classroom-policies](https://sites.google.com/a/evergreenps.org/ms-griffin-s-math-classes/calculus-notes-worksheets-and-classroom-policies)
- [https://sites.google.com/site/dgrahamcalculus/ap-calculus-ab/calculus-worksheets](https://sites.google.com/site/dgrahamcalculus/ap-calculus-ab/calculus-worksheets)
- [http://home.cvc.org/math/apcalc/apcalc.htm](http://home.cvc.org/math/apcalc/apcalc.htm)
- [http://www.mathwithmrwood.com/ap-calculus](http://www.mathwithmrwood.com/ap-calculus)
- [http://www.analyzemath.com/ap_calculus.html](http://www.analyzemath.com/ap_calculus.html)
- [https://online.math.uh.edu/apcalculus/exams/](https://online.math.uh.edu/apcalculus/exams/)
## Field Trips

**SIX FLAGS GREAT ADVENTURE:** This educational event includes workbooks and special science and math related shows throughout the day. Your students will leave with a better understanding of real world applications of the material they have learned in the classroom. Each student will have the opportunity to experience different rides and attractions linking mathematical and scientific concepts to what they are experiencing. [www.sixflags.com](http://www.sixflags.com)

**MUSEUM of MATHEMATICS:** Mathematics illuminates the patterns that abound in our world. The National Museum of Mathematics strives to enhance public understanding and perception of mathematics. Its dynamic exhibits and programs stimulate inquiry, spark curiosity, and reveal the wonders of mathematics. The Museum’s activities lead a broad and diverse audience to understand the evolving, creative, human, and aesthetic nature of mathematics. [www.momath.org](http://www.momath.org)

**LIBERTY SCIENCE CENTER:** An interactive science museum and learning center located in Liberty State Park. The center, which first opened in 1993 as New Jersey's first major state science museum, has science exhibits, the largest IMAX Dome Theater in the United States, numerous educational resources, and the original Hoberman sphere. [http://lsc.org/plan-your-visit/](http://lsc.org/plan-your-visit/)
Marking Period 3 – Suggested Project
* Project begins at the start of the marking period and finishes at the end.

**Optimization Project**
A cylindrical can is to be made to hold 1 L of oil. Find the dimensions that will minimize the cost of the metal to manufacture the can. The can still must hold 1 L of oil, but you want the amount of metal used to make the can to be minimized. Less metal used=money saved. 1L=a cube of liquid with side 10cm. Find the dimensions of the can that will minimize the cost of the metal to manufacture the can.

1) Find the dimensions, showing all work in an organized fashion.

2) Explain in complete sentences how derivatives can help you solve problems in other disciplines such as saving manufacturing costs for a company. Give a specific example.

3) Come up with your own optimization word problem to solve a real world problem in another discipline such as physics or economics, then solve your problem. Research on the internet to get ideas.

* See MP 3 Project folder for full project and rubric.