AP Calculus

Unit 3: Derivatives & Application of Derivatives
Course Philosophy/Description

AP Calculus AB is a course in single-variable calculus that includes techniques and applications of the derivative, techniques and applications of the definite integral, and the Fundamental Theorem of Calculus. It is equivalent to at least a semester of calculus at most colleges and universities, perhaps to a year of calculus at some. Algebraic, numerical, and graphical representations are emphasized throughout the course.

By the end of the course students will have learned Limits and Continuity; the Fundamental Theorems of Calculus; Definition of the Derivative of a Function and Techniques of Differentiation; Applications of the Derivative to maximize or minimize a function; the Chain Rule, Mean Value Theorem; Rate of Change problems; Curve Sketching; Definite and Indefinite integration of algebraic, trigonometric, and transcendental functions.

The connections between numerical, graphical, analytical, and verbal communication of Calculus topics are a major theme in the class along with the overall conceptual understanding of the Calculus topics covered. The use of technology reinforces these approaches to confirm and interpret the results. Calculus is a transition course linking the mathematical and algebraic procedures taught in previous classes with the higher-level skills required in post-secondary technical programs.

PREREQUISITES:

Before studying Calculus, all students must successfully complete coursework for Algebra I, Geometry, Algebra II, and Pre-Calculus. Students must be familiar with the properties, the algebra, the graphs and the language of functions. Attitude prerequisites include a willingness to work both in and out of class, a willingness to collaborate with classmates to foster mutual understanding, and a sincere intent to place out of the first semester of college calculus rather than repeat it.
ESL Framework

This ESL framework was designed to be used by bilingual, dual language, ESL and general education teachers. Bilingual and dual language programs use the home language and a second language for instruction. ESL teachers and general education or bilingual teachers may use this document to collaborate on unit and lesson planning to decide who will address certain components of the SLO and language objective. ESL teachers may use the appropriate leveled language objective to build lessons for ELLs which reflects what is covered in the general education program. In this way, whether it is a pull-out or push-in model, all teachers are working on the same Student Learning Objective connected to the Common Core standard. The design of language objectives are based on the alignment of the World-Class Instructional Design Assessment (WIDA) Consortium’s English Language Development (ELD) standards with the Common Core State Standards (NJSLS). WIDA’s ELD standards advance academic language development across content areas ultimately leading to academic achievement for English learners. As English learners are progressing through the six developmental linguistic stages, this framework will assist all teachers who work with English learners to appropriately identify the language needed to meet the requirements of the content standard. At the same time, the language objectives recognize the cognitive demand required to complete educational tasks. Even though listening and reading (receptive) skills differ from speaking and writing (expressive) skills across proficiency levels the cognitive function should not be diminished. For example, an Entering Level One student only has the linguistic ability to respond in single words in English with significant support from their home language. However, they could complete a Venn diagram with single words which demonstrates that they understand how the elements compare and contrast with each other or they could respond with the support of their home language (L1) with assistance from a teacher, para-professional, peer or a technology program.

http://www.state.nj.us/education/modelcurriculum/ela/ELLOverview.pdf
<table>
<thead>
<tr>
<th>#</th>
<th>Student Learning Objective</th>
<th>NJSLS</th>
<th>Calculus for AP Correlation</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>• Calculate slopes and derivatives using the definition of derivative.</td>
<td>F-IF.A.1-3</td>
<td>2.2, 2.3</td>
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<td></td>
<td>• Graph $f$ from the graph of $f'$, graph $f''$ from the graph of $f$.</td>
<td>F-IF.B.4, APR.D.6-7</td>
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<td></td>
<td>• Find where a function is not differentiable and distinguish between corners, cups,</td>
<td>ASSE.A.1a,b</td>
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<td></td>
<td>discontinuities, and vertical tangents.</td>
<td>F-FT.A.1,3</td>
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<td></td>
<td>• Approximate derivatives numerically and graphically.</td>
<td>F.TF.B.5,6</td>
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<td></td>
<td>• Use rules of differentiation to calculate derivatives, including second and higher</td>
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<td>order derivatives.</td>
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<td>2</td>
<td>• Use derivatives to analyze straight-line motion and solve other problems involving</td>
<td>F-IF.A.1-3, F-IF.B.4,</td>
<td>2.3, 2.4</td>
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<tr>
<td></td>
<td>rates of change.</td>
<td>APR.D.6-7, ASSE.A.1a,b</td>
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<td></td>
<td>• Use the rules for differentiating the six basic trigonometric functions.</td>
<td>F-FT.A.1,3, F.TF.B.5,6</td>
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<td></td>
<td>• Differentiate composite functions using the Chain Rule.</td>
<td>F.TF.C.8</td>
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<td></td>
<td>• Find the slope of parameterized curves.</td>
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<tr>
<td>3</td>
<td>• Find the derivatives using implicit differentiation.</td>
<td>F-IF.A.1-3; F-IF.B.4,</td>
<td>2.5, 2.6</td>
</tr>
<tr>
<td></td>
<td>• Find derivatives using the Power Rule for Rational Powers of $x$.</td>
<td>APR.D.6-7, ASSE.A.1a,b</td>
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<tr>
<td></td>
<td>• Calculate derivatives of functions involving the inverse trigonometric functions.</td>
<td>F-FT.A.1,3, F.TF.B.5,6</td>
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<tr>
<td></td>
<td>• Calculate derivatives of exponential and logarithmic functions.</td>
<td>F.TF.C.8</td>
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<tr>
<td>4</td>
<td>• Determine the local or global extreme values of a function.</td>
<td>F-IF.A.1-3, F-IF.B.4,</td>
<td>2.7, 3.1, 3.2, 3.3,</td>
</tr>
<tr>
<td></td>
<td>• Apply the Mean Value Theorem and find the intervals on which a function is increasing</td>
<td>APR.D.6-7, ASSE.A.1a,b</td>
<td>3.4, 3.5, 3.6</td>
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<td></td>
<td>or decreasing.</td>
<td>F-FT.A.1,3, F.TF.B.5,6</td>
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<td></td>
<td>• Use the First and Second Derivative Test to determine local extreme values of a</td>
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<tr>
<td></td>
<td>function.</td>
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</tbody>
</table>
### Pacing Chart – Unit 3

| • Determine the concavity of a function and locate the points of inflection by analyzing the second derivative. | F.TF.C.8 |
| • Solve application problems involving finding minimum or maximum values of functions. |  |

F.TF.C.8
Research about Teaching and Learning Mathematics

Structure teaching of mathematical concepts and skills around problems to be solved (Checkly, 1997; Wood & Sellars, 1996; Wood & Sellars, 1997)
Encourage students to work cooperatively with others (Johnson & Johnson, 1975; Davidson, 1990)
Use group problem-solving to stimulate students to apply their mathematical thinking skills (Artzt & Armour-Thomas, 1992)
Students interact in ways that support and challenge one another’s strategic thinking (Artzt, Armour-Thomas, & Curcio, 2008)
Activities structured in ways allowing students to explore, explain, extend, and evaluate their progress (National Research Council, 1999)
There are three critical components to effective mathematics instruction (Shellard & Moyer, 2002):

- Teaching for conceptual understanding
- Developing children’s procedural literacy
- Promoting strategic competence through meaningful problem-solving investigations

Teachers should be:

- Demonstrating acceptance and recognition of students’ divergent ideas
- Challenging students to think deeply about the problems they are solving, extending thinking beyond the solutions and algorithms required to solve the problem
- Influencing learning by asking challenging and interesting questions to accelerate students’ innate inquisitiveness and foster them to examine concepts further
- Projecting a positive attitude about mathematics and about students’ ability to “do” mathematics

Students should be:

- Actively engaging in “doing” mathematics
- Solving challenging problems
- Investigating meaningful real-world problems
- Making interdisciplinary connections
- Developing an understanding of mathematical knowledge required to “do” mathematics and connect the language of mathematical ideas with numerical representations
- Sharing mathematical ideas, discussing mathematics with one another, refining and critiquing each other’s ideas and understandings
- Communicating in pairs, small group, or whole group presentations
- Using multiple representations to communicate mathematical ideas
- Using connections between pictures, oral language, written symbols, manipulative models, and real-world situations
- Using technological resources and other 21st century skills to support and enhance mathematical understanding
Mathematics is not a stagnate field of textbook problems; rather, it is a dynamic way of constructing meaning about the world around us, generating knowledge and understanding about the real world every day. Students should be metaphorically rolling up their sleeves and “doing mathematics” themselves, not watching others do mathematics for them or in front of them. (Protheroe, 2007)

Balanced Mathematics Instructional Model

Balanced math consists of three different learning opportunities; guided math, shared math, and independent math. Ensuring a balance of all three approaches will build conceptual understanding, problem solving, computational fluency, and procedural fluency. Building conceptual understanding is the focal point of developing mathematical proficiency. Students should frequently work on rigorous tasks, talk about the math, explain their thinking, justify their answer or process, build models with graphs or charts or manipulatives, and use technology.

When balanced math is used in the classroom it provides students opportunities to:

- solve problems
- make connections between math concepts and real-life situations
- communicate mathematical ideas (orally, visually and in writing)
- choose appropriate materials to solve problems
- reflect and monitor their own understanding of the math concepts
- practice strategies to build procedural and conceptual confidence

Teacher builds conceptual understanding by modeling through demonstration, explicit instruction, and think alouds, as well as guiding students as they practice math strategies and apply problem solving strategies. (whole group or small group instruction)

Teacher and students practice mathematics processes together through interactive activities, problem solving, and discussion. (whole group or small group instruction)

Students practice math strategies independently to build procedural and computational fluency. Teacher assesses learning and reteaches as necessary. (whole group instruction, small group instruction, or centers)
## Effective Pedagogical Routines/Instructional Strategies

<table>
<thead>
<tr>
<th>Collaborative Problem Solving</th>
<th>Analyze Student Work</th>
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<tbody>
<tr>
<td>Connect Previous Knowledge to New Learning</td>
<td>Identify Student’s Mathematical Understanding</td>
</tr>
<tr>
<td>Making Thinking Visible</td>
<td>Identify Student’s Mathematical Misunderstandings</td>
</tr>
<tr>
<td>Develop and Demonstrate Mathematical Practices</td>
<td>Interviews</td>
</tr>
<tr>
<td>Inquiry-Oriented and Exploratory Approach</td>
<td>Role Playing</td>
</tr>
<tr>
<td>Multiple Solution Paths and Strategies</td>
<td>Diagrams, Charts, Tables, and Graphs</td>
</tr>
<tr>
<td>Use of Multiple Representations</td>
<td>Anticipate Likely and Possible Student Responses</td>
</tr>
<tr>
<td>Explain the Rationale of your Math Work</td>
<td>Collect Different Student Approaches</td>
</tr>
<tr>
<td>Quick Writes</td>
<td>Multiple Response Strategies</td>
</tr>
<tr>
<td>Pair/Trio Sharing</td>
<td>Asking Assessing and Advancing Questions</td>
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<tr>
<td>Turn and Talk</td>
<td>Revoicing</td>
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<tr>
<td>Charting</td>
<td>Marking</td>
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<tr>
<td>Gallery Walks</td>
<td>Recapping</td>
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<tr>
<td>Small Group and Whole Class Discussions</td>
<td>Challenging</td>
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<tr>
<td>Student Modeling</td>
<td>Pressing for Accuracy and Reasoning</td>
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<td></td>
<td>Maintain the Cognitive Demand</td>
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</tbody>
</table>
Educational Technology

Standards


➢ Technology Operations and Concepts
  • Construct a spreadsheet workbook with multiple worksheets, rename tabs to reflect the data on the worksheet, and use mathematical or logical functions, charts and data from all worksheets to convey the results.

  Example: Students will use appropriate digital tools to and software to problem solve, check for accuracy and support answers.

➢ Digital Citizenship
  • Analyze the capabilities and limitations of current and emerging technology resources and assess their potential to address personal, social, lifelong learning and career needs.

  Example: Students will be able to use critical thinking skills to plan, conduct, research, manage projects, solve problems, and make informed decisions using appropriate digital tools and resources.

➢ Research and Information Literacy
  • Produce a position statement about a real world problem by developing a systematic plan of investigation with peers and experts synthesizing information from multiple sources.

  Example: Students will produce a position statement about a real world problem by developing a systematic plan of investigation with peers and experts synthesizing information from multiple sources.

➢ Design: Critical Thinking, Problem Solving, and Decision Making
  • Evaluate the strengths and limitations of emerging technologies and their impact on educational, career, personal and or social needs.

  Example: A problem-solving approach will allow students to construct their own idea about mathematics and to take responsibility for their own learning.
**Career Ready Practices**

Career Ready Practices describe the career-ready skills that all educators in all content areas should seek to develop in their students. They are practices that have been linked to increase college, career, and life success. Career Ready Practices should be taught and reinforced in all career exploration and preparation programs with increasingly higher levels of complexity and expectation as a student advances through a program of study.

- **CRP2. Apply appropriate academic and technical skills.**  
  Career-ready individuals readily access and use the knowledge and skills acquired through experience and education to be more productive. They make connections between abstract concepts with real-world applications, and they make correct insights about when it is appropriate to apply the use of an academic skill in a workplace situation.  
  **Example:** Students are able to apply concepts in the classroom to solve real life application problems and analyzing their solutions for accuracy.

- **CRP4. Communicate clearly and effectively and with reason.**  
  Career-ready individuals communicate thoughts, ideas, and action plans with clarity, whether using written, verbal, and/or visual methods. They communicate in the workplace with clarity and purpose to make maximum use of their own and others’ time. They are excellent writers; they master conventions, word choice, and organization, and use effective tone and presentation skills to articulate ideas. They are skilled at interacting with others; they are active listeners and speak clearly and with purpose. Career-ready individuals think about the audience for their communication and prepare accordingly to ensure the desired outcome.  
  **Example:** Students are able to listen, communicate and have constructed arguments to justify their approach and conclusion to a problem.

- **CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.**  
  Career-ready individuals readily recognize problems in the workplace, understand the nature of the problem, and devise effective plans to solve the problem. They are aware of problems when they occur and take action quickly to address the problem; they thoughtfully investigate the root cause of the problem prior to introducing solutions. They carefully consider the options to solve the problem. Once a solution is agreed upon, they follow through to ensure the problem is solved, whether through their own actions or the actions of others.  
  **Example:** Students are able to utilized concepts learnt in the classroom to analyze problems, create a plan to solve the problem, solve the problem and check their solution for correctness.

- **CRP11. Use technology to enhance productivity.**  
  Career-ready individuals find and maximize the productive value of existing and new technology to accomplish workplace tasks and solve workplace problems. They are flexible and adaptive in acquiring new technology. They are proficient with ubiquitous technology applications. They understand the inherent risks-personal and organizational-of technology applications, and they take actions to prevent or mitigate these risks.  
  **Example:** Students are able to use a variety of technology such as the TI-84 and online resources to effectively to help in problem solving.
## WIDA Proficiency Levels

At the given level of English language proficiency, English language learners will process, understand, produce or use

<table>
<thead>
<tr>
<th>Level</th>
<th>Proficiency Details</th>
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</table>
| **6- Reaching** | - Specialized or technical language reflective of the content areas at grade level  
- A variety of sentence lengths of varying linguistic complexity in extended oral or written discourse as required by the specified grade level  
- Oral or written communication in English comparable to proficient English peers |
| **5- Bridging** | - Specialized or technical language of the content areas  
- A variety of sentence lengths of varying linguistic complexity in extended oral or written discourse, including stories, essays or reports  
- Oral or written language approaching comparability to that of proficient English peers when presented with grade level material. |
| **4- Expanding** | - Specific and some technical language of the content areas  
- A variety of sentence lengths of varying linguistic complexity in oral discourse or multiple, related sentences or paragraphs  
- Oral or written language with minimal phonological, syntactic or semantic errors that may impede the communication, but retain much of its meaning, when presented with oral or written connected discourse, with sensory, graphic or interactive support |
| **3- Developing** | - General and some specific language of the content areas  
- Expanded sentences in oral interaction or written paragraphs  
- Oral or written language with phonological, syntactic or semantic errors that may impede the communication, but retain much of its meaning, when presented with oral or written, narrative or expository descriptions with sensory, graphic or interactive support |
| **2- Beginning** | - General language related to the content area  
- Phrases or short sentences  
- Oral or written language with phonological, syntactic, or semantic errors that often impede of the communication when presented with one to multiple-step commands, directions, or a series of statements with sensory, graphic or interactive support |
| **1- Entering** | - Pictorial or graphic representation of the language of the content areas  
- Words, phrases or chunks of language when presented with one-step commands directions, WH-, choice or yes/no questions, or statements with sensory, graphic or interactive support |
Language Development Supports For English Language Learners To Increase Comprehension and Communication Skills

<table>
<thead>
<tr>
<th>Environment</th>
<th>Integration and meaningful way</th>
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</thead>
<tbody>
<tr>
<td>Welcoming and stress-free</td>
<td>Provides opportunities to practice and refine receptive and productive skills in English as a new language</td>
</tr>
<tr>
<td>Respectful of linguistic and cultural diversity</td>
<td>Integrates meaning and purposeful tasks/activities that:</td>
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<tr>
<td>Honors students’ background knowledge</td>
<td>• Are accessible by all students through multiple entry points</td>
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<tr>
<td>Sets clear and high expectations</td>
<td>• Are relevant to students’ lives and cultural experiences</td>
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<tr>
<td>Includes routines and norms</td>
<td>• Build on prior mathematical learning</td>
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<td>Is thinking-focused vs. answer-seeking</td>
<td>• Demonstrate high cognitive demand</td>
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<tr>
<td>Offers multiple modalities to engage in content learning and to demonstrate understanding</td>
<td>• Offer multiple strategies for solutions</td>
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<tr>
<td>Includes explicit instruction of specific language targets</td>
<td>• Allow for a language learning experience in addition to content</td>
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<tr>
<td>Provides participation techniques to include all learners</td>
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<table>
<thead>
<tr>
<th>Sensory Supports*</th>
<th>Graphic Supports*</th>
<th>Interactive Supports*</th>
<th>Verbal and Textual Supports</th>
</tr>
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<tbody>
<tr>
<td>Real-life objects (realia) or concrete objects</td>
<td>Graphs</td>
<td>In a whole group</td>
<td>Labeling</td>
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<tr>
<td>Physical models</td>
<td>Charts</td>
<td>In a small group</td>
<td>Students’ native language</td>
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<tr>
<td>Manipulatives</td>
<td>Timelines</td>
<td>With a partner such as Turn-and-Talk</td>
<td>Modeling</td>
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<tr>
<td>Pictures &amp; photographs</td>
<td>Number lines</td>
<td>In pairs as a group (first, two pairs work independently, then they form a group of four)</td>
<td>Repetitions</td>
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<tr>
<td>Visual representations or models such as diagrams or drawings</td>
<td>Graphic organizers</td>
<td>In triads</td>
<td>Paraphrasing</td>
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<tr>
<td>Videos &amp; films</td>
<td>Graphing paper</td>
<td>Cooperative learning structures such as Think-Pair-Share</td>
<td>Summarizing</td>
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<tr>
<td>Newspapers or magazines</td>
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<td>Interactive websites or software</td>
<td>Guiding questions</td>
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<tr>
<td>Gestures</td>
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<td>With a mentor or coach</td>
<td>Clarifying questions</td>
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<td>Physical movements</td>
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<td>Probing questions</td>
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<tr>
<td>Music &amp; songs</td>
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<td>Leveled questions such as What? When? Where? How? Why?</td>
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<td>Questioning prompts &amp; cues</td>
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<td>Word Banks</td>
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<td></td>
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<td>Sentence starters</td>
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<td>Sentence frames</td>
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<td>Discussion frames</td>
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<td>Talk moves, including Wait Time</td>
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</tbody>
</table>

BUILDING EQUITY IN YOUR TEACHING PRACTICE

How do the essential questions highlight the connection between the big ideas of the unit and equity in your teaching practice?

**CONTENT INTEGRATION**
Teachers use examples and content from a variety of cultures & groups.

- This unit / lesson is connected to other topics explored with students.
- There are multiple viewpoints reflected in the content of this unit / lesson.
- The materials and resources are reflective of the diverse identities and experiences of students.
- The content affirms students, as well as exposes them to experiences other than their own.

**KNOWLEDGE CONSTRUCTION**
Teachers help students understand how knowledge is created and influenced by cultural assumptions, perspectives & biases.

- This unit / lesson provides context to the history of privilege and oppression.
- This unit / lesson addresses power relationships.
- This unit / lesson help students to develop research and critical thinking skills.
- This curriculum creates windows and mirrors* for students.

**PREJUDICE REDUCTION**
Teachers implement lessons and activities to assert positive images of ethnic groups & improve intergroup relations.

- This unit / lesson help students question and unpack biases & stereotypes.
- This unit / lesson help students examine, research and question information and sources.
- The curriculum encourage discussion and understanding about the groups of people being represented.
- This unit / lesson challenges dominant perspectives.

**EQUITABLE PEDAGOGY**
Teachers modify techniques and methods to facilitate the academic achievement of students from diverse backgrounds.

- The instruction has been modified to meet the needs of each student.
- Students feel respected and their cultural identities are valued.
- Additional supports have been provided for students to become successful and independent learners.
- Opportunities are provided for student to reflect on their learning and provide feedback.

**EMPOWERING SCHOOL CULTURE**
Using the other four dimensions to create a safe and healthy educational environment for all.

- There are opportunities for students to connect with the community.
- My classroom is welcoming and supportive for all students.
- I am aware of and sensitive to the needs of my students and their families.
- There are effective parent communication systems established. Parents can talk to me about issues as they arise in my classroom.

## Culturally Relevant Pedagogy Examples

- **Call on Each Student**: Encourage each student to share his or her thoughts through call-and-response, keeping the class’s attention in the process.  
  **Example**: Foster confidence. Make the assessment process less intimidating by offering different ways to demonstrate skills and understanding. For example, avoid handing out quizzes that are purely multiple choice or fill-in-the-blank. Among other question types, mix in problems that involve explaining the step necessary to get to the answer. After, give students time to assess their own progress and performance, helping them focus on growth.

- **Integrate Relevant Word Problems**: Contextualize equations using word problems that reference student interests and cultures.  
  **Example**: When learning about derivatives grab students’ attention by including optimization problems that are relevant to their interest.

- **Present New Concepts Using Student Vocabulary**: Use student diction to capture attention and build understanding before using academic terms.  
  **Example**: Teach math vocabulary in various modalities for students to remember. Use multi-modal activities, analogies, realia, visual cues, graphic representations, gestures, pictures and cognates. Directly explain and model the idea of vocabulary words having multiple meanings. Students can create the Word Wall with their definitions and examples to foster ownership.

- **Gamify Lessons**: Appeal to gaming culture by, for example, writing instruction manuals for projects and offering rewards such as badges.  
  **Example**: Appeal to students’ personal interest to keep them engaged. Create a game to check students understanding of applying the rules of derivatives.  
  - [https://kahoot.com/](https://kahoot.com/)  
  - [http://www.classtools.net/](http://www.classtools.net/)
## Differentiated Instruction

### Accommodate Based on Students Individual Needs: Strategies

<table>
<thead>
<tr>
<th>Time/General</th>
<th>Processing</th>
<th>Comprehension</th>
<th>Recall</th>
</tr>
</thead>
<tbody>
<tr>
<td>- Extra time for assigned tasks</td>
<td>- Extra Response time</td>
<td>- Precise processes for balanced math instructional model</td>
<td>- Teacher-made checklist</td>
</tr>
<tr>
<td>- Adjust length of assignment</td>
<td>- Have students verbalize steps</td>
<td>- Short manageable tasks</td>
<td>- Use visual graphic organizers</td>
</tr>
<tr>
<td>- Timeline with due dates for reports and projects</td>
<td>- Repeat, clarify or reword directions</td>
<td>- Brief and concrete directions</td>
<td>- Reference resources to promote independence</td>
</tr>
<tr>
<td>- Communication system between home and school</td>
<td>- Mini-breaks between tasks</td>
<td>- Provide immediate feedback</td>
<td>- Visual and verbal reminders</td>
</tr>
<tr>
<td>- Provide lecture notes/outline</td>
<td>- Provide a warning for transitions</td>
<td>- Small group instruction</td>
<td>- Graphic organizers</td>
</tr>
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<td></td>
<td>- Reading partners</td>
<td>- Emphasize multi-sensory learning</td>
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</tr>
</tbody>
</table>

### Assistive Technology

- Computer/whiteboard
- Tape recorder
- Video Tape

### Tests/Quizzes/Grading

- Extended time
- Study guides
- Shortened tests
- Read directions aloud

### Behavior/Attention

- Consistent daily structured routine
- Simple and clear classroom rules
- Frequent feedback

### Organization

- Individual daily planner
- Display a written agenda
- Note-taking assistance
- Color code materials
<table>
<thead>
<tr>
<th>Differentiated Instruction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Accommodate Based on Content Needs: Strategies</td>
</tr>
</tbody>
</table>

- Anchor charts to model strategies
- Review Algebra concepts to ensure students have the information needed to progress in understanding
- Pre-teach pertinent vocabulary
- Provide reference sheets that list formulas, step-by-step procedures, theorems, and modeling of strategies
- Word wall with visual representations of mathematical terms
- Teacher modeling of thinking processes involved in solving, graphing, and writing equations
- Introduce concepts embedded in real-life context to help students relate to the mathematics involved
- Record formulas, processes, and mathematical rules in reference notebooks
- Graphing calculator to assist with computations and graphing of trigonometric functions
- Utilize technology through interactive sites to represent nonlinear data
- Graphic organizers to help students interpret the meaning of terms in an expression or equation in context
- Translation dictionary
- Sentence stems to provide additional language support for ELL students.
## Interdisciplinary Connections

*Model interdisciplinary thinking to expose students to other disciplines.*

### Social Studies Connection: NJSLS 6.1.8.B.1.a, 6.1.8.C.1.b, 6.1.8.D.1.a, 6.1.8.B.2.a

*Name of Task:*

- Landfill


### Science Connection: NJSLS HS-PS2-2, HS-PS2-5, HS-PS4-1

*Name of Task:*

- Water Temperature

Enrichment

What is the purpose of Enrichment?

- The purpose of enrichment is to provide extended learning opportunities and challenges to students who have already mastered, or can quickly master, the basic curriculum. Enrichment gives the student more time to study concepts with greater depth, breadth, and complexity.
- Enrichment also provides opportunities for students to pursue learning in their own areas of interest and strengths.
- Enrichment keeps advanced students engaged and supports their accelerated academic needs.
- Enrichment provides the most appropriate answer to the question, “What do you do when the student already knows it?”

Enrichment is...

- Planned and purposeful
- Different, or differentiated, work – not just more work
- Responsive to students’ needs and situations
- A promotion of high-level thinking skills and making connections within content
- The ability to apply different or multiple strategies to the content
- The ability to synthesize concepts and make real world and cross-curricular connections
- Elevated contextual complexity
- Sometimes independent activities, sometimes direct instruction
- Inquiry based or open ended assignments and projects
- Using supplementary materials in addition to the normal range of resources
- Choices for students
- Tiered/Multi-level activities with flexible groups (may change daily or weekly)

Enrichment is not...

- Just for gifted students (some gifted students may need intervention in some areas just as some other students may need frequent enrichment)
- Worksheets that are more of the same (busywork)
- Random assignments, games, or puzzles not connected to the content areas or areas of student interest
- Extra homework
- A package that is the same for everyone
- Thinking skills taught in isolation
- Unstructured free time
# Assessments

## Required District/State Assessments

- SGO Assessments

## Suggested Formative/Summative Classroom Assessments

- Describe Learning Vertically
- Identify Key Building Blocks
- Make Connections (between and among key building blocks)
- Short/Extended Constructed Response Items
- Multiple-Choice Items (where multiple answer choices may be correct)
- Drag and Drop Items
- Use of Equation Editor
- Quizzes
- Journal Entries/Reflections/Quick-Writes
- Accountable talk
- Projects
- Portfolio
- Observation
- Graphic Organizers/ Concept Mapping
- Presentations
- Role Playing
- Teacher-Student and Student-Student Conferencing
- Homework
New Jersey Student Learning Standards (NJSLS)

F-IF.A.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).

F-TF.A.3 Use special triangles to determine geometrically the values of sine, cosine, tangent for π/3, π/4 and π/6, and use the unit circle to express the values of sine, cosines, and tangent for π–x, π+x, and 2π–x in terms of their values for x, where x is any real number.

F-IF.B.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

F-IF.B.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.

APR.D.6 Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) +r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system.

APR.D.7 Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression.

A-SSE.1a Interpret parts of an expression, such as terms, factors, and coefficients.

A-SSE.1b Interpret complicated expressions by viewing one or more of their parts as a single entity.

A-SSE.A.2 Use the structure of an expression to identify ways to rewrite it. For example, see x? -y? as (x?±y?)?±(y?)?, thus recognizing it as a difference of squares that can be factored (x?±y?)(x?+y?).

F-TF.A.1 Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
F-TF.A.3 Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for $x$, $\pi + x$, and $2\pi - x$ in terms of their values for $x$, where $x$ is any real number.

F-TF.B.5 Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.

F-TF.B.6 Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.

F-TF.C.8 Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle.
Mathematical Practices for AP Calculus (by the College Board)

**MPAC 1: Reasoning with definitions and theorems**

Students can:
- Use definitions and theorems to build arguments, to justify conclusions or answers, and to prove results;
- Confirm that hypotheses have been satisfied in order to apply the conclusion of a theorem;
- Apply definitions and theorems in the process of solving a problem;
- Interpret quantifiers in definitions and theorems (e.g., “for all,” “there exists”);
- Develop conjectures based on exploration with technology; and
- Produce examples and counterexamples to clarify understanding of definitions, to investigate whether converses of theorems are true or false, or to test conjectures.

**MPAC 2: Connecting concepts**

Students can:
- Relate the concept of a limit to all aspects of calculus;
- Use the connection between concepts (e.g., rate of change and accumulation) or processes (e.g., differentiation and its inverse process, anti-differentiation) to solve problems;
- Connect concepts to their visual representations with and without technology; and
- Identify a common underlying structure in problems involving different contextual situations.

**MPAC 3: Implementing algebraic/computational processes**

Students can:
- Select appropriate mathematical strategies;
- Sequence algebraic/computational procedures logically;
- Complete algebraic/computational processes correctly;
- Apply technology strategically to solve problems;
- Attend to precision graphically, numerically, analytically, and verbally and specify units of measure; and
- Connect the results of algebraic/computational processes to the question asked.
MPAC 4: Connecting multiple representations
Students can:
• Associate tables, graphs, and symbolic representations of functions;
• Develop concepts using graphical, symbolical, verbal, or numerical representations with and without technology;
• Identify how mathematical characteristics of functions are related in different representations;
• Extract and interpret mathematical content from any presentation of a function (e.g., utilize information from a table of values);
• Construct one representational form from another (e.g., a table from a graph or a graph from given information); and
• Consider multiple representations (graphical, numerical, analytical, and verbal) of a function to select or construct a useful representation for solving a problem.

MPAC 5: Building notational fluency
Students can:
• Know and use a variety of notations
• Connect notation to definitions (e.g., relating the notation for the definite integral to that of the limit of a Riemann sum);
• Connect notation to different representations (graphical, numerical, analytical, and verbal); and
• Assign meaning to notation, accurately interpreting the notation in a given problem and across different contexts.

MPAC 6: Communicating
Students can:
• Clearly present methods, reasoning, justifications, and conclusions;
• Use accurate and precise language and notation;
• Explain the meaning of expressions, notation, and results in terms of a context (including units);
• Explain the connections among concepts;
• Critically interpret and accurately report information provided by technology; and
• Analyze, evaluate, and compare the reasoning of others.
<table>
<thead>
<tr>
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<th>Topic: Derivatives &amp; Applications of Derivatives</th>
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<tr>
<td>Unit 3 NJSLS: F-IF.A.1, F-IF.A.3, F-IF.B.4, APR.D.1-7, ASSE.A.1a, ASSE.A.1b, F-FT.A.1,3; B.5,6; C.8</td>
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**Student Learning Objective (SLO 1):**
- Calculate slopes and derivatives using the definition of derivative.
- Graph $f$ from the graph of $f'$, graph $f'$ from the graph of $f$.
- Find where a function is not differentiable and distinguish between corners, cups, discontinuities, and vertical tangents.
- Approximate derivatives numerically and graphically.
- Use rules of differentiation to calculate derivatives, including second and higher order derivatives.

**New Jersey Student Learning Standards (NJSLS):**

**F-IF.A.1:** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If $f$ is a function and $x$ is an element of its domain, then $f(x)$ denotes the output of $f$ corresponding to the input $x$. The graph of $f$ is the graph of the equation $y = f(x)$.

**F-IF.A.3:** Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosines, and tangent for $\pi-x$, $\pi+x$, and $2\pi-x$ in terms of their values for $x$, where $x$ is any real number.

**F-IF.B.4:** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

**APR.D.6:** Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.

**APR.D.7:** Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression.

**A-SSE.1a:** Interpret parts of an expression, such as terms, factors, and coefficients.
A-SSE.1b: Interpret complicated expressions by viewing one or more of their parts as a single entity.

F-TF.A.1: Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.

F-TF.A.3: Use special triangles to determine geometrically the values of sine, cosine, tangent for π/3, π/4 and π/6, and use the unit circle to express the values of sine, cosine, and tangent for x, π + x, and 2π - x in terms of their values for x, where x is any real number.

F-TF.B.5: Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.

F-TF.B.6: Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.

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<tr>
<td>MPAC 1</td>
<td>Students will learn about:</td>
<td>• The derivative gives the value of the slope of the tangent line to a curve at a point.</td>
<td>Derivatives of a Function Part 1</td>
</tr>
<tr>
<td>MPAC 2</td>
<td>• Calculate slopes and derivatives using the definition of the derivative.</td>
<td>• Learn to calculate derivatives using the definition.</td>
<td>Derivatives of a Function Part 2</td>
</tr>
<tr>
<td>MPAC 5</td>
<td>• Graph ( f ) from the graph ( f' ), graph ( f' ) from the graph of ( f ), and graph the derivative of a function given numerically with data.</td>
<td>• The derivative of a function is defined as the limit of a difference quotient and can be determined using a variety of strategies.</td>
<td>Matching Functions with Derivatives</td>
</tr>
<tr>
<td>MPAC 6</td>
<td>• Find where a function is not differentiable and distinguish between corners, cups discontinuities, and vertical tangents.</td>
<td>• Graphs of differentiable functions can be approximated by their tangent lines at points where the derivative exists.</td>
<td>Differentiability</td>
</tr>
<tr>
<td></td>
<td>• Approximate derivatives numerically and graphically.</td>
<td>• Rules of derivatives of functions analytically more efficiently</td>
<td>Differentiation Day 1</td>
</tr>
<tr>
<td></td>
<td>• Use rules of differentiation to calculate derivatives, including second and higher order derivatives.</td>
<td></td>
<td>Differentiation Day 2</td>
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</table>

* SEE UNIT 3 TASK FOLDER
- Use derivatives to calculate the instantaneous rate of change.
- Common errors - when calculating derivatives using the definition, students often make errors in evaluating and simplifying the numerator of the difference quotient. When \( f(x) \) us a polynomial or rational function, \( h \) is always a factor of the simplified expression

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**Student Learning Objective (SLO 2):**
- Use derivatives to analyze straight-line motion and solve other problems involving rates of change.
- Use the rules for differentiating the six basic trigonometric functions.
- Differentiate composite functions using the Chain Rule.
- Find the slope of parameterized curves.

**New Jersey Student Learning Standards (NJSLS):** F-IF.A.1, F-IF.A.3, F-IF.B.4, APR.D.1-7, ASSE.A.1a,b, F-FT.A.1,3; B.5,6; C.8

**F-IF.A.1:** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If \( f \) is a function and \( x \) is an element of its domain, then \( f(x) \) denotes the output of \( f \) corresponding to the input \( x \). The graph of \( f \) is the graph of the equation \( y = f(x) \).

**F-IF.A.3:** Use special triangles to determine geometrically the values of sine, cosine, tangent for \( \pi/3, \pi/4 \) and \( \pi/6 \), and use the unit circle to express the values of sine, cosines, and tangent for \( \pi-x, \pi+x, \) and \( 2\pi-x \) in terms of their values for \( x \), where \( x \) is any real number.

**F-IF.B.4:** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
**APR.D.6:** Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.

**APR.D.7:** Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression.

**A-SSE.1a:** Interpret parts of an expression, such as terms, factors, and coefficients.

**A-SSE.1b:** Interpret complicated expressions by viewing one or more of their parts as a single entity.

**F-TF.A.1:** Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.

**F-TF.A.3:** Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for $x$, $\pi + x$, and $2\pi - x$ in terms of their values for $x$, where $x$ is any real number.

**F-TF.B.5:** Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.

**F-TF.B.6:** Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.

**F-TF.C.8:** Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle.

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<td>• MPAC 1</td>
<td>Students will learn about: • Use derivatives to analyze straight-line motion and solve other problems involving rates of change.</td>
<td>• Derivatives give the rates at which things change in the world. • Acceleration due to gravity can be either a positive or a negative</td>
<td>• The Chain Rule 1 &amp; 2 ♦ Velocity &amp; Other Rates of Change 1 &amp; 2</td>
</tr>
<tr>
<td>• MPAC 2</td>
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<td>• MPAC 5</td>
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<tr>
<td>• MPAC 6</td>
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- Use rules to differentiate the six trigonometric functions.
- Differentiate composite functions using the Chain Rule.
- Find slopes of parameterized curve.
- Common errors 1- students have trouble distinguishing between speed and velocity.
- Common errors 2- Students sometimes forget to or misapply the basic trigonometric identities. You may wish to review the reciprocal, Pythagorean, angle sum, and half-angle identities.
- Common errors 3- In applying the outside-inside rule to differentiate \( f(g(x)) \), a common mistake is to omit the factor \( g'(x) \) in the answer.

number, depending on how one defines a coordinate system.
- Velocity is the rate of change of position, and acceleration is the rate of change of velocity.
- The derivatives of sines and cosines play a key role in describing periodic change.
- Chain Rule is the most widely used differentiation rule in mathematics.
- Direct application of the definition of the derivative can be used to find the derivative for selected functions, including polynomial, power, sine, cosine, exponential, and logarithmic functions.
- The Chain Rule provides a way to differentiate composite functions.
- The Chain Rule is the basis for implicit differentiation and find the derivative of inverse functions.

- Velocity of a Particle
- Derivatives of Trigonometry Functions 1 & 2
- Derivatives MC AP Exam
- Mixed Review
- AP Calculus Exam Review
- Function of \( f & g \) (AP Exam)

* SEE UNIT 3 TASK FOLDER
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**Student Learning Objective (SLO 3):**
- Find the derivatives using implicit differentiation.
- Find derivatives using the Power Rule for Rational Powers of $x$.
- Calculate derivatives of functions involving the inverse trigonometric functions.
- Calculate derivatives of exponential and logarithmic functions.

**New Jersey Student Learning Standards (NJSLS):** F-IF.A.1, F-IF.A.3, F-IF.B.4, F-IF.C.7a-e

**F-IF.A.1:** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If $f$ is a function and $x$ is an element of its domain, then $f(x)$ denotes the output of $f$ corresponding to the input $x$. The graph of $f$ is the graph of the equation $y = f(x)$.

**F-IF.A.3:** Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosines, and tangent for $\pi-x$, $\pi+x$, and $2\pi-x$ in terms of their values for $x$, where $x$ is any real number.

**F-IF.B.4:** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

**APR.D.6:** Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.

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**F-TF.A.3:** Use special triangles to determine geometrically the values of sine, cosine, tangent for π/3, π/4 and π/6, and use the unit circle to express the values of sine, cosine, and tangent for x, π + x, and 2π - x in terms of their values for x, where x is any real number.

**F-TF.B.5:** Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.

**F-TF.B.6:** Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.

**F-TF.C.8:** Prove the Pythagorean identity \( \sin^2(\theta) + \cos^2(\theta) = 1 \) and use it to find \( \sin(\theta), \cos(\theta), \) or \( \tan(\theta) \) given \( \sin(\theta), \cos(\theta), \) or \( \tan(\theta) \) and the quadrant of the angle.

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<td>MPAC 1</td>
<td>You may wish to begin this lesson with informal discussion about graphs of equations of the form ( F(x, y) = c ). Use equations whose graphs can be displayed on a graphing utility, in either parametric mode or rectangular mode, by graphing the separate branches in the same window.</td>
<td>Implicit differentiation allows us to find derivatives of functions that are not defined or written explicitly as a function of a single variable.</td>
<td>Implicit Differentiation 1-2</td>
</tr>
</tbody>
</table>
| MPAC 2 | Students will learn about:  
• Find derivatives using implicit differentiation.  
• Find derivatives using Power Rule for Rational Powers of \( x \).  
• Calculate derivatives of functions involving inverse functions. | Implicit differentiation is a powerful and important technique. It is used in application. | Implicit Differentiation MC- AP Exam |
| MPAC 3 | | The relationship between the graph of a function and its inverse allows us to see the relationship between their derivatives. | Implicit Differentiation OE- AP Exam |
| MPAC 4 | | The relationship between exponential and logarithmic functions provides a powerful differentiation tool called logarithmic differentiation | Implicit-Inverse Trig Practice |
|       | | Derivatives of Inverse Functions | |

---

**Implicit Differentiation 1-2**  
**Implicit Differentiation MC- AP Exam**  
**Implicit Differentiation OE- AP Exam**  
**Implicit-Inverse Trig Practice**  
**Derivatives of Inverse Functions**
- Calculate derivatives of exponential and logarithmic functions.

- Discuss the relationship between slopes of linear functions and their inverses.

- Help students understand how derivatives of inverse functions are derived.

- A discussion of logarithmic differentiation is a good way to conclude this lesson. Make sure students understand why the derivative of \( y = x^x \) cannot be found directly using the Power Rule.

- Common errors - It is easy to make mistakes in taking derivatives, which requires the Product Rule and/or the Chain Rule. Students forget to use the Product Rule when differentiating the term \( xy \).

- Derivatives of Logs & Exponential Functions

- 2017 AP Exam OE

* SEE UNIT 3 TASK FOLDER
**Grade:** 12th  
**Unit:** 3 (Three)  
**Topic:** Derivatives & Applications of Derivatives

**Student Learning Objective (SLO 4):**
- Determine the local or global extreme values of a function.
- Apply the Mean Value Theorem and find the intervals on which a function is increasing or decreasing.
- Use the First and Second Derivative Test to determine local extreme values of a function.
- Determine the concavity of a function and locate the points of inflection by analyzing the second derivative.
- Solve application problems involving finding minimum or maximum values of functions.

**New Jersey Student Learning Standards (NJSLS): F-IF.A.1, F-IF.A.3, F-IF.B.4, F-FT.A.1,3; B.5,6; C.8**

**F-IF.A.1:** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If \( f \) is a function and \( x \) is an element of its domain, then \( f(x) \) denotes the output of \( f \) corresponding to the input \( x \). The graph of \( f \) is the graph of the equation \( y = f(x) \).

**F-IF.A.3:** Use special triangles to determine geometrically the values of sine, cosine, tangent for \( \pi/3 \), \( \pi/4 \) and \( \pi/6 \), and use the unit circle to express the values of sine, cosines, and tangent for \( \pi-x \), \( \pi+x \), and \( 2\pi-x \) in terms of their values for \( x \), where \( x \) is any real number.

**F-IF.B.4:** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

**APR.D.6:** Rewrite simple rational expressions in different forms; write \( a(x)/b(x) \) in the form \( q(x) + r(x)/b(x) \), where \( a(x) \), \( b(x) \), \( q(x) \), and \( r(x) \) are polynomials with the degree of \( r(x) \) less than the degree of \( b(x) \), using inspection, long division, or, for the more complicated examples, a computer algebra system.

**APR.D.7:** Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression.

**A-SSE.1a:** Interpret parts of an expression, such as terms, factors, and coefficients.
**A-SSE.1b:** Interpret complicated expressions by viewing one or more of their parts as a single entity.

**F-TF.A.1:** Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.

**F-TF.A.3:** Use special triangles to determine geometrically the values of 

\[
\sin \left( \frac{\pi}{3} \right), \sin \left( \frac{\pi}{4} \right) \text{ and } \sin \left( \frac{\pi}{6} \right), \\
\cos \left( \frac{\pi}{3} \right), \cos \left( \frac{\pi}{4} \right) \text{ and } \cos \left( \frac{\pi}{6} \right), \\
\tan \left( \frac{\pi}{3} \right), \tan \left( \frac{\pi}{4} \right) \text{ and } \tan \left( \frac{\pi}{6} \right).
\]

and use the unit circle to express the values of sine, cosine, and tangent for \( x, \pi + x, \) and \( 2\pi - x \) in terms of their values for \( x, \) where \( x \) is any real number.

**F-TF.B.5:** Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.

**F-TF.B.6:** Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.

**F-TF.C.8:** Prove the Pythagorean identity

\[
\sin^2(\theta) + \cos^2(\theta) = 1
\]

and use it to find \( \sin(\theta), \cos(\theta), \) or \( \tan(\theta) \) given \( \sin(\theta), \cos(\theta), \) or \( \tan(\theta) \) and the quadrant of the angle.

<table>
<thead>
<tr>
<th>MPs</th>
<th>Skills, Strategies &amp; Concepts</th>
<th>Essential Understandings/ Questions</th>
<th>Tasks/Activities</th>
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<tr>
<td>MPAC 1</td>
<td>Students will learn about:</td>
<td>• Finding maximum and minimum values of functions, called optimization, is an important issue in real-world problems.</td>
<td></td>
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<tr>
<td>MPAC 2</td>
<td>• Determine the local and global extreme values of a function.</td>
<td>• The Mean Value Theorem is an important theoretical to connect the average and instantaneous rates of change.</td>
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<tr>
<td>MPAC 3</td>
<td>• Apply the Mean Value Theorem and find the intervals on which a function is increasing or decreasing.</td>
<td>• Differential calculus is a powerful problem-solving tool precisely because of its usefulness for analyzing functions.</td>
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<tr>
<td>MPAC 4</td>
<td>• Use the First and Second Derivative Test to determine the local extreme values of a function.</td>
<td>• Historically, optimization problems were among the earliest</td>
<td></td>
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<tr>
<td>MPAC 5</td>
<td>• Determine the concavity of a function and locate the points of inflection by analyzing the second derivatives</td>
<td>• Extreme Values of Functions 1 &amp; 2</td>
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<td></td>
<td></td>
<td>• Mean Value Theorem 1-3</td>
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<td></td>
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<td>• Connecting Graphs 1 &amp;2</td>
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<td>• Optimization 1-4</td>
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<td></td>
<td>• 2011 AP Calculus OE # 4</td>
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<td>• 2014 AP Calculus OE # 6</td>
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<td>• 2016 AP Calculus OE # 6</td>
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<td></td>
<td></td>
<td>• It’s a Match Up</td>
<td></td>
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<tr>
<td></td>
<td></td>
<td>• Application of Derivatives</td>
<td></td>
</tr>
<tr>
<td>Graph $f$ using information about $f'$.</td>
<td>One way to begin the lesson is to sketch an arbitrary function and discuss the local and global (absolute) minima and maxima of the function.</td>
<td>You can motivate the First Derivative Test presenting several graphs and having students discuss the derivatives and how they relate to the extreme values of the function.</td>
<td></td>
</tr>
<tr>
<td>Common errors 1- Students will neglect to find the points where the derivative is undefined.</td>
<td>Common error 2- Students have trouble with the use of closed intervals in describing where a function is increasing or decreasing.</td>
<td>applications of what we now call differential calculus.</td>
<td></td>
</tr>
</tbody>
</table>

* SEE UNIT 3 TASK FOLDER
### Vocabulary

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<th>Area below the curve</th>
<th>First Derivative Test</th>
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<tr>
<td>Area between the curve</td>
<td>Fundamental Theorem of Calculus</td>
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<td>Area under the curve</td>
<td>Higher Derivative</td>
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<td>Asymptote</td>
<td>Implicit Differentiation</td>
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<td>Average rate of change</td>
<td>Infinite Limit</td>
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<td>Axis of rotation</td>
<td>Inflection Point</td>
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<td>Bounds of integration</td>
<td>Instantaneous Velocity</td>
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<td>Chain rule</td>
<td>Integral</td>
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<td>Concave up</td>
<td>Integration by parts</td>
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<tr>
<td>Concave down</td>
<td>Integration by substitution</td>
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<td>Continuous function</td>
<td>Intermediate Value Theorem</td>
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<td>Continuous differentiable function</td>
<td>L’Hopital’s Rule</td>
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<td>Critical number</td>
<td>Limit</td>
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<td>Critical point</td>
<td>Right Limit/ Left Limit</td>
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<td>Critical value</td>
<td>Local maximum/ Local minimum</td>
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<td>Curve sketching</td>
<td>Mean Value Theorem</td>
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<td>Cylindrical Shell Method</td>
<td>One-sided limit</td>
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<td>Decreasing function</td>
<td>Product Rule</td>
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<td>Definite Integral</td>
<td>Projectile Motion</td>
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<td>Definite integral rules</td>
<td>Quotient Rule</td>
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<td>Derivative</td>
<td>Relative maximum/ Relative minimum</td>
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<td>Derivative of a Power Series</td>
<td>Riemann Sum</td>
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<td>Derivative Rules</td>
<td>Rolle’s Theorem</td>
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<td>Discontinuity</td>
<td>Sandwich Theorem</td>
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<td>Disk method</td>
<td>U-Substitution</td>
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<td>Divergent Sequence and Series</td>
<td>Volume by parallel cross section</td>
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<tr>
<td>Explicit Differentiation</td>
<td>Extreme Value Theorem</td>
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<td>Factorial</td>
<td>Integral</td>
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# References & Suggested Instructional Websites

- [http://www.mathwords.com/index_calculus.htm](http://www.mathwords.com/index_calculus.htm)
- [http://www.hershey.k12.pa.us/Page/3607](http://www.hershey.k12.pa.us/Page/3607)
- [https://sites.google.com/a/evergreenps.org/ms-griffin-s-math-classes/calculus-notes-worksheets-and-classroom-policies](https://sites.google.com/a/evergreenps.org/ms-griffin-s-math-classes/calculus-notes-worksheets-and-classroom-policies)
- [https://sites.google.com/site/dgrahamcalculus/ap-calculus-ab/calculus-worksheets](https://sites.google.com/site/dgrahamcalculus/ap-calculus-ab/calculus-worksheets)
- [http://home.cvc.org/math/apcalc/apcalc.htm](http://home.cvc.org/math/apcalc/apcalc.htm)
- [http://www.mathwithmrwood.com/ap-calculus](http://www.mathwithmrwood.com/ap-calculus)
- [http://www.analyzemath.com/ap_calculus.html](http://www.analyzemath.com/ap_calculus.html)
- [https://online.math.uh.edu/apcalculus/exams/](https://online.math.uh.edu/apcalculus/exams/)
<table>
<thead>
<tr>
<th>Site Name</th>
<th>Description</th>
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<tr>
<td><strong>SIX FLAGS GREAT ADVENTURE:</strong></td>
<td>This educational event includes workbooks and special science and math related shows throughout the day. Your students will leave with a better understanding of real world applications of the material they have learned in the classroom. Each student will have the opportunity to experience different rides and attractions linking mathematical and scientific concepts to what they are experiencing. <a href="http://www.sixflags.com">www.sixflags.com</a></td>
</tr>
<tr>
<td><strong>MUSEUM of MATHEMATICS:</strong></td>
<td>Mathematics illuminates the patterns that abound in our world. The National Museum of Mathematics strives to enhance public understanding and perception of mathematics. Its dynamic exhibits and programs stimulate inquiry, spark curiosity, and reveal the wonders of mathematics. The Museum’s activities lead a broad and diverse audience to understand the evolving, creative, human, and aesthetic nature of mathematics. <a href="http://www.momath.org">www.momath.org</a></td>
</tr>
<tr>
<td><strong>LIBERTY SCIENCE CENTER:</strong></td>
<td>An interactive science museum and learning center located in Liberty State Park. The center, which first opened in 1993 as New Jersey's first major state science museum, has science exhibits, the largest IMAX Dome Theater in the United States, numerous educational resources, and the original Hoberman sphere. <a href="http://lsc.org/plan-your-visit/">http://lsc.org/plan-your-visit/</a></td>
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## Optimization Project

A cylindrical can is to be made to hold 1 L of oil. Find the dimensions that will minimize the cost of the metal to manufacture the can. The can still must hold 1 L of oil, but you want the amount of metal used to make the can to be minimized. Less metal used = money saved. 1L = a cube of liquid with side 10cm. Find the dimensions of the can that will minimize the cost of the metal to manufacture the can.

1) Find the dimensions, showing all work in an organized fashion.

2) Explain in complete sentences how derivatives can help you solve problems in other disciplines such as saving manufacturing costs for a company. Give a specific example.

3) Come up with your own optimization word problem to solve a real world problem in another discipline such as physics or economics, then solve your problem. Research on the internet to get ideas.

* See MP 3 Project folder for full project and rubric.
The exam is 3 hours and 15 minutes long and has two sections — multiple choice and free-response. A graphing calculator is required for parts of the exam (see below). You may not take both the Calculus AB and Calculus BC exams within the same year.

**Section I: Multiple Choice** | 45 Questions | 1 hour and 45 minutes | 50% of Final Exam Score  
Part A — 30 questions | 60 minutes (calculator not permitted)  
Part B — 15 questions | 45 minutes (graphing calculator required)

**Section II: Free-Response** | 6 Questions | 1 hour and 30 minutes | 50% of Final Exam Score  
Part A — 2 problems | 30 minutes (graphing calculator required)  
Part B — 4 problems | 60 minutes (calculator not permitted)

### Completing Section II: Free-Response Questions
- During the second timed portion of the free-response section (Part B), you are permitted to continue work on problems in Part A, but you are not permitted to use a calculator during this time. For more information, see this course’s calculator policy and the list of approved graphing calculators.
- As you begin each part of Section II, you may wish to look over the questions before starting to work on them. It is not expected that everyone will be able to complete all parts of all questions.
- Show all of your work, even though a question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Justifications require that you give mathematical reasons, and that you verify the needed conditions under which relevant theorems, properties, definitions, or tests are applied. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.
- Your work must be expressed in standard mathematical notation rather than calculator syntax. For example, \[ \int_{1}^{5} x^2 \, dx \] may not be written as \( \text{fnInt}(X^2, X, 1, 5) \).
- Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If you use decimal approximations in calculations, your work will be scored on accuracy. Unless otherwise specified, your final answers should be accurate to three places after the decimal point.
- Unless otherwise specified, the domain of a function \( f \) is assumed to be the set of all real numbers \( x \) for which \( f(x) \) is a real number.
AP Calculus Test Preparation Strategies

Keep an eye on your time.
Monitor your time carefully. Make sure not to spend too much time on any one question so you’ll have enough time to answer all of them. If you do work that you think is incorrect, simply put an “X” through it instead of spending time erasing it completely: crossed-out work won’t be graded.

Show your work, even when you’re using a calculator.
Show all the steps you took to reach your solution on questions involving calculations. The exam reader wants to see if you know how to solve the problem. If you use your calculator to solve an equation, compute a numerical derivative, or find a definite integral, then be sure to write the equation, derivative, or integral first: an answer without this information might not get full credit, even if the answer is correct. Remember to write your work in standard notation (e.g. $\int_{1}^{5} x^2 \, dx$) rather than calculator syntax (e.g. fnInt($X^2$,X,1,5)), as calculator syntax is not acceptable.

Try to solve each part of the question.
Many free-response questions are divided into parts such as (a), (b), (c), and (d), with each part calling for a different response. Credit for each part is awarded independently, so you should attempt to solve each part. For example, you may receive no credit for your answer to part (a), but still receive full credit for part (b), (c), or (d). If the answer to a later part of a question depends on the answer to an earlier part, you may still be able to receive full credit for the later part, even if that earlier answer is wrong.

Be sure to fully answer the question being asked.
For example, if a question asks for the maximum value of a function, do not stop after finding the x-value at which the maximum value occurs. Be sure to express your answer in correct units if units are given and always provide a justification when it is asked for.

When asked to justify or to explain an answer, think about how that can be done.
For example, if you are asked to justify a point of inflection, you need to show that the sign of the second derivative changes. Simply saying that the second derivative equals zero or is undefined is not a justification.

Do not round partial answers.
Store partial answers in your calculator so that you can use them unrounded in further calculations.

Practice!
Answering free-response questions from previous AP Exams is a great way to practice: it allows you to compare your own responses with those that have already been evaluated and scored. Free-response questions and scoring guidelines are available on the Exam Practice page for AP Calculus AB.