AP Calculus

Unit 1: Functions
Course Philosophy/Description

AP Calculus AB is a course in single-variable calculus that includes techniques and applications of the derivative, techniques and applications of the definite integral, and the Fundamental Theorem of Calculus. It is equivalent to at least a semester of calculus at most colleges and universities, perhaps to a year of calculus at some. Algebraic, numerical, and graphical representations are emphasized throughout the course.

By the end of the course students will have learned Limits and Continuity; the Fundamental Theorems of Calculus; Definition of the Derivative of a Function and Techniques of Differentiation; Applications of the Derivative to maximize or minimize a function; the Chain Rule, Mean Value Theorem; Rate of Change problems; Curve Sketching; Definite and Indefinite integration of algebraic, trigonometric, and transcendental functions. The connections between numerical, graphical, analytical, and verbal communication of Calculus topics are a major theme in the class along with the overall conceptual understanding of the Calculus topics covered. The use of technology reinforces these approaches to confirm and interpret the results. Calculus is a transition course linking the mathematical and algebraic procedures taught in previous classes with the higher-level skills required in post-secondary technical programs.

**PREREQUISITES:**

Before studying Calculus, all students must have successfully completed coursework for Algebra I, Geometry, Algebra II, and Pre-Calculus. Students must be familiar with the properties, the algebra, the graphs and the language of functions. Attitude prerequisites include a willingness to work both in and out of class, a willingness to collaborate with classmates to foster mutual understanding, and a sincere intent to place out of the first semester of college calculus rather than repeat it.
This ESL framework was designed to be used by bilingual, dual language, ESL and general education teachers. Bilingual and dual language programs use the home language and a second language for instruction. ESL teachers and general education or bilingual teachers may use this document to collaborate on unit and lesson planning to decide who will address certain components of the SLO and language objective. ESL teachers may use the appropriate leveled language objective to build lessons for ELLs which reflects what is covered in the general education program. In this way, whether it is a pull-out or push-in model, all teachers are working on the same Student Learning Objective connected to the Common Core standard. The design of language objectives are based on the alignment of the World-Class Instructional Design Assessment (WIDA) Consortium’s English Language Development (ELD) standards with the Common Core State Standards (NJSLS). WIDA’s ELD standards advance academic language development across content areas ultimately leading to academic achievement for English learners. As English learners are progressing through the six developmental linguistic stages, this framework will assist all teachers who work with English learners to appropriately identify the language needed to meet the requirements of the content standard. At the same time, the language objectives recognize the cognitive demand required to complete educational tasks. Even though listening and reading (receptive) skills differ from speaking and writing (expressive) skills across proficiency levels the cognitive function should not be diminished. For example, an Entering Level One student only has the linguistic ability to respond in single words in English with significant support from their home language. However, they could complete a Venn diagram with single words which demonstrates that they understand how the elements compare and contrast with each other or they could respond with the support of their home language (L1) with assistance from a teacher, para-professional, peer or a technology program.

http://www.state.nj.us/education/modelcurriculum/ela/ELLOverview.pdf
<table>
<thead>
<tr>
<th>#</th>
<th>Student Learning Objective</th>
<th>NJSLS</th>
<th>Calculus for AP Correlation</th>
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| 1  | • Determine whether a relation represents a function.  
• Identify and obtain information from or about a graph of a function.  
• Graph linear, constant, identity, square, cube, square-root, reciprocal and greatest-integer functions.                                                                                                                | F-IF.A.1  
F-IF.B.4, B.5  
F-IF.C.7a-7c                                                                 | P.1, P.2, P.3                                                          |
| 2  | • Form the sum, difference, product, and quotient of two functions.  
• Form a composite function and find its domain.  
• Construct, analyze and solve problems by constructing functions.  
• Graph transformations of functions.                                                                                                                                                                                                                                      | F-BF.A.1a-c                                                                 | P.3                                                                       |
| 3  | • Analyze and graph quadratic, power, polynomial rational, and logarithms functions.  
• Determine the vertical, horizontal, or oblique asymptotes of a rational function.  
• Work with properties of logarithms.  
• Solve and logarithmic and exponential equation.                                                                                                                                                                      | F-IF.B.4, B.5  
F-IF.C.7d-7e                                                                 | P.3, P.5                                                                |
| 4  | • Use a graph to determine where a function is increasing, decreasing or constant and locate its local maxima or minima.  
• Determine odd and even functions from a graph or equation.  
• Determine the domain, range, period, and signs of the trigonometric functions.  
• Find the values of a trigonometric function utilizing fundamental identities.  
• Find the average rate of change of a function.                                                                                                                                                                       | F-TF.A.1, A.3  
F-TF.B.5, B.6  
F-TF.C.8                                                                 | P.2, P.3                                                                |
Research about Teaching and Learning Mathematics

Structure teaching of mathematical concepts and skills around problems to be solved (Checkly, 1997; Wood & Sellars, 1996; Wood & Sellars, 1997)

Encourage students to work cooperatively with others (Johnson & Johnson, 1975; Davidson, 1990)

Use group problem-solving to stimulate students to apply their mathematical thinking skills (Artzt & Armour-Thomas, 1992)

Students interact in ways that support and challenge one another’s strategic thinking (Artzt, Armour-Thomas, & Curcio, 2008)

Activities structured in ways allowing students to explore, explain, extend, and evaluate their progress (National Research Council, 1999)

There are three critical components to effective mathematics instruction (Shellard & Moyer, 2002):

- Teaching for conceptual understanding
- Developing children’s procedural literacy
- Promoting strategic competence through meaningful problem-solving investigations

Teachers should be:

- Demonstrating acceptance and recognition of students’ divergent ideas
- Challenging students to think deeply about the problems they are solving, extending thinking beyond the solutions and algorithms required to solve the problem
- Influencing learning by asking challenging and interesting questions to accelerate students’ innate inquisitiveness and foster them to examine concepts further
- Projecting a positive attitude about mathematics and about students’ ability to “do” mathematics

Students should be:

- Actively engaging in “doing” mathematics
- Solving challenging problems
- Investigating meaningful real-world problems
- Making interdisciplinary connections
- Developing an understanding of mathematical knowledge required to “do” mathematics and connect the language of mathematical ideas with numerical representations
- Sharing mathematical ideas, discussing mathematics with one another, refining and critiquing each other’s ideas and understandings
- Communicating in pairs, small group, or whole group presentations
- Using multiple representations to communicate mathematical ideas
- Using connections between pictures, oral language, written symbols, manipulative models, and real-world situations
- Using technological resources and other 21st century skills to support and enhance mathematical understanding
Mathematics is not a stagnant field of textbook problems; rather, it is a dynamic way of constructing meaning about the world around us, generating knowledge and understanding about the real world every day. Students should be metaphorically rolling up their sleeves and “doing mathematics” themselves, not watching others do mathematics for them or in front of them. (Protheroe, 2007)

Balanced Mathematics Instructional Model

Balanced math consists of three different learning opportunities; guided math, shared math, and independent math. Ensuring a balance of all three approaches will build conceptual understanding, problem solving, computational fluency, and procedural fluency. Building conceptual understanding is the focal point of developing mathematical proficiency. Students should frequently work on rigorous tasks, talk about the math, explain their thinking, justify their answer or process, build models with graphs or charts or manipulatives, and use technology.

When balanced math is used in the classroom it provides students opportunities to:

- solve problems
- make connections between math concepts and real-life situations
- communicate mathematical ideas (orally, visually and in writing)
- choose appropriate materials to solve problems
- reflect and monitor their own understanding of the math concepts
- practice strategies to build procedural and conceptual confidence

Teacher builds conceptual understanding by modeling through demonstration, explicit instruction, and think alouds, as well as guiding students as they practice math strategies and apply problem solving strategies. (whole group or small group instruction)

Teacher and students practice mathematics processes together through interactive activities, problem solving, and discussion. (whole group or small group instruction)

Students practice math strategies independently to build procedural and computational fluency. Teacher assesses learning and reteaches as necessary. (whole group instruction, small group instruction, or centers)
<table>
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<th>Effective Pedagogical Routines/Instructional Strategies</th>
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<td>Collaborative Problem Solving</td>
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<td>Connect Previous Knowledge to New Learning</td>
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<td>Making Thinking Visible</td>
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<td>Develop and Demonstrate Mathematical Practices</td>
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<td>Inquiry-Oriented and Exploratory Approach</td>
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<td>Multiple Solution Paths and Strategies</td>
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<td>Explain the Rationale of your Math Work</td>
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<td>Quick Writes</td>
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<td>Analyze Student Work</td>
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<td>Identify Student’s Mathematical Understanding</td>
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<td>Identify Student’s Mathematical Misunderstandings</td>
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<td>Interviews</td>
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<td>Role Playing</td>
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<tr>
<td>Diagrams, Charts, Tables, and Graphs</td>
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<td>Anticipate Likely and Possible Student Responses</td>
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<td>Collect Different Student Approaches</td>
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<td>Multiple Response Strategies</td>
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<td>Asking Assessing and Advancing Questions</td>
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<td>Revoicing</td>
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<td>Marking</td>
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<td>Recapping</td>
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<td>Challenging</td>
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<td>Pressing for Accuracy and Reasoning</td>
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<td>Maintain the Cognitive Demand</td>
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Educational Technology

Standards


➢ Technology Operations and Concepts
  • Construct a spreadsheet workbook with multiple worksheets, rename tabs to reflect the data on the worksheet, and use mathematical or logical functions, charts and data from all worksheets to convey the results.

  **Example:** Students will use appropriate digital tools and software to problem solve, check for accuracy and support answers.

➢ Digital Citizenship
  • Analyze the capabilities and limitations of current and emerging technology resources and assess their potential to address personal, social, lifelong learning and career needs.

  **Example:** Students will be able to use critical thinking skills to plan and conduct research, manage projects, solve problems, and make informed decisions using appropriate digital tools and resources.

➢ Research and Information Literacy
  • Produce a position statement about a real world problem by developing a systematic plan of investigation with peers and experts synthesizing information from multiple sources.

  **Example:** Students will produce a position statement about a real world problem by developing a systematic plan of investigation with peers and experts synthesizing information from multiple sources.

➢ Design: Critical Thinking, Problem Solving, and Decision Making
  • Evaluate the strengths and limitations of emerging technologies and their impact on educational, career, personal and or social needs.

  **Example:** A problem-solving approach will allow students to construct their own idea about mathematics and to take responsibility for their own learning.
Career Ready Practices

Career Ready Practices describe the career-ready skills that all educators in all content areas should seek to develop in their students. They are practices that have been linked to increase college, career, and life success. Career Ready Practices should be taught and reinforced in all career exploration and preparation programs with increasingly higher levels of complexity and expectation as a student advances through a program of study.

- **CRP2. Apply appropriate academic and technical skills.**
  Career-ready individuals readily access and use the knowledge and skills acquired through experience and education to be more productive. They make connections between abstract concepts with real-world applications, and they make correct insights about when it is appropriate to apply the use of an academic skill in a workplace situation.
  **Example:** Students are able to apply concepts in the classroom to solve real-life application problems and analyze their solutions for accuracy.

- **CRP4. Communicate clearly and effectively and with reason.**
  Career-ready individuals communicate thoughts, ideas, and action plans with clarity, whether using written, verbal, and/or visual methods. They communicate in the workplace with clarity and purpose to make maximum use of their own and others’ time. Interacting with others; they are active listeners and speak clearly and with purpose. Career-ready individuals think about the audience for their communication and prepare accordingly to ensure the desired outcome.
  **Example:** Students are able to listen, communicate and have constructed arguments to justify their approach and conclusion to a problem.

- **CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.**
  Career-ready individuals readily recognize problems in the workplace, understand the nature of the problem, and devise effective plans to solve the problem. They are aware of problems when they occur and take action quickly to address the problem; they thoughtfully investigate the root cause of the problem prior to introducing solutions. They carefully consider the options to solve the problem. Once a solution is agreed upon, they follow through to ensure the problem is solved, whether through their own actions or the actions of others.
  **Example:** Students are able to utilize concepts learned in the classroom to analyze problems, create a plan to solve the problem, and check their solution for correctness.

- **CRP11. Use technology to enhance productivity.**
  Career-ready individuals find and maximize the productive value of existing and new technology to accomplish workplace tasks and solve workplace problems. They are flexible and adaptive in acquiring new technology. They are proficient with ubiquitous technology applications. They understand the inherent risks-personal and organizational-of technology applications, and they take actions to prevent or mitigate these risks.
  **Example:** Students are able to use a variety of technology such as the TI-84 and online resources to effectively help them in problem solving.
WIDA Proficiency Levels

At the given level of English language proficiency, English language learners will process, understand, produce or use:

<table>
<thead>
<tr>
<th>Level</th>
<th>Language Proficiency</th>
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</table>
| 6- Reaching | • Specialized or technical language reflective of the content areas at grade level  
• A variety of sentence lengths of varying linguistic complexity in extended oral or written discourse as required by the specified grade level  
• Oral or written communication in English comparable to proficient English peers |
| 5- Bridging | • Specialized or technical language of the content areas  
• A variety of sentence lengths of varying linguistic complexity in extended oral or written discourse, including stories, essays or reports  
• Oral or written language approaching comparability to that of proficient English peers when presented with grade level material. |
| 4- Expanding | • Specific and some technical language of the content areas  
• A variety of sentence lengths of varying linguistic complexity in oral discourse or multiple, related sentences or paragraphs  
• Oral or written language with minimal phonological, syntactic or semantic errors that may impede the communication, but retain much of its meaning, when presented with oral or written connected discourse, with sensory, graphic or interactive support |
| 3- Developing | • General and some specific language of the content areas  
• Expanded sentences in oral interaction or written paragraphs  
• Oral or written language with phonological, syntactic or semantic errors that may impede the communication, but retain much of its meaning, when presented with oral or written, narrative or expository descriptions with sensory, graphic or interactive support |
| 2- Beginning | • General language related to the content area  
• Phrases or short sentences  
• Oral or written language with phonological, syntactic, or semantic errors that often impede the communication when presented with one to multiple-step commands, directions, or a series of statements with sensory, graphic or interactive support |
| 1- Entering | • Pictorial or graphic representation of the language of the content areas  
• Words, phrases or chunks of language when presented with one-step commands directions, WH-, choice or yes/no questions, or statements with sensory, graphic or interactive support |
## Language Development Supports For English Language Learners

To Increase Comprehension and Communication Skills

### Environment

- Welcoming and stress-free
- Respectful of linguistic and cultural diversity
- Honors students' background knowledge
- Sets clear and high expectations
- Includes routines and norms
- Is thinking-focused vs. answer-seeking
- Offers multiple modalities to engage in content learning and to demonstrate understanding
- Includes explicit instruction of specific language targets
- Provides participation techniques to include all learners
- Integrates learning centers and games in a meaningful way
- Provides opportunities to practice and refine receptive and productive skills in English as a new language
- Integrates meaningful and purposeful tasks/activities that:
  - Are accessible by all students through multiple entry points
  - Are relevant to students' lives and cultural experiences
  - Build on prior mathematical learning
  - Demonstrate high cognitive demand
  - Offer multiple strategies for solutions
  - Allow for a language learning experience in addition to content

### Sensory Supports*

- Real-life objects (realia) or concrete objects
- Physical models
- Manipulatives
- Pictures & photographs
- Visual representations or models such as diagrams or drawings
- Videos & films
- Newspapers or magazines
- Gestures
- Physical movements
- Music & songs

### Graphic Supports*

- Graphs
- Charts
- Timelines
- Number lines
- Graphic organizers
- Graphing paper

### Interactive Supports*

- In a whole group
- In a small group
- With a partner such as Turn-and-Talk
- In pairs as a group (first, two pairs work independently, then they form a group of four)
- In triads
- Cooperative learning structures such as Think-Pair-Share
- Interactive websites or software
- With a mentor or coach

### Verbal and Textual Supports

- Labeling
- Students' native language
- Modeling
- Repetitions
- Paraphrasing
- Summarizing
- Guiding questions
- Clarifying questions
- Probing questions
- Leveled questions such as What? When? Where? How? Why?
- Questioning prompts & cues
- Word Banks
- Sentence starters
- Sentence frames
- Discussion frames
- Talk moves including Wait Time

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BUILDING EQUITY IN YOUR TEACHING PRACTICE

How do the essential questions highlight the connection between the big ideas of the unit and equity in your teaching practice?

**CONTENT INTEGRATION**
Teachers use examples and content from a variety of cultures & groups.

- This unit / lesson is connected to other topics explored with students.
- There are multiple viewpoints reflected in the content of this unit / lesson.
- The materials and resources are reflective of the diverse identities and experiences of students.
- The content affirms students, as well as exposes them to experiences other than their own.

**KNOWLEDGE CONSTRUCTION**
Teachers help students understand how knowledge is created and influenced by cultural assumptions, perspectives & biases.

- This unit / lesson provides context to the history of privilege and oppression.
- This unit / lesson addresses power relationships.
- This unit / lesson help students to develop research and critical thinking skills.
- This curriculum creates windows and mirrors* for students.

**PREJUDICE REDUCTION**
Teachers implement lessons and activities to assert positive images of ethnic groups & improve intergroup relations.

- This unit / lesson help students question and unpack biases & stereotypes.
- This unit / lesson help students examine, research and question information and sources.
- The curriculum encourage discussion and understanding about the groups of people being represented.
- This unit / lesson challenges dominant perspectives.

**EQUITABLE PEDAGOGY**
Teachers modify techniques and methods to facilitate the academic achievement of students from diverse backgrounds.

- The instruction has been modified to meet the needs of each student.
- Students feel respected and their cultural identities are valued.
- Additional supports have been provided for students to become successful and independent learners.
- Opportunities are provided for student to reflect on their learning and provide feedback.

**EMPOWERING SCHOOL CULTURE**
Using the other four dimensions to create a safe and healthy educational environment for all.

- There are opportunities for students to connect with the community.
- My classroom is welcoming and supportive for all students.
- I am aware of and sensitive to the needs of my students and their families.
- There are effective parent communication systems established. Parents can talk to me about issues as they arise in my classroom.

Culturally Relevant Pedagogy Examples

• **Everyone has a Voice:** Create a classroom environment where students know that their contributions are expected and valued.
  
  **Example:** Norms for sharing are established that communicate a growth mindset for mathematics. All students are capable of expressing mathematical thinking and contributing to the classroom community. Students learn new ways of looking at problem solving by working with and listening to each other.

• **Integrate Relevant Word Problems:** Contextualize equations using word problems that reference student interests and cultures.
  
  **Example:** When learning to interpret functions, problems that relate to student interests such as music, sports and art enable the students to understand and relate to the concept in a more meaningful way.

• **Present New Concepts Using Student Vocabulary:** Use student diction to capture attention and build understanding before using academic terms.
  
  **Example:** Work with students to create a variety of sorting and match games of vocabulary words in this unit. Students can work in teams or individually to play these games for approximately 10-15 minutes each week. This will give students a different way of becoming familiar with the vocabulary rather than just looking up the words or writing the definition down.

• **Call on Each Student:** Encourage each student to share his or her thoughts through call-and-response, keeping the class’s attention in the process.
  
  **Example:** Foster confidence. Make the assessment process less intimidating by offering different ways to demonstrate skills and understanding. For example, avoid handing out quizzes that are purely multiple choice or fill-in-the-blank. Among other question types, mix in problems that involve explaining the step necessary to get to the answer. After, give students time to assess their own progress and performance, helping them focus on growth.
## Differentiated Instruction

**Accommodate Based on Students Individual Needs: Strategies**

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<tr>
<th>Time/General</th>
<th>Processing</th>
<th>Comprehension</th>
<th>Recall</th>
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<tbody>
<tr>
<td>• Extra time for assigned tasks</td>
<td>• Extra Response time</td>
<td>• Precise processes for balanced math instructional model</td>
<td>• Teacher-made checklist</td>
</tr>
<tr>
<td>• Adjust length of assignment</td>
<td>• Have students verbalize steps</td>
<td>• Short manageable tasks</td>
<td>• Use visual graphic organizers</td>
</tr>
<tr>
<td>• Timeline with due dates for reports and projects</td>
<td>• Repeat, clarify or reword directions</td>
<td>• Brief and concrete directions</td>
<td>• Reference resources to promote independence</td>
</tr>
<tr>
<td>• Communication system between home and school</td>
<td>• Mini-breaks between tasks</td>
<td>• Provide immediate feedback</td>
<td>• Visual and verbal reminders</td>
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<tr>
<td>• Provide lecture notes/outline</td>
<td>• Provide a warning for transitions</td>
<td>• Small group instruction</td>
<td>• Graphic organizers</td>
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<td></td>
<td>• Reading partners</td>
<td>• Emphasize multi-sensory learning</td>
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<th>Assistive Technology</th>
<th>Tests/Quizzes/Grading</th>
<th>Behavior/Attention</th>
<th>Organization</th>
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<tr>
<td>• Computer/whiteboard</td>
<td>• Extended time</td>
<td>• Consistent daily structured routine</td>
<td>• Individual daily planner</td>
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<tr>
<td>• Tape recorder</td>
<td>• Study guides</td>
<td>• Simple and clear classroom rules</td>
<td>• Display a written agenda</td>
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<tr>
<td>• Video Tape</td>
<td>• Shortened tests</td>
<td>• Frequent feedback</td>
<td>• Note-taking assistance</td>
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<td></td>
<td>• Read directions aloud</td>
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<td>• Color code materials</td>
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<td></td>
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## Differentiated Instruction

### Accommodate Based on Content Needs: Strategies

- Anchor charts to model strategies
- Review Algebra concepts to ensure students have the information needed to progress in understanding
- Pre-teach pertinent vocabulary
- Provide reference sheets that list formulas, step-by-step procedures, theorems, and modeling of strategies
- Word wall with visual representations of mathematical terms
- Teacher modeling of thinking processes involved in solving, graphing, and writing equations
- Introduce concepts embedded in real-life context to help students relate to the mathematics involved
- Record formulas, processes, and mathematical rules in reference notebooks
- Graphing calculator to assist with computations and graphing of trigonometric functions
- Utilize technology through interactive sites to represent nonlinear data
- Graphic organizers to help students interpret the meaning of terms in an expression or equation in context
- Translation dictionary
- Sentence stems to provide additional language support for ELL students.
Interdisciplinary Connections

Model interdisciplinary thinking to expose students to other disciplines.

Social Studies Connection: - NJSLS 6.1.8.B.1.a, 6.1.8.C.1.b, 6.1.8.D.1.a, 6.1.8.B.2.a

Name of Task:

- Rainforest Deforestation- Problem or Myth?
  
  http://illuminations.nctm.org/Lesson.aspx?id=3820

- Rock Concert
  
  http://apcentral.collegeboard.com/apc/public/repository/ap09_calculus_ab_q2.pdf

Science Connection: - NJSLS HS-PS2-2, HS-PS2-5, HS-PS4-1

Name of Task:

- Moving Particle
  
Enrichment

What is the purpose of Enrichment?

- The purpose of enrichment is to provide extended learning opportunities and challenges to students who have already mastered, or can quickly master, the basic curriculum. Enrichment gives the student more time to study concepts with greater depth, breadth, and complexity.
- Enrichment also provides opportunities for students to pursue learning in their own areas of interest and strengths.
- Enrichment keeps advanced students engaged and supports their accelerated academic needs.
- Enrichment provides the most appropriate answer to the question, “What do you do when the student already knows it?”

Enrichment is…

- Planned and purposeful
- Different, or differentiated, work – not just more work
- Responsive to students’ needs and situations
- A promotion of high-level thinking skills and making connections within content
- The ability to apply different or multiple strategies to the content
- The ability to synthesize concepts and make real world and cross-curricular connections
- Elevated contextual complexity
- Sometimes independent activities, sometimes direct instruction
- Inquiry based or open ended assignments and projects
- Using supplementary materials in addition to the normal range of resources
- Choices for students
- Tiered/Multi-level activities with flexible groups (may change daily or weekly)

Enrichment is not…

- Just for gifted students (some gifted students may need intervention in some areas just as some other students may need frequent enrichment)
- Worksheets that are more of the same (busywork)
- Random assignments, games, or puzzles not connected to the content areas or areas of student interest
- Extra homework
- A package that is the same for everyone
- Thinking skills taught in isolation
- Unstructured free time
Assessments

**Required District/State Assessments**
SGO Assessments

**Suggested Formative/Summative Classroom Assessments**
Describe Learning Vertically
Identify Key Building Blocks
Make Connections (between and among key building blocks)
Short/Extended Constructed Response Items
Multiple-Choice Items (where multiple answer choices may be correct)
Drag and Drop Items
Use of Equation Editor
Quizzes
Journal Entries/Reflections/Quick-Writes
Accountable talk
Projects
Portfolio
Observation
Graphic Organizers/Concept Mapping
Presentations
Role Playing
Teacher-Student and Student-Student Conferencing
Homework
F-IF.A.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If $f$ is a function and $x$ is an element of its domain, then $f(x)$ denotes the output of $f$ corresponding to the input $x$. The graph of $f$ is the graph of the equation $y = f(x)$.

F-IF.B.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

F-IF.B.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.

F-IF.C.7a Graph linear and quadratic functions and show intercepts, maxima, and minima.

F-IF.C.7b Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

F-IF.C.7c Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.

F-IF.C.7d Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.

F-IF.C.7e Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

F-BF.A.1a Determine an explicit expression, a recursive process, or steps for calculation from a context.

F-BF.A.1b Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.

F-BF.A.1c Compose functions. For example, if $T(y)$ is the temperature in the atmosphere as a function of height, and $h(t)$ is the height of a weather balloon as a function of time, then $T(h(t))$ is the temperature at the location of the weather balloon as a function of time.

F-TF.A.1 Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
<table>
<thead>
<tr>
<th>New Jersey Student Learning Standards (NJSLS)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>F-TF.A.3</strong> Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosines, and tangent for $\pi-x$, $\pi+x$, and $2\pi-x$ in terms of their values for $x$, where $x$ is any real number.</td>
</tr>
<tr>
<td><strong>F-TF.B.5</strong> Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.</td>
</tr>
<tr>
<td><strong>F-TF.B.6</strong> Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.</td>
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<tr>
<td><strong>F-TF.C.8</strong> Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle.</td>
</tr>
</tbody>
</table>
## Mathematical Practices for AP Calculus (by the College Board)

### MPAC 1: Reasoning with definitions and theorems

Students can:
- Use definitions and theorems to build arguments, to justify conclusions or answers, and to prove results;
- Confirm that hypotheses have been satisfied in order to apply the conclusion of a theorem;
- Apply definitions and theorems in the process of solving a problem;
- Interpret quantifiers in definitions and theorems (e.g., “for all,” “there exists”);
- Develop conjectures based on exploration with technology; and
- Produce examples and counterexamples to clarify understanding of definitions, to investigate whether converses of theorems are true or false, or to test conjectures.

### MPAC 2: Connecting concepts

Students can:
- Relate the concept of a limit to all aspects of calculus;
- Use the connection between concepts (e.g., rate of change and accumulation) or processes (e.g., differentiation and its inverse process, anti-differentiation) to solve problems;
- Connect concepts to their visual representations with and without technology; and
- Identify a common underlying structure in problems involving different contextual situations.

### MPAC 3: Implementing algebraic/computational processes

Students can:
- Select appropriate mathematical strategies;
- Sequence algebraic/computational procedures logically;
- Complete algebraic/computational processes correctly;
- Apply technology strategically to solve problems;
- Attend to precision graphically, numerically, analytically, and verbally and specify units of measure; and
- Connect the results of algebraic/computational processes to the question asked.
MPAC 4: Connecting multiple representations
Students can:
• Associate tables, graphs, and symbolic representations of functions;
• Develop concepts using graphical, symbolical, verbal, or numerical representations with and without technology;
• Identify how mathematical characteristics of functions are related in different representations;
• Extract and interpret mathematical content from any presentation of a function (e.g., utilize information from a table of values);
• Construct one representational form from another (e.g., a table from a graph or a graph from given information); and
• Consider multiple representations (graphical, numerical, analytical, and verbal) of a function to select or construct a useful representation for solving a problem.

MPAC 5: Building notational fluency
Students can:
• Know and use a variety of notations
• Connect notation to definitions (e.g., relating the notation for the definite integral to that of the limit of a Riemann sum);
• Connect notation to different representations (graphical, numerical, analytical, and verbal); and
• Assign meaning to notation, accurately interpreting the notation in a given problem and across different contexts.

MPAC 6: Communicating
Students can:
• Clearly present methods, reasoning, justifications, and conclusions;
• Use accurate and precise language and notation;
• Explain the meaning of expressions, notation, and results in terms of a context (including units);
• Explain the connections among concepts;
• Critically interpret and accurately report information provided by technology; and
• Analyze, evaluate, and compare the reasoning of others.
### Grade: 12  
### Unit: 1 (One)  
### Topic: Functions

**Unit 1 NJSLS:** F-IF.A.1, F-IF.B.4, F-IF.B.5, F-IF.C.7a-e, F-BF.A.1a-c, F-TF.A.1, F-TF.A.3, F-TF.B.5, F-TF.B.6, F-TF.C.8

#### Student Learning Objective (SLO 1):
- Determine whether a relation represents a function.
- Identify and obtain information from or about a graph of a function.
- Graph linear, constant, identity, square, cube, square-root, reciprocal and greatest-integer functions

#### New Jersey Student Learning Standards (NJSLS):

**F-IF.A.1:** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If \( f \) is a function and \( x \) is an element of its domain, then \( f(x) \) denotes the output of \( f \) corresponding to the input \( x \). The graph of \( f \) is the graph of the equation \( y = f(x) \).

**F-IF.B.4:** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

**F-IF.B.5:** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.

**F-IF.C.7a:** Graph linear and quadratic functions and show intercepts, maxima, and minima.

**F-IF.C.7b:** Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

**F-IF.C.7c:** Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.

<table>
<thead>
<tr>
<th>MPs</th>
<th>Skills, Strategies &amp; Concepts</th>
<th>Essential Understandings/Questions</th>
<th>Tasks/Activities</th>
</tr>
</thead>
<tbody>
<tr>
<td>• MPAC 3</td>
<td>• Students are expected to draw on previous knowledge in finding the graphs of various kinds of functions. It may be necessary to</td>
<td>• Numerical, algebraic, and graphical models provide different methods to visualize, analyze, and understand data.</td>
<td>• Ivy Global</td>
</tr>
<tr>
<td>• MPAC 5</td>
<td></td>
<td></td>
<td>• Understanding Functions</td>
</tr>
<tr>
<td>• MPAC 6</td>
<td></td>
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<td>• Hoisting the Flag 1</td>
</tr>
<tr>
<td>Discuss what important features are expected to be shown in the graphs.</td>
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<tr>
<td>• Represent functions numerically, algebraically, and graphically, determine the domain and range for functions, and analyze function characteristics such as extreme value, symmetry, asymptotes, and end behavior.</td>
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<tr>
<td>• Encourage students to apply prior knowledge and intuition to determine whether functions are odd or even.</td>
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<tr>
<td>• Review the concept of the vertical line test for recognizing graphs of functions.</td>
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</tr>
<tr>
<td>• Functions and graphs form the basis for understanding the mathematics and applications you see both in the work place and coursework in college.</td>
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<tr>
<td>• Influenza Epidemic</td>
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<tr>
<td>• Interpreting Graphs</td>
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<tr>
<td>* SEE UNIT 1 TASK FOLDER</td>
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</tr>
</tbody>
</table>
**Student Learning Objective (SLO 2):**

- Form the sum, difference, product, and quotient of two functions.
- Form a composite function and find its domain.
- Construct, analyze and solve problems by constructing functions.
- Graph transformations of functions.

**New Jersey Student Learning Standards (NJSLS):**

**F-BF.A.1a:** Determine an explicit expression, a recursive process, or steps for calculation from a context.

**F-BF.A.1b:** Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.

**F-BF.A.1c:** Compose functions. For example, if \( T(y) \) is the temperature in the atmosphere as a function of height, and \( h(t) \) is the height of a weather balloon as a function of time, then \( T(h(t)) \) is the temperature at the location of the weather balloon as a function of time.

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<tr>
<td>MPAC 3</td>
<td>Students often expect the domain of the sum, difference, product, or quotient function to be obvious from looking at the new function equation, however this is not always the case. Give examples for instance: ((fg)(x), \text{where } f(x) = g(x) = \sqrt{x}. ) Then ((fg)(x) = x, ) but the domain of ( fg ) is the set of nonnegative real numbers only.</td>
<td>Most functions that you encounter in calculus and in real life can be created by combining or modifying other functions.</td>
<td>The Canoe Trip 1</td>
</tr>
<tr>
<td>MPAC 5</td>
<td></td>
<td>Studying transformation helps to understand the relationships between graphs that have similarities but are not the same</td>
<td>The Canoe Trip 2</td>
</tr>
<tr>
<td>MPAC 6</td>
<td></td>
<td></td>
<td>Building Explicit Quadratic Functions by Composition</td>
</tr>
</tbody>
</table>
<pre><code>                                                                                           |                                                                                                   | Transformations of the Graphs of Logarithmic and Exponential Functions            |
</code></pre>
- Point out and review that the domain of the sum, difference, product, or quotient function is the intersection of the individual domains (less the zeros of g, in the case of the quotient function.

- Explore the connection between transformation and composition of functions. For example, if \( g(x) = x + 3 \), then \( f(x + 3) = f(g(x)) \).

- Graphing utilities are great tools for showing the transformation applied to parent functions.

* SEE UNIT 1 TASK FOLDER
Grade: 12  
Unit: 1 (One)  
Topic: Functions

**Student Learning Objective (SLO 3):**
- Analyze and graph quadratic, power, polynomial rational, and logarithms functions.
- Determine the vertical, horizontal, or oblique asymptotes of a rational function.
- Work with properties of logarithms.
- Solve and logarithmic and exponential equation.

**New Jersey Student Learning Standards (NJSLS):**

**F-IF.B.4:** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

**F-IF.B.5:** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.

**F-IF.C.7d:** Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.

**F-IF.C.7e:** Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

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<tbody>
<tr>
<td>MPAC 1</td>
<td>Students will learn about:</td>
<td>- Numerical, algebraic, and graphical models provide different methods to visualize, analyze, and understand data.</td>
<td></td>
</tr>
<tr>
<td>MPAC 3</td>
<td>- Transformations</td>
<td>- Many business and economic problems can be modeled by linear functions. Quadratic and higher</td>
<td></td>
</tr>
<tr>
<td>MPAC 5</td>
<td>- Vertical and Horizontal Transformations</td>
<td></td>
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<tr>
<td>MPAC 6</td>
<td>- Reflections Across the Axes</td>
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<td></td>
<td>- Vertical and Horizontal Stretches</td>
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<tr>
<td></td>
<td>- Combining Functions</td>
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</tr>
</tbody>
</table>

- **Invertible or Not?**
- **Oakland Coliseum**
- **Graphing Rational Functions**
- **Exponential Kiss**
| • Use functions to model application problems. |
| • Sketch power functions in the form of $f(x) = kx^a$ (where $k$ and $a$ are rational numbers). |
| • Graph polynomial functions, predict their end behavior, and find their real zeros using a graphing and algebraic method. |
| • Describe graphs |
| • Determine the algebraic representation and graphical representation of functions and their inverse. |
| • Use properties of logarithms to solve problems |

degree polynomial functions can be used to model some manufacturing applications.

• Power functions specify the proportional relationships of geometry, chemistry, and physics.

• Polynomial functions of higher degree with modeling can be used to provide approximations to more complicated functions which will be explored in calculus.

• Rational functions are used in calculus and in scientific applications such as Kepler’s Law.

• Logarithmic functions are used in many applications, including the measurement of the relative intensity of sounds

| • Graphs of Composition Functions |
| • Graphs of Logarithmic Functions |

* SEE UNIT 1 TASK FOLDER
### Student Learning Objective (SLO 4):

- Use a graph to determine where a function is increasing, decreasing or constant and locate its local maxima or minima.
- Determine odd and even functions from a graph or equation.
- Determine the domain, range, period, and signs of the trigonometric functions.
- Find the values of a trigonometric function utilizing fundamental identities.
- Find the average rate of change of a function.

### New Jersey Student Learning Standards (NJSLS):

**F-TF.A.1:** Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.

**F-TF.A.3:** Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosines, and tangent for $\pi-x$, $\pi+x$, and $2\pi-x$ in terms of their values for $x$, where $x$ is any real number.

**F-TF.B.5:** Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.

**F-TF.B.6:** Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.

**F-TF.C.8:** Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle.

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<tbody>
<tr>
<td>MPAC 1</td>
<td>Students will learn about:</td>
<td>• Angles are the domain elements of the trigonometric functions.</td>
<td>• What exactly is a radian?</td>
</tr>
<tr>
<td>MPAC 3</td>
<td>• Convert between degree and radians.</td>
<td></td>
<td>• Special Triangles 1</td>
</tr>
<tr>
<td>MPAC 5</td>
<td>MPAC 6</td>
<td>MPAC 5</td>
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<tr>
<td>Identify the periodicity and even-odd properties of the trigonometric functions.</td>
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<tr>
<td>Generate graphs of trigonometric functions and explore various transformation upon these graphs.</td>
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<tr>
<td>Use the inverse of trigonometric functions to solve problems.</td>
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<tr>
<td>Calculate average velocity.</td>
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<td></td>
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</tr>
<tr>
<td>Calculate instantaneous velocity.</td>
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<tr>
<td>Use the fundamental identities to simplify trigonometric expressions and solve trigonometric equations.</td>
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</tr>
<tr>
<td>Have students graph $y = \sin^3 x + \cos^2 x \sin x$ in standard trigonometric viewing window. Then discuss why the graph looks as it does and ask students to name the function being viewed.</td>
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<tr>
<td>Have students find calculate the instantaneous velocities and derivatives using limits</td>
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<tr>
<td>Extending trigonometric functions beyond triangle ratios opens a new world of applications.</td>
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<tr>
<td>Sine and cosine gain added significance when used to model waves and periodic behavior.</td>
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<tr>
<td>Identities are important when working with trigonometric functions in calculus.</td>
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<tr>
<td>Derivative allows us to analyze rate of change, which are fundamental to understanding physics, economics, engineering, and even history</td>
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</tbody>
</table>

*Foxes & Rabbits 2*

*Hours of Daylight 1*

*Calculation with Sine & Cosine*

*Finding Trigonometric Values*

*Trigonometric Ratio & Pythagorean Theorem*

*SEE UNIT 1 TASK FOLDER*
### Vocabulary

<table>
<thead>
<tr>
<th>Area below the curve</th>
<th>First Derivative Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>Area between the!</td>
<td>Fundamental Theorem of Calculus</td>
</tr>
<tr>
<td>Area under the curve</td>
<td>Higher Derivative</td>
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<tr>
<td>Asymptote</td>
<td>Implicit Differentiation</td>
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<tr>
<td>Average rate of change</td>
<td>Infinite Limit</td>
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<td>Axis of rotation</td>
<td>Inflection Point</td>
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<td>Bounds of integration</td>
<td>Instantaneous Velocity</td>
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<td>Chain rule</td>
<td>Integral</td>
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<td>Concave up</td>
<td>Integration by parts</td>
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<tr>
<td>Concave down</td>
<td>Integration by substitution</td>
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<tr>
<td>Continuous function</td>
<td>Intermediate Value Theorem</td>
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<tr>
<td>Continuous differentiable function</td>
<td>L’Hopital’s Rule</td>
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<tr>
<td>Critical number</td>
<td>Limit</td>
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<tr>
<td>Critical point</td>
<td>Right Limit/ Left Limit</td>
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<tr>
<td>Critical value</td>
<td>Local maximum/ Local minimum</td>
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<tr>
<td>Curve sketching</td>
<td>Mean Value Theorem</td>
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<td>Cylindrical Shell Method</td>
<td>One-sided limit</td>
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<td>Decreasing function</td>
<td>Product Rule</td>
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<td>Definite Integral</td>
<td>Projectile Motion</td>
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<tr>
<td>Definite integral rules</td>
<td>Quotient Rule</td>
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<tr>
<td>Derivative</td>
<td>Relative maximum/ Relative minimum</td>
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<tr>
<td>Derivative of a Power Series</td>
<td>Riemann Sum</td>
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<td>Derivative Rules</td>
<td>Rolle’s Theorem</td>
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<td>Discontinuity</td>
<td>Sandwich Theorem</td>
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<td>Disk method</td>
<td>U-Substitution</td>
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<td>Divergent Sequence and Series</td>
<td>Volume by parallel cross section</td>
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<tr>
<td>Explicit Differentiation</td>
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<td>Projectile Motion</td>
<td>Volume by parallel cross section</td>
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## References & Suggested Instructional Websites

- [https://www.khanacademy.org/math/ap-calculus-ab](https://www.khanacademy.org/math/ap-calculus-ab)
- [http://www.mathwords.com/index_calculus.htm](http://www.mathwords.com/index_calculus.htm)
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- [http://www.mathwithmrwood.com/ap-calculus](http://www.mathwithmrwood.com/ap-calculus)
- [http://www.analyzemath.com/ap_calculus.html](http://www.analyzemath.com/ap_calculus.html)
- [https://online.math.uh.edu/apcalculus/exams/](https://online.math.uh.edu/apcalculus/exams/)
### Field Trips

<table>
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<tr>
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<td><strong>SIX FLAGS GREAT ADVENTURE</strong>:</td>
<td>This educational event includes workbooks and special science and math related shows throughout the day. Your students will leave with a better understanding of real world applications of the material they have learned in the classroom. Each student will have the opportunity to experience different rides and attractions linking mathematical and scientific concepts to what they are experiencing. <a href="http://www.sixflags.com">www.sixflags.com</a></td>
</tr>
<tr>
<td><strong>MUSEUM of MATHEMATICS</strong>:</td>
<td>Mathematics illuminates the patterns that abound in our world. The National Museum of Mathematics strives to enhance public understanding and perception of mathematics. Its dynamic exhibits and programs stimulate inquiry, spark curiosity, and reveal the wonders of mathematics. The Museum’s activities lead a broad and diverse audience to understand the evolving, creative, human, and aesthetic nature of mathematics. <a href="http://www.momath.org">www.momath.org</a></td>
</tr>
<tr>
<td><strong>LIBERTY SCIENCE CENTER</strong>:</td>
<td>An interactive science museum and learning center located in Liberty State Park. The center, which first opened in 1993 as New Jersey's first major state science museum, has science exhibits, the largest IMAX Dome Theater in the United States, numerous educational resources, and the original Hoberman sphere. <a href="http://lsc.org/plan-your-visit/">http://lsc.org/plan-your-visit/</a></td>
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</table>
# Marking Period 1 – Suggested Project

*Projects beginnings at the start of the marking period and finishes at the end.*

<table>
<thead>
<tr>
<th>Task</th>
<th>Example(s)/Details</th>
<th>Portion of Project</th>
<th>Due Date</th>
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</thead>
<tbody>
<tr>
<td>Choose one topic that will be covered during MP1</td>
<td>Modeling and Equation Solving, Graphs of rational functions, Logarithmic Functions etc.</td>
<td>Proposal (submit your top topic 3- I will TRY to give you your first choice!)</td>
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<td>Submit using Google Docs</td>
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| Using your topic- identify the skills that you will need to understand the concept (past & present) | **Topic:** Twelve Basic Functions  
  - Identify the graphs of common functions  
  - Sketch the graphs of common functions  
  - Identify the key characteristics of common graphs  
  - Domain and range  
  - Continuity  
  - Boundedness, etc.. | Key Concepts  
Submit using Google Docs |          |
| Create a lesson that you would use to teach the concepts to your peers |  
  - Identify the graphs of common functions  
  - Show graph examples of the 12 basic functions, etc…….. | Use Google Docs or Google Slides |          |
| Use at least 10 students and one teacher to teach your lesson. (11th/10th grade student currently enrolled in Algebra II) |  
  - You will create a Pre/Post test using Google Forms (Multiple Choice)  
  - minimum 4 questions  
  - Participants will take the test on Google Forms prior to receiving your lesson  
  - At the conclusion of your lesson participants will take your test again | Submission of Pre/Post Test  
Submit using Google Forms |          |
| Data Analysis                                                       |  
  - You MUST use all of the following: Measure of Central Tendency (Mean, Mode, Median) and Standard Deviation. | Submission Included Correction on ALL above parts. |          |
<table>
<thead>
<tr>
<th>Task</th>
<th>Instructions</th>
<th>Google Docs</th>
<th>Final Project</th>
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</thead>
<tbody>
<tr>
<td>Discuss your conjecture in depth. (Were</td>
<td>Discuss all outliners. Discuss any correlations (connections); DO NOT</td>
<td>Submission Included on Running Google Doc</td>
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<td>you right or wrong, did you find anything</td>
<td>makes conclusions about CAUSATION.</td>
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<td>Final Project</td>
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<td>surprising? Etc.)</td>
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<tr>
<td>o Discuss your conjecture in depth.</td>
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<td>o Discuss all outliners.</td>
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<td>o Discuss any correlations (connections);</td>
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<td>DO NOT makes conclusions about CAUSATION.</td>
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<td>Create Visual Representation</td>
<td>o Use graphical representation to present your data. Example: Pre/Post Test,</td>
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<td>Skill of question, etc.</td>
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<td></td>
<td>*You may graph You may use <a href="https://www.meta-chart.com">https://www.meta-chart.com</a> for your graphing essentials and add it to your project</td>
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<td>Submit</td>
<td>Each person must type their project in Google Docs and share it with_______</td>
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<td>Final Project</td>
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<td>Each of the analysis of the data must be on separate pages. The project</td>
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<td>must include a title page and table of contents. Be sure all parts of your</td>
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<td>project are properly labeled with appropriate capitalization and grammar.</td>
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<td></td>
<td>Us the equation editor in Google Docs to assist you. You may hand draw your</td>
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<td>graphs and take pictures and add them to Google docs or use <a href="https://www.meta-chart.com">https://www.meta-chart.com</a> and add it to your Google Docs.</td>
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<tr>
<td>Presentation</td>
<td>Each person will make a 10-minutes presentation about their project, including</td>
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<td>Presentation</td>
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<td>the data found. You will then field questions from the class and myself, and</td>
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<td>you will be expected to understand the data well enough to answer the questions</td>
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<td>coherently.</td>
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<td>TASK</td>
<td>DUE DATE</td>
<td>POSSIBLE POINTS</td>
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<tr>
<td>Choose one topic from that will be covered during Mp1</td>
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<td>(5 Points) Student submitted topic for approval on time using Google Docs</td>
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</table>
| Using your topic- identify the skills that you will need to understand the concept (past & present) |                                       | (20 Points) • Student identified past and current skills that are important to understand the concept.  
• Student give a brief explanation of why each skill is necessary  
• Student submitted on time using Google Docs                        |
| Create a lesson that you would use to teach the concepts to your peers |                                       | (20 Points) Student created a well thought out lesson to teach their selected concept                 |
| Create a pre-test that students will take before lesson- use google forms.  
Create a post-test that students will take after the lesson- use google forms. (minimum 5 questions) | | (20 Points) • Student created a Pre/Post Test that is relevant to their topic and content that will be presented to students  
• Students created Pre/Post Test using Google Forms                    |
| Use at least 10 students and one teacher to teach your lesson. (11th/10th grade student currently enrolled in Algebra II) |                                       | (20 Points) Lesson was well presented, peers were engaged and multiple approaches were used to present the lesson, for example verbal, visual, etc. |
| Data analysis                                                       |                                       | (20 Points) Student use appropriate statistical measures to analyze data and technology to create visual graphs of Pre/Post test results |
| Create visual representation                                        |                                       | (15 Points) Student creativity using technology or Arts skills. Visual must be pleasing to eyes and easy to read. |
| Submit (Entire Presentation from ALL above steps)                   |                                       | (15 Points) Student submitted one project with all the components listed above. Google Submission      |
| Presentation                                                        |                                       | (15 Points) • Student was able to their topic and relevance  
• Student discussed the results from their Pre/Post Test  
• Student was able to answer questions from the class about their lesson |
The exam is 3 hours and 15 minutes long and has two sections — multiple choice and free-response. A graphing calculator is required for parts of the exam (see below). You may not take both the Calculus AB and Calculus BC exams within the same year.

**Section I: Multiple Choice** | 45 Questions | 1 hour and 45 minutes | 50% of Final Exam Score
Part A — 30 questions | 60 minutes (calculator not permitted)
Part B — 15 questions | 45 minutes (graphing calculator required)

**Section II: Free-Response** | 6 Questions | 1 hour and 30 minutes | 50% of Final Exam Score
Part A — 2 problems | 30 minutes (graphing calculator required)
Part B — 4 problems | 60 minutes (calculator not permitted)

Completing Section II: Free-Response Questions
- During the second timed portion of the free-response section (Part B), you are permitted to continue work on problems in Part A, but you are not permitted to use a calculator during this time. For more information, see this course’s calculator policy and the list of approved graphing calculators.

- As you begin each part of Section II, you may wish to look over the questions before starting to work on them. It is not expected that everyone will be able to complete all parts of all questions.

- Show all of your work, even though a question may not explicitly remind you to do so. Clearly label any functions, graphs, tables, or other objects that you use. Justifications require that you give mathematical reasons, and that you verify the needed conditions under which relevant theorems, properties, definitions, or tests are applied. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit.

- Your work must be expressed in standard mathematical notation rather than calculator syntax. For example, $\int_{1}^{5} x^2 \, dx$ may not be written as fnInt($X^2$, X, 1, 5).

- Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If you use decimal approximations in calculations, your work will be scored on accuracy. Unless otherwise specified, your final answers should be accurate to three places after the decimal point.

- Unless otherwise specified, the domain of a function $f$ is assumed to be the set of all real numbers $x$ for which $f(x)$ is a real number.
AP Calculus Test Preparation Strategies

Keep an eye on your time.
Monitor your time carefully. Make sure not to spend too much time on any one question so you’ll have enough time to answer all of them. If you do work that you think is incorrect, simply put an “X” through it instead of spending time erasing it completely: crossed-out work won’t be graded.

Show your work, even when you’re using a calculator.
Show all the steps you took to reach your solution on questions involving calculations. The exam reader wants to see if you know how to solve the problem. If you use your calculator to solve an equation, compute a numerical derivative, or find a definite integral, then be sure to write the equation, derivative, or integral first: an answer without this information might not get full credit, even if the answer is correct. Remember to write your work in standard notation (e.g. \( \int_1^6 x^2\,dx \)) rather than calculator syntax (e.g. fnInt(X^2, X, 1, 5), as calculator syntax is not acceptable.

Try to solve each part of the question.
Many free-response questions are divided into parts such as (a), (b), (c), and (d), with each part calling for a different response. Credit for each part is awarded independently, so you should attempt to solve each part. For example, you may receive no credit for your answer to part (a), but still receive full credit for part (b), (c), or (d). If the answer to a later part of a question depends on the answer to an earlier part, you may still be able to receive full credit for the later part, even if that earlier answer is wrong.

Be sure to fully answer the question being asked.
For example, if a question asks for the maximum value of a function, do not stop after finding the x-value at which the maximum value occurs. Be sure to express your answer in correct units if units are given and always provide a justification when it is asked for.

When asked to justify or to explain an answer, think about how that can be done.
For example, if you are asked to justify a point of inflection, you need to show that the sign of the second derivative changes. Simply saying that the second derivative equals zero or is undefined is not a justification.

Do not round partial answers.
Store partial answers in your calculator so that you can use them unrounded in further calculations.

Practice!
Answering free-response questions from previous AP Exams is a great way to practice: it allows you to compare your own responses with those that have already been evaluated and scored. Free-response questions and scoring guidelines are available on the Exam Practice page for AP Calculus AB.