MATHEMATICS

Grade 8: Unit 4
Statistics and Probability: Scatterplots and Association
Course Philosophy/Description

In mathematics, students will learn to address a range of tasks focusing on the application of concepts, skills and understandings. Students will be asked to solve problems involving the key knowledge and skills for their grade level as identified by the NJSLS; express mathematical reasoning and construct a mathematical argument and apply concepts to solve model real world problems. The balanced math instructional model will be used as the basis for all mathematics instruction.

1) Eighth grade Mathematics consists of the following domains: The Number System (NS), Expressions and Equations (EE), Functions (F), Geometry (G), and Statistics and Probability (SP). In eighth grade, instructional time should focus on three critical areas: (1) formulating and reasoning about expressions and equations, including modeling an association in bivariate data with a linear equation, and solving linear equations and systems of linear equations; (2) grasping the concept of a function and using functions to describe quantitative relationships; (3) analyzing two- and three-dimensional space and figures using distance, angle, similarity, and congruence, and understanding and applying the Pythagorean Theorem. Student use linear equations and systems of linear equations to represent, analyze, and solve a variety of problems. Students recognize equations for proportions ($y/x = m$ or $y = mx$) as special linear equations ($y = mx + b$), understanding that the constant of proportionality ($m$) is the slope, and the graphs are lines through the origin. They understand that the slope ($m$) of a line is a constant rate of change, so that if the input or x-coordinate changes by an amount A, the output or y-coordinate changes by the amount $m \cdot A$. Students also use a linear equation to describe the association between two quantities in bivariate data (such as arm span vs. height for students in a classroom). At this grade, fitting the model, and assessing its fit to the data are done informally. Interpreting the model in the context of the data requires students to express a relationship between the two quantities in question and to interpret components of the relationship (such as slope and y-intercept) in terms of the situation. Students strategically choose and efficiently implement procedures to solve linear equations in one variable, understanding that when they use the properties of equality and the concept of logical equivalence, they maintain the solutions of the original equation. Students solve systems of two linear equations in two variables and relate the systems to pairs of lines in the plane; these intersect, are parallel, or are the same line. Students use linear equations, 3 systems of linear equations, linear functions, and their understanding of slope of a line to analyze situations and solve problems.

2) Students grasp the concept of a function as a rule that assigns to each input exactly one output. They understand that functions describe situations where one quantity determines another. They can translate among representations and partial representations of functions (noting that tabular and graphical representations may be partial representations), and they describe how aspects of the function are reflected in the different representations.

3) Students use ideas about distance and angles, how they behave under translations, rotations, reflections, and dilations, and ideas about congruence and similarity to describe and analyze two-dimensional figures and to solve problems. Students show that the sum of the angles in a triangle is the angle formed by a straight line, and that various configurations of lines give rise to similar triangles because of the angles created when a transversal cuts parallel lines. Students understand the statement of the Pythagorean Theorem and its converse, and can explain why the Pythagorean Theorem holds, for example, by decomposing a square in two different ways. They apply the Pythagorean Theorem to find distances between points on the coordinate plane, to find lengths, and to analyze polygons. Students complete their work on volume by solving problems involving cones, cylinders, and spheres.
ESL Framework

This ESL framework was designed to be used by bilingual, dual language, ESL and general education teachers. Bilingual and dual language programs use the home language and a second language for instruction. ESL teachers and general education or bilingual teachers may use this document to collaborate on unit and lesson planning to decide who will address certain components of the SLO and language objective. ESL teachers may use the appropriate leveled language objective to build lessons for ELLs which reflects what is covered in the general education program. In this way, whether it is a pull-out or push-in model, all teachers are working on the same Student Learning Objective connected to the New Jersey Student Learning Standards. The design of language objectives are based on the alignment of the World-Class Instructional Design Assessment (WIDA) Consortium’s English Language Development (ELD) standards with the New Jersey Student Learning Standards (NJSLS). WIDA’s ELD standards advance academic language development across content areas ultimately leading to academic achievement for English learners. As English learners are progressing through the six developmental linguistic stages, this framework will assist all teachers who work with English learners to appropriately identify the language needed to meet the requirements of the content standard. At the same time, the language objectives recognize the cognitive demand required to complete educational tasks. Even though listening and reading (receptive) skills differ from speaking and writing (expressive) skills across proficiency levels the cognitive function should not be diminished. For example, an Entering Level One student only has the linguistic ability to respond in single words in English with significant support from their home language. However, they could complete a Venn diagram with single words which demonstrates that they understand how the elements compare and contrast with each other or they could respond with the support of their home language (L1) with assistance from a teacher, para-professional, peer or a technology program.

http://www.state.nj.us/education/modelcurriculum/ela/ELLOverview.pdf
<table>
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<tr>
<th>#</th>
<th>Student Learning Objective</th>
<th>NJSLS</th>
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<tr>
<td>1</td>
<td>Construct and interpret scatter plots for bivariate measurement data and describe visual patterns of association (clusters, outliers, positive or negative association, linear association and nonlinear association, strong, weak, and no association).</td>
<td>8.SP.A.1</td>
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<tr>
<td>2</td>
<td>For scatter plots that suggest a linear association, informally fit a straight line and informally assess the model’s fit.</td>
<td>8.SP.A.2</td>
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<tr>
<td>3</td>
<td>Use a linear model (equation) representing measurement data to solve problems, interpreting the slope and intercept in the context of the situation.</td>
<td>8.SP.A.3</td>
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<tr>
<td>4</td>
<td>Construct two-way frequency tables and two-way relative frequency tables, and describe possible associations between two variables.</td>
<td>8.SP.A.4</td>
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<tr>
<td>5</td>
<td>Model a linear relationship by constructing a function from two (x,y) values. Interpret the rate of change and initial value of the linear function in terms of the situation it models, and in terms of its graph or a table of values.</td>
<td>8.F.B.4*</td>
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<tr>
<td>6</td>
<td>Apply the Pythagorean Theorem to determine unknown side lengths of right triangles in two and three dimensions to solve real-world and mathematical problems and to determine the distance between two points in the coordinate plane.</td>
<td>8.G.B.7, 8.G.B.8*</td>
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<tr>
<td>7</td>
<td>Solve real world and mathematical problems leading to two linear equations in two variables, interpreting solutions in context.</td>
<td>8.EE.C.8c*</td>
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</table>
Research about Teaching and Learning Mathematics

Structure teaching of mathematical concepts and skills around problems to be solved (Checkly, 1997; Wood & Sellars, 1996; Wood & Sellars, 1997)
Encourage students to work cooperatively with others (Johnson & Johnson, 1975; Davidson, 1990)
Use group problem-solving to stimulate students to apply their mathematical thinking skills (Artzt & Armour-Thomas, 1992)
Students interact in ways that support and challenge one another’s strategic thinking (Artzt, Armour-Thomas, & Curcio, 2008)
Activities structured in ways allowing students to explore, explain, extend, and evaluate their progress (National Research Council, 1999)

There are three critical components to effective mathematics instruction (Shellard & Moyer, 2002):

- Teaching for conceptual understanding
- Developing children’s procedural literacy
- Promoting strategic competence through meaningful problem-solving investigations

Teachers should be:

- Demonstrating acceptance and recognition of students’ divergent ideas
- Challenging students to think deeply about the problems they are solving, extending thinking beyond the solutions and algorithms required to solve the problem
- Influencing learning by asking challenging and interesting questions to accelerate students’ innate inquisitiveness and foster them to examine concepts further
- Projecting a positive attitude about mathematics and about students’ ability to “do” mathematics

Students should be:

- Actively engaging in “doing” mathematics
- Solving challenging problems
- Investigating meaningful real-world problems
- Making interdisciplinary connections
- Developing an understanding of mathematical knowledge required to “do” mathematics and connect the language of mathematical ideas with numerical representations
- Sharing mathematical ideas, discussing mathematics with one another, refining and critiquing each other’s ideas and understandings
- Communicating in pairs, small group, or whole group presentations
- Using multiple representations to communicate mathematical ideas
- Using connections between pictures, oral language, written symbols, manipulative models, and real-world situations
- Using technological resources and other 21st century skills to support and enhance mathematical understanding
Mathematics is not a stagnate field of textbook problems; rather, it is a dynamic way of constructing meaning about the world around us, generating knowledge and understanding about the real world every day. Students should be metaphorically rolling up their sleeves and “doing mathematics” themselves, not watching others do mathematics for them or in front of them. (Protheroe, 2007)

Balanced Mathematics Instructional Model

Balanced math consists of three different learning opportunities: guided math, shared math, and independent math. Ensuring a balance of all three approaches will build conceptual understanding, problem solving, computational fluency, and procedural fluency. Building conceptual understanding is the focal point of developing mathematical proficiency. Students should frequently work on rigorous tasks, talk about the math, explain their thinking, justify their answer or process, build models with graphs or charts or manipulatives, and use technology.

When balanced math is used in the classroom it provides students opportunities to:

- solve problems
- make connections between math concepts and real-life situations
- communicate mathematical ideas (orally, visually and in writing)
- choose appropriate materials to solve problems
- reflect and monitor their own understanding of the math concepts
- practice strategies to build procedural and conceptual confidence

Teacher builds conceptual understanding by modeling through demonstration, explicit instruction, and think alouds, as well as guiding students as they practice math strategies and apply problem solving strategies. (whole group or small group instruction)

Students practice math strategies independently to build procedural and computational fluency. Teacher assesses learning and reteaches as necessary. (whole group instruction, small group instruction, or centers)

Teacher and students practice mathematics processes together through interactive activities, problem solving, and discussion. (whole group or small group instruction)
## Effective Pedagogical Routines/Instructional Strategies

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<th>Analyze Student Work</th>
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<td>Connect Previous Knowledge to New Learning</td>
<td>Identify Student’s Mathematical Understanding</td>
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<td>Making Thinking Visible</td>
<td>Identify Student’s Mathematical Misunderstandings</td>
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<td>Develop and Demonstrate Mathematical Practices</td>
<td>Interviews</td>
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<td>Inquiry-Oriented and Exploratory Approach</td>
<td>Role Playing</td>
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<td>Multiple Solution Paths and Strategies</td>
<td>Diagrams, Charts, Tables, and Graphs</td>
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<td>Use of Multiple Representations</td>
<td>Anticipate Likely and Possible Student Responses</td>
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<td>Explain the Rationale of your Math Work</td>
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<td>Maintain the Cognitive Demand</td>
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<td>Technology Operations and Concepts</td>
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| • Use and/or develop a simulation that provides an environment to solve a real world problem or theory.  
  Example: Students can use math game websites to simulate and solve Pythagorean Theorem real world and mathematical problems.  
| • Graph and calculate data within a spreadsheet and present a summary of the results.  
  Example: Students can create graphs from a spreadsheet to interpret frequency tables and data information.  
  [http://www.meta-calculator.com/online/](http://www.meta-calculator.com/online/) |

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<th>Design</th>
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| • Develop a proposal for a chosen solution that include models (physical, graphical or mathematical) to communicate the solution to peers.  
  Example: Students can construct and develop a model of scatter plots and described visual patterns of association.  

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| • Use digital tools and online resources to explore a problem or issue.  
  Example: Students can search through Learnzillion, Imagine Math Facts, and other interactive sites for appropriate instructional videos and/or information pertaining to strategies and modeling for relationships between quantities, applying the Pythagorean Theorem and analyzing and solving linear equations. |
Career Ready Practices

Career Ready Practices describe the career-ready skills that all educators in all content areas should seek to develop in their students. They are practices that have been linked to increase college, career, and life success. Career Ready Practices should be taught and reinforced in all career exploration and preparation programs with increasingly higher levels of complexity and expectation as a student advances through a program of study.

- **CRP2. Apply appropriate academic and technical skills.**
  Career-ready individuals readily access and use the knowledge and skills acquired through experience and education to be more productive. They make connections between abstract concepts with real-world applications, and they make correct insights about when it is appropriate to apply the use of an academic skill in a workplace situation.

  **Example:** Students will apply prior knowledge when solving real world problems. Students will make sound judgements about the use of specific tools, such as graphs and tables to explore and deepen understanding the concepts of scatter plots for bivariate measurement data to investigate patterns of association between two quantities.

- **CRP4. Communicate clearly and effectively and with reason.**
  Career-ready individuals communicate thoughts, ideas, and action plans with clarity, whether using written, verbal, and/or visual methods. They communicate in the workplace with clarity and purpose to make maximum use of their own and others’ time. They are excellent writers; they master conventions, word choice, and organization, and use effective tone and presentation skills to articulate ideas. They are skilled at interacting with others; they are active listeners and speak clearly and with purpose. Career-ready individuals think about the audience for their communication and prepare accordingly to ensure the desired outcome.

  **Example:** Students will on a daily basis communicate their reasoning behind their solution paths by making connections to the context and the quantities, using proper vocabulary, along with decontextualizing and/or contextualizing the problem. Students will create representations using tables and graphs to display bivariate distributions of data on two quantitative variables. They will also explain the meaning behind the quantities and units involved. Students will also ask probing questions to clarify and improve arguments.

- **CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.**
  Career-ready individuals readily recognize problems in the workplace, understand the nature of the problem and devise effective plans to solve the problem. They are aware of problems when they occur and take action quickly to address the problem; they thoughtfully investigate the root cause of the
problem prior to introducing solutions. They carefully consider the options to solve the problem. Once a solution is agreed upon, they follow through to ensure the problem is solved, whether through their own actions or the actions of others.

**Example:** Throughout their daily lessons, students will understand the meaning of a problem and look for entry points into solving their problems by analyzing the relationships of the quantities, constraints and goals of the task. Plans for solution paths will be made and have meaning. Students will self-monitor, evaluate and critique their process and progress as they are working and make changes as necessary.

- **CRP12. Work productively in teams while using cultural global competence.**
  Career-ready individuals positively contribute to every team, whether formal or informal. They apply an awareness of cultural difference to avoid barriers to productive and positive interaction. They find ways to increase the engagement and contribution of all team members. They plan and facilitate effective team meetings.

  **Example:** Students will work in collaborative and whole group settings to develop various solutions to math tasks that are presented to them. They will work together to understand the terms of the problem, ask clarifying and challenging questions among each other, and develop agreed upon solutions using a variety of strategies and models. Students will listen to, read and discuss arguments with each other with respect and courtesy at all times and will be willing to assist those that may need assistance. Students will demonstrate and explain to a peer or small group how to construct two-way frequency tables and the possible associations between two variables.
### WIDA Proficiency Levels

At the given level of English language proficiency, English language learners will process, understand, produce or use:

<table>
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<th>Level</th>
<th>Description</th>
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| 6- Reaching | - Specialized or technical language reflective of the content areas at grade level  
  - A variety of sentence lengths of varying linguistic complexity in extended oral or written discourse as required by the specified grade level  
  - Oral or written communication in English comparable to proficient English peers |
| 5- Bridging | - Specialized or technical language of the content areas  
  - A variety of sentence lengths of varying linguistic complexity in extended oral or written discourse, including stories, essays or reports  
  - Oral or written language approaching comparability to that of proficient English peers when presented with grade level material. |
| 4- Expanding | - Specific and some technical language of the content areas  
  - A variety of sentence lengths of varying linguistic complexity in oral discourse or multiple, related sentences or paragraphs  
  - Oral or written language with minimal phonological, syntactic or semantic errors that may impede the communication, but retain much of its meaning, when presented with oral or written connected discourse, with sensory, graphic or interactive support |
| 3- Developing | - General and some specific language of the content areas  
  - Expanded sentences in oral interaction or written paragraphs  
  - Oral or written language with phonological, syntactic or semantic errors that may impede the communication, but retain much of its meaning, when presented with oral or written narrative or expository descriptions with sensory, graphic or interactive support |
| 2- Beginning | - General language related to the content area  
  - Phrases or short sentences  
  - Oral or written language with phonological, syntactic, or semantic errors that often impede the communication when presented with one to multiple-step commands, directions, or a series of statements with sensory, graphic or interactive support |
| 1- Entering | - Pictorial or graphic representation of the language of the content areas  
  - Words, phrases or chunks of language when presented with one-step commands directions, WH-, choice or yes/no questions, or statements with sensory, graphic or interactive support |
# Language Development Supports For English Language Learners

**To Increase Comprehension and Communication Skills**

## Environment
- Welcoming and stress-free
- Respectful of linguistic and cultural diversity
- Honors students’ background knowledge
- Sets clear and high expectations
- Includes routines and norms
- Is thinking-focused vs. answer-seeking
- Offers multiple modalities to engage in content learning and to demonstrate understanding
- Includes explicit instruction of specific language targets
- Provides participation techniques to include all learners
- Integrates learning centers and games in a meaningful way
- Provides opportunities to practice and refine receptive and productive skills in English as a new language
- Integrates meaning and purposeful tasks/activities that:
  - Are accessible by all students through multiple entry points
  - Are relevant to students’ lives and cultural experiences
  - Build on prior mathematical learning
  - Demonstrate high cognitive demand
  - Offer multiple strategies for solutions
  - Allow for a language learning experience in addition to content

### Sensory Supports*
- Real life objects (realia) or concrete objects
- Physical models
- Manipulatives
- Pictures & photographs
- Visual representations or models such as diagrams or drawings
- Videos & films
- Newspapers or magazines
- Gestures
- Physical movements
- Music & songs

### Graphic Supports*
- Graphs
- Charts
- Timelines
- Number lines
- Graphic organizers
- Graphing paper

### Interactive Supports*
- In a whole group
- In a small group
- With a partner such as *Turn-and-Talk*
- In pairs as a group (first, two pairs work independently, then they form a group of four)
- In triads
- Cooperative learning structures such as *Think-Pair-Share*
- Interactive websites or software
- With a mentor or coach

### Verbal and Textual Supports
- Labeling
- Students’ native language
- Modeling
- Repetitions
- Paraphrasing
- Summarizing
- Guiding questions
- Clarifying questions
- Probing questions
- Leveled questions such as *What? When? Where? How? Why?*
- Questioning prompts & cues
- Word Banks
- Sentence starters
- Sentence frames
- Discussion frames
- Talk moves, including *Wait Time*

BUILDING EQUITY IN YOUR TEACHING PRACTICE

How do the essential questions highlight the connection between the big ideas of the unit and equity in your teaching practice?

**CONTENT INTEGRATION**
Teachers use examples and content from a variety of cultures & groups.

This unit / lesson is connected to other topics explored with students.

There are multiple viewpoints reflected in the content of this unit / lesson.

The materials and resources are reflective of the diverse identities and experiences of students.

The content affirms students, as well as exposes them to experiences other than their own.

**KNOWLEDGE CONSTRUCTION**
Teachers help students understand how knowledge is created and influenced by cultural assumptions, perspectives & biases.

This unit / lesson provides context to the history of privilege and oppression.

This unit / lesson addresses power relationships.

This unit / lesson help students to develop research and critical thinking skills.

This curriculum creates windows and mirrors* for students.

**PREJUDICE REDUCTION**
Teachers implement lessons and activities to assert positive images of ethnic groups & improve intergroup relations.

This unit / lesson help students question and unpack biases & stereotypes.

This unit / lesson help students examine, research and question information and sources.

The curriculum encourage discussion and understanding about the groups of people being represented.

This unit / lesson challenges dominant perspectives.

**EQUITABLE PEDAGOGY**
The instruction has been modified to meet the needs of each student.

Students feel respected and their cultural identities are valued.

Additional supports have been provided for students to become successful and independent learners.

Opportunities are provided for student to reflect on their learning and provide feedback.

**EMPOWERING SCHOOL CULTURE**
Using the other four dimensions to create a safe and healthy educational environment for all.

There are opportunities for students to connect with the community.

My classroom is welcoming and supportive for all students?

I am aware of and sensitive to the needs of my students and their families.

There are effective parent communication systems established. Parents can talk to me about issues as they arise in my classroom.

### Culturally Relevant Pedagogy Examples

- **Integrate Relevant Word Problems:** Contextualize equations using word problems that reference student interests and cultures.
  **Example:** Create and use word problems that students relate to, have prior knowledge of and includes their interest. These can include current events and/or relevant real-world situations. Using content that students can relate to adds meaning, value and connection. The following link provides you with a variety of word problems that are current, relevant to real-world and student interests: [https://www.yummymath.com/](https://www.yummymath.com/)

- **Problem-Based Learning Scenarios:** Present relatable real-world problems for your students to solve, explicitly referencing cultures and communities when applicable.
  **Example:** Explain to the students that they are going to purchase a 65-inch television that needs to fit into their entertainment center. The width of the available space for the television is 40 inches and the height is 58 inches. The widths of three televisions they can choose from are: 25 inches, 33 inches, and 39 inches. Find the dimensions of two different televisions that will fit in the available space.

- **Use Learning Stations:** Provide a range of material by setting up learning stations.
  **Example:** Reinforce understandings of concepts and skills by promoting learning through student interests, modalities, experiences and/or prior knowledge. Encourage the students to make content choices based upon their strengths, needs, values and experiences. Providing students with choice boards will give them a sense of ownership to their learning and understanding.

- **Present New Concepts Using Student Vocabulary:** Use student diction to capture attention and build understanding before using academic terms.
  **Example:** Teach math vocabulary in various modalities to increase students’ retention. Use multi-modal activities, analogies, realia, visual cues, graphic representations, gestures, pictures, practice and cognates. Inform students that some vocabulary words have multiple meanings. Have students create the Word Wall with their definitions and examples to foster ownership. Work with students to create a sorting and matching game using vocabulary words from within the unit. Students can work in teams or individually to play these games. This will allow students to familiarize themselves with the vocabulary words within the unit.
## Differentiated Instruction

### Accommodate Based on Students Individual Needs: Strategies

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<th>Processing</th>
<th>Comprehension</th>
<th>Recall</th>
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<tr>
<td>Extra time for assigned tasks</td>
<td>Extra Response time</td>
<td>Precise processes for balanced math instructional model</td>
<td>Teacher-made checklist</td>
</tr>
<tr>
<td>Adjust length of assignment</td>
<td>Have students verbalize steps</td>
<td>Short manageable tasks</td>
<td>Use visual graphic organizers</td>
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<tr>
<td>Timeline with due dates for reports and projects</td>
<td>Repeat, clarify or reword directions</td>
<td>Brief and concrete directions</td>
<td>Reference resources to promote independence</td>
</tr>
<tr>
<td>Communication system between home and school</td>
<td>Mini-breaks between tasks</td>
<td>Provide immediate feedback</td>
<td>Visual and verbal reminders</td>
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<td>Provide lecture notes/outline</td>
<td>Provide a warning for transitions</td>
<td>Small group instruction</td>
<td>Graphic organizers</td>
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<td></td>
<td>Partnering</td>
<td>Emphasize multi-sensory learning</td>
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<th>Assistive Technology</th>
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<th>Behavior/Attention</th>
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<td>Tape recorder</td>
<td>Study guides</td>
<td>Simple and clear classroom rules</td>
<td>Display a written agenda</td>
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<tr>
<td>Video Tape</td>
<td>Shortened tests</td>
<td>Frequent feedback</td>
<td>Note-taking assistance</td>
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<td></td>
<td>Read directions aloud</td>
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<td>Color code materials</td>
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</table>

- **Tests/Quizzes/Grading**
  - Extended time
  - Study guides
  - Shortened tests
  - Read directions aloud

- **Behavior/Attention**
  - Consistent daily structured routine
  - Simple and clear classroom rules
  - Frequent feedback

- **Organization**
  - Individual daily planner
  - Display a written agenda
  - Note-taking assistance
  - Color code materials
## Differentiated Instruction

**Accommodate Based on Content Specific Needs: Strategies**

- Anchor charts to model strategies and use of formulas
- Reference sheets that list formulas, step-by-step procedures and model strategies
- Conceptual word wall that contains definition, translation, pictures and/or examples
- Graphic organizers, examples include: Venn Diagrams, Four Square, KWL, etc.
- Translation dictionary
- Teacher modeling
- Graphing calculator to assist with computations
- Students can utilize math journals to write notes, copy solution steps, and translate terms and key vocabulary
- Highlight and label the solution steps for multi-step problems in different colors
- Utilize technological programs which provide verbal and visual instruction in native and/or second language
- Algebra tiles to compare functions algebraically and solve system of linear equations
- Graph paper to create coordinate planes and scatter plots
- Tables and graphs to investigate patterns of association in bivariate data
- Area models to understand and apply the Pythagorean Theorem
Interdisciplinary Connections

Model interdisciplinary thinking to expose students to other disciplines.

ELA Connection:

Various Tasks: (RL.8.10)
- Students will be able to read, analyze, and cite informational text to solve problems and explain their reasoning of how the task was solved. Students will also focus on vocabulary, mechanics and grammar in effective writing.

Science Connection:

Bird Eggs: (MS-LS2-1)
- Students will analyze and draw conclusions based on graphs relating bird eggs lengths and widths.

Sugar Prices (MS-LS4-6)
- Students will take their own pulse and time how many times their heart beats in a minute. They will then use that information and put it into a table and graph it to draw conclusions.
Enrichment

What is the Purpose of Enrichment?

- The purpose of enrichment is to provide extended learning opportunities and challenges to students who have already mastered, or can quickly master, the basic curriculum. Enrichment gives the student more time to study concepts with greater depth, breadth, and complexity.
- Enrichment also provides opportunities for students to pursue learning in their own areas of interest and strengths.
- Enrichment keeps advanced students engaged and supports their accelerated academic needs.
- Enrichment provides the most appropriate answer to the question, “What do you do when the student already knows it?”

Enrichment is...

- Planned and purposeful
- Different, or differentiated, work – not just more work
- Responsive to students’ needs and situations
- A promotion of high-level thinking skills and making connections within content
- The ability to apply different or multiple strategies to the content
- The ability to synthesize concepts and make real world and cross-curricular connections
- Elevated contextual complexity
- Sometimes independent activities, sometimes direct instruction
- Inquiry based or open ended assignments and projects
- Using supplementary materials in addition to the normal range of resources
- Choices for students
- Tiered/Multi-level activities with flexible groups (may change daily or weekly)

Enrichment is not...

- Just for gifted students (some gifted students may need intervention in some areas just as some other students may need frequent enrichment)
- Worksheets that are more of the same (busywork)
- Random assignments, games, or puzzles not connected to the content areas or areas of student interest
- Extra homework
- A package that is the same for everyone
- Thinking skills taught in isolation
- Unstructured free time
Assessments

Required District/State Assessments
- Unit Assessments
- PARCC
- SGO Assessments

Suggested Formative/Summative Classroom Assessments
- Describe Learning Vertically
- Identify Key Building Blocks
- Make Connections (between and among key building blocks)
- Short/Extended Constructed Response Items
- Multiple-Choice Items (where multiple answer choices may be correct)
- Drag and Drop Items
- Use of Equation Editor
- Quizzes
- Journal Entries/Reflections/Quick-Writes
- Accountable talk
- Projects
- Portfolio
- Observation
- Graphic Organizers/Concept Mapping
- Presentations
- Role Playing
- Teacher-Student and Student-Student Conferencing
- Homework
New Jersey Student Learning Standards (NJSLS)

8.SP.A.1: Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.

8.SP.A.2: Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit (e.g. line of best fit) by judging the closeness of the data points to the line.

8.SP.A.3: Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.

8.SP.A.4: Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?

8.F.B.4: Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two \((x, y)\) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

8.G.B.7: Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

8.G.B.8: Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

8.EE.C.8: Analyze and solve pairs of simultaneous linear equations.
New Jersey Student Learning Standards (NJSLS)

8.EE.C.8c: Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.*
<table>
<thead>
<tr>
<th>Mathematical Practices</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Make sense of problems and persevere in solving them.</td>
</tr>
<tr>
<td>2. Reason abstractly and quantitatively.</td>
</tr>
<tr>
<td>3. Construct viable arguments and critique the reasoning of others.</td>
</tr>
<tr>
<td>4. Model with mathematics.</td>
</tr>
<tr>
<td>5. Use appropriate tools strategically.</td>
</tr>
<tr>
<td>6. Attend to precision.</td>
</tr>
<tr>
<td>7. Look for and make use of structure.</td>
</tr>
<tr>
<td>8. Look for and express regularity in repeated reasoning.</td>
</tr>
</tbody>
</table>
**Grade:** Eight  
**Unit:** 4 (four)  
**Topic:** Statistics and Probability: Scatterplots and Association

**NJSLS:**  

**Unit Focus:**  
- Investigate patterns of association in bivariate data.  
- Use functions to model relationships between quantities.  
- Understand and apply the Pythagorean Theorem.  
- Analyze and solve linear equations and simultaneous linear equations

---

**New Jersey Student Learning Standard(s):**  
8.SP.A.1: Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.

**Student Learning Objective 1:** Construct and interpret scatter plots for bivariate measurement data and describe visual patterns of association (clusters, outliers, positive or negative association, linear association and nonlinear association, strong, weak, and no association).

**Modified Student Learning Objectives/Standards:** N/A

<table>
<thead>
<tr>
<th>MPs</th>
<th>Evidence Statement Key/ Clarifications</th>
<th>Skills, Strategies &amp; Concepts</th>
<th>Essential Understandings/ Questions (Accountable Talk)</th>
<th>Tasks/Activities</th>
</tr>
</thead>
<tbody>
<tr>
<td>MP 3</td>
<td>8.SP.1</td>
<td>Bivariate distributions describe patterns or trends in the variability in data on two variables.</td>
<td>How can I analyze a scatter plot?</td>
<td>Bird Eggs</td>
</tr>
<tr>
<td>MP 5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MP 7</td>
<td></td>
<td></td>
<td></td>
<td>Animal Brains</td>
</tr>
</tbody>
</table>
Apply knowledge of graphing on the coordinate plane in order to create scatter plots.

Integrate technology in order to show how to create scatter plots using technology.

Review how to determine if a graph is positive or negative, outliers, and determining lines of best fit.

Students build on their previous knowledge of scatter plots by examining relationships between variables. They analyze scatter plots to determine positive and negative associations, the degree of association, and type of association.

Describe clustering, outliers, positive or negative association, linear or non-linear association, strong, weak and no association.

Students examine outliers to determine if data points are valid or represent a recording or measurement error.

**Examples:**
Data for 10 students’ Math and Science scores are provided in the chart. Describe the association between the Math and Science scores.

<table>
<thead>
<tr>
<th>Student</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Math</td>
<td>64</td>
<td>50</td>
<td>85</td>
<td>34</td>
<td>56</td>
<td>24</td>
<td>72</td>
<td>63</td>
<td>42</td>
<td>93</td>
</tr>
<tr>
<td>Science</td>
<td>68</td>
<td>70</td>
<td>83</td>
<td>33</td>
<td>60</td>
<td>27</td>
<td>74</td>
<td>63</td>
<td>40</td>
<td>96</td>
</tr>
</tbody>
</table>

Data for 10 students’ Math scores and the distance they live from school are provided in the table below. Describe the association between the Math scores and the distance they live from school.

How can I use bivariate data to solve problems?

Graphs and tables are useful for displaying bivariate distributions of data on two quantitative variables.

A correlation coefficient is a numerical summary of bivariate data that measures the strength of the relationship between two variables.

When the trend in bivariate data on two quantitative variables is generally linear, a centrally located line can be useful for making predictions.

What can I infer from the data?

What information can be learned from a scatter plot? How can we use the data to draw conclusions?
Data from a local fast food restaurant is provided showing the number of staff members and the average time for filling an order are provided in the table below. Describe the association between the number of staff and the average time for filling an order.

<table>
<thead>
<tr>
<th>Number of staff</th>
<th>Average time to fill order (seconds)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>180</td>
</tr>
<tr>
<td>4</td>
<td>138</td>
</tr>
<tr>
<td>5</td>
<td>120</td>
</tr>
<tr>
<td>6</td>
<td>108</td>
</tr>
<tr>
<td>7</td>
<td>96</td>
</tr>
<tr>
<td>8</td>
<td>84</td>
</tr>
</tbody>
</table>

The chart below lists the life expectancy in years for people in the United States every five years from 1970 to 2005. What would you expect the life expectancy of a person in the United States to be in 2010, 2015, and 2020 based upon this data? Explain how you determined your values.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Life Expectancy (in years)</td>
<td>70.6</td>
<td>72.6</td>
<td>73.7</td>
<td>74.7</td>
<td>75.4</td>
<td>75.8</td>
<td>76.8</td>
<td>77.4</td>
</tr>
</tbody>
</table>

The capacity of the fuel tank in a car is 13.5 gallons. The table below shows the number of miles traveled and how many gallons of gas are left in the tank. Describe the relationship between the variables. If the data is linear, determine a line of best fit. Do you think the line represents a good fit for the data set? Why or why not? What is the average fuel efficiency of the car in miles per gallon?
**SPED Strategies:**
Pre-teach vocabulary words using verbal descriptions and illustrations to foster association or connection with previous learning (i.e. bivariate, positive correlation, negative correlation, outliers, scatter plot).

Model how to create scatter plots from a list or ordered pairs or a table of x and y values to visualize the relationship.

Model how scatter plots can be used to draw conclusions about relationships based on visualizing a trend or association in data.

Encourage students to practice drawing conclusions made from a scatter plot by working with a partner on a scatter plot about something of interest to students.

Create a reference sheet with students that outlines the skills, concepts, and thinking involved in analyzing scatter plots.

**ELL Strategies:**
Provide and develop hands-on scatter-plots with steps to graph data.
Have students create models and visualize actual functions and create their own.

Pre-teach vocabulary words using verbal descriptions and illustrations to foster association or connection with previous learning (i.e. bivariate, positive correlation, negative correlation, outliers, scatter plot).

**Website:** Teachers First Adapt a Strategy. Adjusting Lessons for ESL/ELL students
http://www.teachersfirst.com/content/esl/adaptstrat.cfm

**New Jersey Student Learning Standard(s):**

**8.SP.A.2:** Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit (e.g. line of best fit) by judging the closeness of the data points to the line.

**Student Learning Objective 2:** For scatter plots that suggest a linear association, informally fit a straight line and informally assess the model’s fit.

**Modified Student Learning Objectives/Standards:** N/A

<table>
<thead>
<tr>
<th>MPs</th>
<th>Evidence Statement Key/ Clarifications</th>
<th>Skills, Strategies &amp; Concepts</th>
<th>Essential Understandings/ Questions (Accountable Talk)</th>
<th>Tasks/Activities</th>
</tr>
</thead>
<tbody>
<tr>
<td>MP 2</td>
<td><strong>8.SP.2</strong></td>
<td>Apply knowledge of graphing on the coordinate plane in order to create scatter plots.</td>
<td>How can I analyze a scatter plot?</td>
<td>Scatter Plots/Teacher Resources</td>
</tr>
<tr>
<td>MP 5</td>
<td><strong>8.SP.2</strong></td>
<td>Sketch a line on a scatter plot that best fits the data.</td>
<td>A correlation coefficient is a numerical summary of</td>
<td></td>
</tr>
<tr>
<td>MP 7</td>
<td><strong>8.SP.2</strong></td>
<td>Recognize when a scatter plot represents a linear relationship.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Informally assess the model fit by judging the closeness of the data to the points on the line.

Integrate technology in order to show how to create scatter plots using technology.

Review how to determine if a graph is positive or negative, outliers, and determining lines of best fit.

Students build on their previous knowledge of scatter plots examine relationships between variables. They analyze scatter plots to determine positive and negative associations, the degree of association, and type of association. Students examine outliers to determine if data points are valid or represent a recording or measurement error.

Students can use tools such as those at the National Center for Educational Statistics to create a graph or generate data sets.

**Examples:**

Data for 10 students’ Math and Science scores are provided in the chart. Describe the association between the Math and Science scores.

<table>
<thead>
<tr>
<th>Student</th>
<th>Math</th>
<th>Science</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>64</td>
<td>88</td>
</tr>
<tr>
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<td>50</td>
<td>70</td>
</tr>
<tr>
<td>3</td>
<td>86</td>
<td>83</td>
</tr>
<tr>
<td>4</td>
<td>34</td>
<td>33</td>
</tr>
<tr>
<td>5</td>
<td>56</td>
<td>60</td>
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<td>24</td>
<td>27</td>
</tr>
<tr>
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<td>74</td>
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<td>10</td>
<td>80</td>
<td>96</td>
</tr>
</tbody>
</table>

Data for 10 students’ Math scores and the distance they live from school are provided in the table below. Describe the association between the Math scores and the distance they live from school.

bivariate data that measures the strength of the relationship between two variables.

What information can be learned from a scatter plot? How can we use the data to draw conclusions?

How is the line-of-best fit used to assess data?

**Scatter Plots/Student Materials**

**Nonlinear Models in Data Context**

**Nonlinear Models in data Context**

**Taxi Fares**
Data from a local fast food restaurant is provided showing the number of staff members and the average time for filling an order are provided in the table below. Describe the association between the number of staff and the average time for filling an order.

<table>
<thead>
<tr>
<th>Number of staff</th>
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<td>84</td>
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The chart below lists the life expectancy in years for people in the United States every five years from 1970 to 2005. What would you expect the life expectancy of a person in the United States to be in 2010, 2015, and 2020 based upon this data? Explain how you determined your values.

<table>
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<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Life Expectancy (in years)</td>
<td>70.8</td>
<td>72.6</td>
<td>73.7</td>
<td>74.7</td>
<td>75.4</td>
<td>75.8</td>
<td>76.8</td>
<td>77.4</td>
</tr>
</tbody>
</table>

The capacity of the fuel tank in a car is 13.5 gallons. The table below shows the number of miles traveled and how many gallons of gas are left in the tank. Describe the relationship between the variables. If the data is linear, determine a line of best fit. Do you think the line represents a
good fit for the data set? Why or why not? What is the average fuel efficiency of the car in miles per gallon?

<table>
<thead>
<tr>
<th>Miles Traveled</th>
<th>0</th>
<th>75</th>
<th>120</th>
<th>160</th>
<th>250</th>
<th>300</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gallons Used</td>
<td>0</td>
<td>2.3</td>
<td>4.5</td>
<td>5.7</td>
<td>9.7</td>
<td>10.7</td>
</tr>
</tbody>
</table>

**SPED Strategies:**
Model how to draw a line of best fit and what it reveals about the data.

Review and analyze data to determine the trend (increasing or decreasing points (i.e. sports, scores, rates, stock market etc.).

**ELL Strategies:**
Model to students the thinking process in analyzing scatter plots.

Model how to draw a line of best fit and what it reveals about the data.

**Website:**
Teachers First Adapt a Strategy. Adjusting Lessons for ESL/ELL students
- [http://www.teachersfirst.com/content/esl/adaptstrat.cfm](http://www.teachersfirst.com/content/esl/adaptstrat.cfm)
New Jersey Student Learning Standard(s):
8.SP.A.3: Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.

Student Learning Objective 3: Use a linear model (equation) representing measurement data to solve problems, interpreting the slope and intercept in the context of the situation.

Modified Student Learning Objectives/Standards: N/A

<table>
<thead>
<tr>
<th>MPs</th>
<th>Evidence Statement Key/ Clarifications</th>
<th>Skills, Strategies &amp; Concepts</th>
<th>Essential Understandings/ Questions (Accountable Talk)</th>
<th>Tasks/Activities</th>
</tr>
</thead>
<tbody>
<tr>
<td>MP 2</td>
<td></td>
<td>Emphasize the importance of being able to relate the slope and intercepts to the word problems and their meaning in the problems.</td>
<td>How can I use a linear model to solve problems?</td>
<td>Walking Race and Making Money</td>
</tr>
<tr>
<td>MP 4</td>
<td></td>
<td>Given the equation for a linear model (line of best fit), interpret the slope and intercept.</td>
<td>What do the slope and intercepts mean in the context of the problem?</td>
<td>Walk the Graph</td>
</tr>
<tr>
<td>MP 6</td>
<td></td>
<td>Given the equation for a linear model, solve problems in the context of the measurement data.</td>
<td>What information is needed from a real life problem in order to create a linear equation?</td>
<td>Task Obesity Rates</td>
</tr>
<tr>
<td>MP 7</td>
<td></td>
<td>Examples:</td>
<td>How can the equations for the line-of-best-fit be used to solve mathematical and real-world problems.</td>
<td>Obesity Rates in Maryland</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Given data from students’ math scores and absences, make a scatter plot.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Draw a line of best fit, paying attention to the closeness of the data points on either side of the line.
<table>
<thead>
<tr>
<th>SPED Strategies:</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Review, demonstrate and practice how to write an equation in slope-intercept form using real life examples.</td>
<td></td>
</tr>
<tr>
<td>Review and practice how to determine slope using two points on a coordinate grid.</td>
<td></td>
</tr>
<tr>
<td>Model how to find the y-intercept by looking at a graph and determine its meaning in the context of the problem.</td>
<td></td>
</tr>
<tr>
<td>Provide students with opportunities to practice the highlighted skills: determining slope and y-intercept and reflecting on their meanings within the problem.</td>
<td></td>
</tr>
</tbody>
</table>

**Resources UDL - Visual and Auditory Learner(s):**
Linear vs nonlinear relations
[https://youtu.be/F5RZak0dVi4](https://youtu.be/F5RZak0dVi4)

**ELL Strategies:**
Analyze problems in order to have students identify the important information.
Review with students how to find the slope of the line and the intercept.
Provide visual models as a point of reference for the students.
New Jersey Student Learning Standard(s):  
8.SP.A.4: Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?

Student Learning Objective 4: Construct two-way frequency tables and two-way relative frequency tables, and describe possible associations between two variables.

Modified Student Learning Objectives/Standard:  
M.EE.8.SP.A.4: Construct a graph or table from given categorical data and compare data categorized in the graph or table.

<table>
<thead>
<tr>
<th>MPs</th>
<th>Evidence Statement Key/ Clarifications</th>
<th>Skills, Strategies &amp; Concepts</th>
<th>Essential Understandings/ Questions (Accountable Talk)</th>
<th>Tasks/Activities</th>
</tr>
</thead>
<tbody>
<tr>
<td>MP 2</td>
<td>8.SP.4</td>
<td>Carefully analyze frequency tables and make sure that students read the correct column based on the questions.</td>
<td>Conditional relative frequency distributions are useful for establishing an association between two categorical variables.</td>
<td>Halloween Survey</td>
</tr>
<tr>
<td>MP 4</td>
<td></td>
<td>Build on previous knowledge of tables in order to have students construct frequency tables given data.</td>
<td></td>
<td>Summer Camp</td>
</tr>
<tr>
<td>MP 5</td>
<td></td>
<td>Recognize that categorical data can also be described numerically through the use of a two-way table.</td>
<td></td>
<td>Two Way Tables</td>
</tr>
<tr>
<td>MP 7</td>
<td></td>
<td>Construct a two-way table summarize data on two categorical variables collected from the same subjects.</td>
<td></td>
<td>Sports and Musical Instruments</td>
</tr>
<tr>
<td></td>
<td>An equal number of tasks require students to:</td>
<td></td>
<td>How can I determine if there is an association between two given sets of data?</td>
<td>Mini Problems</td>
</tr>
<tr>
<td></td>
<td>• Answer basic comprehension questions about a two-way table, or;</td>
<td></td>
<td></td>
<td>Music and Sports</td>
</tr>
<tr>
<td></td>
<td>• To compute marginal sums or marginal percentages, or;</td>
<td></td>
<td></td>
<td>Walking Race and Making Money</td>
</tr>
<tr>
<td></td>
<td>• To interpret patterns or association.</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Use relative frequencies calculated for rows or columns to describe possible association between the two variables.

**Example:**
The table illustrates the results when 100 students were asked the survey questions: Do you have a curfew? Do you have assigned chores? Is there evidence that those who have a curfew also tend to have chores?

<table>
<thead>
<tr>
<th></th>
<th>Curfew</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yes</td>
<td>40</td>
</tr>
<tr>
<td>No</td>
<td>10</td>
</tr>
<tr>
<td>Chores</td>
<td></td>
</tr>
<tr>
<td>Yes</td>
<td>10</td>
</tr>
<tr>
<td>No</td>
<td>40</td>
</tr>
</tbody>
</table>

Solution: Of the students who answered that they had a curfew, 40 had chores and 10 did not. Of the students who answered they did not have a curfew, 10 had chores and 40 did not. From this sample, there appears to be a positive correlation between having a curfew and having chores.

**SPED Strategies:**
Model the thinking and procedures behind creating two-way frequency tables.

Make connection for students between previous learning about scatter plots and two-way frequency tables: What are the similarities and differences?

How can I find the relative frequency using two-way tables?
Provide students with examples for them to analyze.

**ELL Strategies:**
Provides students real life examples that they can relate to.

Create a two-way frequency table together with the students; have students write notes as you walk them through the process.

Link this to prior learning experiences.

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**New Jersey Student Learning Standard(s):**

8.F.B.4: Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two \((x, y)\) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

**Student Learning Objective 5:** Model a linear relationship by constructing a function from two \((x, y)\) values. Interpret the rate of change and initial value of the linear function in terms of the situation it models, and in terms of its graph or a table of values.

**Modified Student Learning Objectives/Standards:**

M.EE.8.F.B.4: Determine the values or rule of a function using a graph or a table

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<tr>
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<th>Essential Understandings/ Questions (Accountable Talk)</th>
<th>Tasks/Activities</th>
</tr>
</thead>
<tbody>
<tr>
<td>MP 2</td>
<td>8.F.B.4</td>
<td>As with equations, two ((x,y)) values can be used to construct a function. Construct a function in order to model a linear relationship.</td>
<td>How can real world data be transformed into graphs and tables?</td>
<td>Comparing Lines and Linear Equations</td>
</tr>
<tr>
<td>MP 6</td>
<td>Tasks may or may not have a context.</td>
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<td>MP 7</td>
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<td>Sandy’s Candy Corporation</td>
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<tr>
<td><strong>Interpret the rate of change and initial value of a linear function in context.</strong></td>
<td><strong>What are important characteristics needed to construct a linear model?</strong></td>
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<tr>
<td>Identify and calculate slope.</td>
<td>How are functions of linear relationships modeled?</td>
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<tr>
<td>Identify ( y )-intercept.</td>
<td>Once the linear function is modeled, how are the initial value and the rate of change determined?</td>
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<tr>
<td>Create graphs from the given data.</td>
<td><strong>Bow Wow Barkley</strong></td>
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<tr>
<td>Analyze graphs. Make predictions given a graph.</td>
<td><strong>The Picture Frame Problem</strong></td>
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<tr>
<td>Review that rate of change and slope are the same.</td>
<td><strong>Matching Situations Graphs and Linear</strong></td>
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<tr>
<td>Emphasize the important characteristics of linear functions.</td>
<td><strong>Resources:</strong> UDL - Visual and Auditory Learner(s): Linear vs nonlinear relations</td>
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<tr>
<td>Explain any constraints on the domain of the relationship.</td>
<td><strong>SPED Strategies:</strong> Review and practice how to determine slope using two points.</td>
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</tr>
<tr>
<td><strong>Review and practice how to find the ( y )-intercept by looking at a graph.</strong></td>
<td><strong>Model the thinking process and steps involved in creating a function based on a set of data.</strong></td>
<td></td>
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<tr>
<td><strong>Illustrate for students how to interpret the slope from a graph or a table.</strong></td>
<td><strong>UDL - Visual and Auditory Learner(s): Linear vs nonlinear relations</strong></td>
<td></td>
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</tr>
</tbody>
</table>
ELL Strategies:
Review with students how to find the slope of the line and the y-intercept from a graph, equation or two points.

Create anchor charts/posters with strategies for finding slope of a line and the y-intercept.

New Jersey Student Learning Standard(s):
8.G.B.7: Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

8.G.B.8: Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

Student Learning Objective 6: Apply the Pythagorean Theorem to determine unknown side lengths of right triangles in two and three dimensions to solve real-world and mathematical problems and to determine the distance between two points in the coordinate plane.

Modified Student Learning Objectives/Standards: N/A

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<tr>
<td>MP 2</td>
<td>8.G.7-1 8.G.7-2 8.G.7-2</td>
<td>Students should apply the Pythagorean Theorem to determine unknown side lengths in right triangles. Students should be able to determine if a triangle is a right triangle and identify a Pythagorean triple.</td>
<td>Right triangles have a special relationship among the side lengths which can be represented by a model and a formula.</td>
<td>Applications of the Pythagorean Theorem Teacher/Student</td>
</tr>
</tbody>
</table>

https://youtu.be/F5RZak0dVi4
| 8.G.8 | Review squaring numbers and taking square roots so that students will effectively be able to apply the Pythagorean Theorem and find missing side lengths.

Through authentic experiences and exploration, students should use the Pythagorean Theorem to solve problems. Problems can include working in both two and three dimensions. Students should be familiar with the common Pythagorean Theorem (e.g. $a^2 + b^2 = c^2$).

Students will apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

Students will solve problems involving the Pythagorean Theorem.

Students will create a right triangle from the two points given (as shown in the diagram below) and then use the Pythagorean Theorem to find the distance between the two given points.

![Diagram](image)

**SPED Strategies:**

Pythagorean Triples can be used to construct right triangles.

How can the Pythagorean Theorem be used to solve problems?

Both the Pythagorean Theorem and distance formula can be used to find missing side lengths in a coordinate plane and real-world situation.

How can we use the coordinate plane to determine the distance between two points using the Pythagorean Theorem?

How is the Pythagorean Theorem used to determine unknown side length in right triangles?

How is the Pythagorean Theorem used to find distance between two points in a coordinate system?

<table>
<thead>
<tr>
<th>Using Pythagorean Theorem and Exponents</th>
<th>Area of a Trapezoid</th>
</tr>
</thead>
<tbody>
<tr>
<td>Two Triangles' Area</td>
<td></td>
</tr>
</tbody>
</table>
Review the how the Pythagorean Theorem is used to find missing side lengths and the distance between two points as covered in Unit 3.

Model how to use the Pythagorean Theorem to solve real world problems involving right triangles in two and three dimensions.

**Resources: UDL - Visual and Auditory Learner(s)**
Linear vs nonlinear relations
[https://youtu.be/F5RZak0dVi4](https://youtu.be/F5RZak0dVi4)

**ELL Strategies:**
Utilize authentic experiences and explanations to help students understand Pythagorean Theorem.

Review the thinking process used to calculate missing side lengths of a right triangle with students. Provide examples.

Review key vocabulary with the students; present pictorial representation when available.
New Jersey Student Learning Standard(s):
8.EE.C.8: Analyze and solve pairs of simultaneous linear equations.

8.EE.C.8c: Solve real-world and mathematical problems leading to two linear equations in two variables. For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.

Student Learning Objective 7: Solve real world and mathematical problems leading to two linear equations in two variables, interpreting solutions in context.

Modified Student Learning Objectives/Standards: N/A

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<tr>
<td>MP 1 MP 2 MP 6 MP 7</td>
<td>8.EE.8c</td>
<td>Simultaneous linear equations may have an infinite number of solutions, no solutions or a single solution. Solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs. Estimate solutions of a linear system by graphing. Solve simple cases of a linear system of two equations by inspection. Solve systems of two linear equations in two variables algebraically.</td>
<td>How is a solution to a system of equations derived? What type of real world problems would lead to two linear equations in two variables?</td>
<td>Fixing the Furnace Picking Apples Cookie Calorie Conundrum Maximizing Profit/Selling Boomerangs Text and Talk Plans Let’s Race Baseball Shop</td>
</tr>
<tr>
<td></td>
<td>Tasks may have three equations, but students are only required to analyze two equations at a time.</td>
<td>SPED Strategies: Pre-teach vocabulary words using verbal descriptions and illustrations.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Model the thinking and mathematical processes involved in creating a system of equations using real-life high interest examples.

Provide students with the opportunity to work in small groups or pairs to create, solve and interpret systems of equations.

Create a reference document with students that summarizes their learning of key concepts related to systems of equations.

**ELL Strategies:**
Review and practice how to determine which method to use when solving a system of equations.

Review and practice using systems of equations to solve real-world problems; it is helpful when comparing items that are related to each other. For example, trying to decide between two cell phone plans.

There are multiple methods available to use when solving a system of equations.

Create and practice writing an equation based on a real-world situation.

Provide students with exemplars of linear equations with one solution, infinitely many solutions or no solutions.
## Integrated Evidence Statements

### 8.C.1.2: Base reasoning on the principle that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane. Content Scope: Knowledge and skills articulated in 8.EE.8a.

### 8.C.2: Given an equation or system of equations, present the solution steps as a logical argument that concludes with the set of solutions (if any). Content Scope: Knowledge and skills articulated in 8.EE.7a, 8.EE.7b, 8.EE.8b.

- Tasks may have three equations, but students are only required to analyze two equations at a time.

### 8.C.4.1: Present solutions to multi-step problems in the form of valid chains of reasoning, using symbols such as equals signs appropriately (for example, rubrics award less than full credit for the presence of nonsense statements such as $1 + 4 = 5 + 7 = 12$, even if the final answer is correct), or identify or describe errors in solutions to multi-step problems and present corrected solutions. Content Scope: Knowledge and skills articulated in 8.EE.8c.

### 8. EE.C.Int.1: Solve word problems leading to linear equations in one variable whose solutions require expanding expressions using the distributive property and collecting like terms.

- Most tasks involve contextual real-world problems.

### 8. C.6: Construct, autonomously, chains of reasoning that will justify or refute propositions or conjectures. Content Scope: Knowledge and skills articulated in 7.RP.A, 7.NS.A, 7.EE.A.

- Some of the tasks may use scaffolding.

### D.1: Solve multi-step contextual word problems with degree of difficulty appropriate to Grade 8, requiring application of knowledge and skills articulated in Type I, Sub-Claim A Evidence Statements.

- Some of the tasks may use scaffolding.

### 8.D.2: Solve multi-step contextual problems with degree of difficulty appropriate to grade 8, requiring application of knowledge and skills articulated in 7.RP.A, 7.NS.3, 7.EE, 7.G, and 7.SP.B.

- Some of the tasks may use scaffolding.

### 8.D.3- Micro-models: Autonomously apply a technique from pure mathematics to a real-world situation in which the technique yields valuable results even though it is obviously not applicable in a strict mathematical sense (e.g., profitably applying proportional relationships...
Integrated Evidence Statements

to a phenomenon that is obviously nonlinear or statistical in nature). Content Scope: Knowledge and skills articulated in Type I, Sub-Claim A Evidence Statements.

- Some of the tasks may use scaffolding.

8. D.4- Reasoned estimates: Use reasonable estimates of known quantities in a chain of reasoning that yields an estimate of an unknown quantity. Content Scope: Knowledge and skills articulated in Type I, Sub-Claim A Evidence Statements.

- Some of the tasks may use scaffolding.
# Unit 4 Vocabulary

| Altitude of a Triangle Base (of a polygon) | Negative association |
| Bivariate data                           | Non-linear association |
| Categorical data                         | Nonlinear |
| Clustering                               | Outliers |
| Clustering linear association            | Outliers linear |
| Converse of Pythagorean Theorem          | Patterns |
| Coordinate Plane                         | Perfect Cubes |
| Cubed Root                               | Positive association |
| Deductive Reasoning                      | Pythagorean Triples |
| Diameter                                 | Radius |
| Distance Formula                         | Scatter plot |
| Equation of linear model                 | Scatter plots |
| Frequencies                              | Slope and intercept |
| Geometric Solid                          | Sphere |
| Height of Solids                         | Two-way table |
| Hypotenuse Leg of a Triangle             | Two-way table |
| Line of best fit                         | Variables |
| Linear model                             | Volume |
| Literal Equation                         |
## References & Suggested Instructional Websites

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<td><a href="https://illuminations.nctm.org/">https://illuminations.nctm.org/</a></td>
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<tr>
<td><a href="http://www.internet4classrooms.com">www.internet4classrooms.com</a></td>
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<tr>
<td><a href="https://www.georgiastandards.org/Georgia-Standards/Pages/Math-6-8.aspx">https://www.georgiastandards.org/Georgia-Standards/Pages/Math-6-8.aspx</a></td>
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<td><a href="http://www.illustrativemathematics.org/">www.illustrativemathematics.org/</a></td>
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<tr>
<td><a href="http://www.ncpublicschools.org/">http://www.ncpublicschools.org/</a></td>
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</table>
Field Trip Ideas

**THE BOUNCE FACTORY (Warren, NJ)** - STEM- Inspired FUN Field Trips The Bounce Factory, Bricks 4 Kidz of Hunterdon Somerset and Team Makers of North Jersey have combined to create a unique and exciting Field Trip for students in grades 1st – 8th. It integrates STEM learning with fun, hands on activities that will focus on Science, Engineering and Math concepts. The students will build motorized models with LEGO® bricks and discuss engineering and physics principals; enter the Bounce rooms for activities that will set in motion discussions of how physics impacts their play; learn about Math and Science concepts while playing integrative teambuilding activities that build their skills and promote working together; learn strategy and the power of collaboration while playing laser tag in a state of the art facility.

http://www.bouncefactorynj.com/

**LIBERTY SCIENCE CENTER (Jersey City, NJ)** - An interactive science museum and learning center with math connections. There is a math guidebook for teachers to make connections with math: http://lsc.org/plan-your-visit/

**NATIONAL MUSEUM OF MATHEMATICS (New York, NY)** - Mathematics illuminates the patterns and structures all around us. Our dynamic exhibits, gallery, and programs will stimulate inquiry, spark curiosity, and reveal the wonders of mathematics. MoMath has innovative exhibits that will engage folks from 105 to 5 years old (and sometimes younger), but with a special emphasis on activities for 4th through 8th graders. **Requires approval from Unit Superintendent**

http://momath.org/

**NEW JERSEY JACKALS (Little Falls, NJ)** – Students will be able to watch a live minor league baseball game while figuring out the players batting averages, the earned run average, determine the win to loss ratio for the season, the pitch count, and other player statistics.

http://njjackals.pointstreaksites.com/view/njjackals/home-page-657

**BUEHLER’S CHALLENGER & SCIENCE CENTER (Paramus, NJ)** - Fly a space mission beyond your wildest dreams in the challenger simulator! Students will work on teams to complete their mission, while conducting experiments, monitoring life support, and implementing navigation orders. In this dynamic environment, students use principles of science, mathematics, and technology to complete their tasks. There
Field Trip Ideas

are 3 missions to choose from: “Rendezvous with Comet Halley”, “Return to the Moon”, “Voyage to Mars”.**Requires approval from Unit Superintendent**
http://www.bcsc.org/5-9th-grade/

**MUSEUM OF AMERICAN FINANCE (New York, NY)** – For more than 20 years, educators from around the country have been bringing students to the Museum to help them understand how finance impacts their daily lives. The Museum offers discounted admission for pre-booked groups of eight or more, as well as a variety of classes for students in middle school through college.
http://www.moaf.org/index

**BRANCH BROOK PARK SKATING RINK (Newark, NJ)** - A unique educational experience that gets students excited about learning! Students will learn how the concepts of Science, Technology, Engineering and Math can be found in everyday experiences, even FUN experiences like roller skating! Our professional STEM Educators teach visiting students about how STEM principles exist in just about every part of life. The lessons focus on hands on activities that are both educational and fun! Lessons are customized based on teachers needs to directly relate back to classroom learning making this program completely unique! Following the completion of the 1hour STEM Lesson, the students roller skate for physical fitness. While Roller Skating the concepts students learned are continually reinforced. Our lessons are designed not only to help students overcome their fear of learning STEM concepts but to show how STEM is both FUN & EXCITING!