MATHEMATICS

Grade 6: Unit 4
Variability, Distributions, and Relationships Between Quantities
Course Philosophy/Description

In mathematics, students will learn to address a range of tasks focusing on the application of concepts, skills and understandings. Students will be asked to solve problems involving the key knowledge and skills for their grade level as identified by the NJSL; express mathematical reasoning and construct a mathematical argument and apply concepts to solve model real world problems. The balanced math instructional model will be used as the basis for all mathematics instruction.

Sixth Grade Mathematics consists of the following domains: Ratios and Proportional Relationships (RP), The Number System (NS), Expressions and Equations (EE), Geometry (G), and Statistics and Probability (SP). In sixth grade, instructional time should focus on four critical areas: (1) connecting ratio and rate to whole number multiplication and division and using concepts of ratio and rate to solve problems; (2) completing understanding of division of fractions and extending the notion of number to the system of rational numbers, which includes negative numbers; (3) writing, interpreting, and using expressions and equations; and (4) developing understanding of statistical thinking.

1) Students use reasoning about multiplication and division to solve ratio and rate problems about quantities. By viewing equivalent ratios and rates as deriving from, and extending, pairs of rows (or columns) in the multiplication table, and by analyzing simple drawings that indicate the relative size of quantities, students connect their understanding of multiplication and division with ratios and rates. Thus students expand the scope of problems for which they can use multiplication and division to solve problems, and they connect ratios and fractions. Students solve a wide variety of problems involving ratios and rates.

2) Students use the meaning of fractions, the meanings of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for dividing fractions make sense. Students use these operations to solve problems. Students extend their previous understandings of number and the ordering of numbers to the full system of rational numbers, which includes negative rational numbers, and in particular negative integers. They reason about the order and absolute value of rational numbers and about the location of points in all four quadrants of the coordinate plane.

3) Students understand the use of variables in mathematical expressions. They write expressions and equations that correspond to given situations, evaluate expressions, and use expressions and formulas to solve problems. Students understand that expressions in different forms can be equivalent, and they use the properties of operations to rewrite expressions in equivalent forms. Students know that the solutions of an equation are the values of the variables that make the equation true. Students use properties of operations and the idea of maintaining the equality of both sides of an equation to solve simple one-step equations. Students construct and analyze tables, such as tables of quantities that are in equivalent ratios, and they use equations (such as $3x = y$) to describe relationships between quantities.

4) Building on and reinforcing their understanding of number, students begin to develop their ability to think statistically. Students recognize that a data distribution may not have a definite center and that different ways to measure center yield different values. The median measures center in the sense that it is roughly the middle value. The mean measures center in the sense that it is the value that each data point would take on if the total of the data values were redistributed equally, and also in the sense that it is a balance point. Students recognize that a measure of variability (interquartile range or mean absolute deviation) can also be useful for summarizing data because two very different sets of data can have the same mean and median yet be distinguished by their variability.
Students learn to describe and summarize numerical data sets, identifying clusters, peaks, gaps, and symmetry, considering the context in which the data were collected. Students in Grade 6 also build on their work with area in elementary school by reasoning about relationships among shapes to determine area, surface area, and volume. They find areas of right triangles, other triangles, and special quadrilaterals by decomposing these shapes, rearranging or removing pieces, and relating the shapes to rectangles. Using these methods, students discuss, develop, and justify formulas for areas of triangles and parallelograms. Students find areas of polygons and surface areas of prisms and pyramids by decomposing them into pieces whose area they can determine. They reason about right rectangular prisms with fractional side lengths to extend formulas for the volume of a right rectangular prism to fractional side lengths. They prepare for work on scale drawings and constructions in Grade 7 by drawing polygons in the coordinate plane.
This ESL framework was designed to be used by bilingual, dual language, ESL and general education teachers. Bilingual and dual language programs use the home language and a second language for instruction. ESL teachers and general education or bilingual teachers may use this document to collaborate on unit and lesson planning to decide who will address certain components of the SLO and language objective. ESL teachers may use the appropriate leveled language objective to build lessons for ELLs which reflects what is covered in the general education program. In this way, whether it is a pull-out or push-in model, all teachers are working on the same Student Learning Objective connected to the New Jersey Student Learning Standards. The design of language objectives are based on the alignment of the World-Class Instructional Design Assessment (WIDA) Consortium’s English Language Development (ELD) standards with the New Jersey Student Learning Standards (NJSLS). WIDA’s ELD standards advance academic language development across content areas ultimately leading to academic achievement for English learners. As English learners are progressing through the six developmental linguistic stages, this framework will assist all teachers who work with English learners to appropriately identify the language needed to meet the requirements of the content standard. At the same time, the language objectives recognize the cognitive demand required to complete educational tasks. Even though listening and reading (receptive) skills differ from speaking and writing (expressive) skills across proficiency levels the cognitive function should not be diminished. For example, an Entering Level One student only has the linguistic ability to respond in single words in English with significant support from their home language. However, they could complete a Venn diagram with single words which demonstrates that they understand how the elements compare and contrast with each other or they could respond with the support of their home language (L1) with assistance from a teacher, para-professional, peer or a technology program.

http://www.state.nj.us/education/modelcurriculum/ela/ELLOverview.pdf
<table>
<thead>
<tr>
<th>#</th>
<th>Student Learning Objective</th>
<th>NJSLS</th>
<th>Instruction: 8 weeks</th>
<th>Assessment: 1 week</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Write an equation using two variables (independent and dependent) to represent two quantities that change in relationship to one another in a real world problem.</td>
<td>6.EE.C.9</td>
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<tr>
<td>2</td>
<td>Analyze the relationship between the dependent and independent variables and relate the equation to a given graph and to its table of values.</td>
<td>6.EE.C.9</td>
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<td>3</td>
<td>Distinguish questions that are statistical (anticipate variability in data) from those that are not.</td>
<td>6.SP.A.1</td>
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<td>4</td>
<td>Display numerical data in plots on the number line (including dot plots, histograms, and box plots) and summarize in relation to their context.</td>
<td>6.SP.A.2, 6.SP.A.3, 6.SP.B.4</td>
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<tr>
<td>5</td>
<td>Summarize numerical data in relation to their context by identifying the number of observations and describing how the data was measured.</td>
<td>6.SP.B.5a, 5b</td>
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<tr>
<td>6</td>
<td>Calculate, and interpret measures of center (mean and median) and variability (interquartile range and mean absolute deviation); report measures of center and variability appropriate to the shape of the distribution and context.</td>
<td>6.SP.B.5c, 5d</td>
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<td>7</td>
<td>Create and complete tables of equivalent ratios to solve real world and mathematical problems using ratio and rate reasoning that include making tables of equivalent ratios, solving unit rate problems, finding percent of a quantity as a rate per 100.</td>
<td>6.RP.A.3a, 3b, 3c</td>
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<td>8</td>
<td>Use ratio and rate reasoning to convert measurement units and to transform units appropriately when multiplying or dividing quantities.</td>
<td>6.RP.A.3d*</td>
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<tr>
<td>9</td>
<td>Solve real world and mathematical problems by graphing points in all four quadrants of the coordinate plane; use the absolute value of the differences of their coordinates to find distances between points with the same first coordinate or same second coordinate.</td>
<td>6.NS.C.8*</td>
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</table>
Research about Teaching and Learning Mathematics

Structure teaching of mathematical concepts and skills around problems to be solved (Checkly, 1997; Wood & Sellars, 1996; Wood & Sellars, 1997)

Encourage students to work cooperatively with others (Johnson & Johnson, 1975; Davidson, 1990)

Use group problem-solving to stimulate students to apply their mathematical thinking skills (Artzt & Armour-Thomas, 1992)

Students interact in ways that support and challenge one another’s strategic thinking (Artzt, Armour-Thomas, & Curcio, 2008)

Activities structured in ways allowing students to explore, explain, extend, and evaluate their progress (National Research Council, 1999)

There are three critical components to effective mathematics instruction (Shellard & Moyer, 2002):

- Teaching for conceptual understanding
- Developing children’s procedural literacy
- Promoting strategic competence through meaningful problem-solving investigations

Teachers should be:

- Demonstrating acceptance and recognition of students’ divergent ideas
- Challenging students to think deeply about the problems they are solving, extending thinking beyond the solutions and algorithms required to solve the problem
- Influencing learning by asking challenging and interesting questions to accelerate students’ innate inquisitiveness and foster them to examine concepts further
- Projecting a positive attitude about mathematics and about students’ ability to “do” mathematics

Students should be:

- Actively engaging in “doing” mathematics
- Solving challenging problems
- Investigating meaningful real-world problems
- Making interdisciplinary connections
- Developing an understanding of mathematical knowledge required to “do” mathematics and connect the language of mathematical ideas with numerical representations
- Sharing mathematical ideas, discussing mathematics with one another, refining and critiquing each other’s ideas and understandings
- Communicating in pairs, small group, or whole group presentations
- Using multiple representations to communicate mathematical ideas
- Using connections between pictures, oral language, written symbols, manipulative models, and real-world situations
- Using technological resources and other 21st century skills to support and enhance mathematical understanding
Mathematics is not a stagnant field of textbook problems; rather, it is a dynamic way of constructing meaning about the world around us, generating knowledge and understanding about the real world every day. Students should be metaphorically rolling up their sleeves and “doing mathematics” themselves, not watching others do mathematics for them or in front of them. (Protheroe, 2007)

**Balanced Mathematics Instructional Model**

Balanced math consists of three different learning opportunities; guided math, shared math, and independent math. Ensuring a balance of all three approaches will build conceptual understanding, problem solving, computational fluency, and procedural fluency. Building conceptual understanding is the focal point of developing mathematical proficiency. Students should frequently work on rigorous tasks, talk about the math, explain their thinking, justify their answer or process, build models with graphs or charts or manipulatives, and use technology.

When balanced math is used in the classroom it provides students opportunities to:

- solve problems
- make connections between math concepts and real-life situations
- communicate mathematical ideas (orally, visually and in writing)
- choose appropriate materials to solve problems
- reflect and monitor their own understanding of the math concepts
- practice strategies to build procedural and conceptual confidence

Teacher builds conceptual understanding by modeling through demonstration, explicit instruction, and think alouds, as well as guiding students as they practice math strategies and apply problem solving strategies. (whole group or small group instruction)

Teacher and students practice mathematics processes together through interactive activities, problem solving, and discussion. (whole group or small group instruction)

Students practice math strategies independently to build procedural and computational fluency. Teacher assesses learning and reteaches as necessary. (whole group instruction, small group instruction, or centers)
<table>
<thead>
<tr>
<th>Effective Pedagogical Routines/Instructional Strategies</th>
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<tbody>
<tr>
<td>Collaborative Problem Solving</td>
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<tr>
<td>Connect Previous Knowledge to New Learning</td>
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<tr>
<td>Making Thinking Visible</td>
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<tr>
<td>Develop and Demonstrate Mathematical Practices</td>
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<tr>
<td>Inquiry-Oriented and Exploratory Approach</td>
</tr>
<tr>
<td>Multiple Solution Paths and Strategies</td>
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<tr>
<td>Use of Multiple Representations</td>
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<tr>
<td>Explain the Rationale of your Math Work</td>
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<tr>
<td>Quick Writes</td>
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<tr>
<td>Pair/Trio Sharing</td>
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<tr>
<td>Turn and Talk</td>
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<tr>
<td>Charting</td>
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<td>Gallery Walks</td>
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<tr>
<td>Small Group and Whole Class Discussions</td>
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<td>Student Modeling</td>
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<tr>
<td>Analyze Student Work</td>
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<tr>
<td>Identify Student’s Mathematical Understanding</td>
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<td>Identify Student’s Mathematical Misunderstandings</td>
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<tr>
<td>Interviews</td>
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<tr>
<td>Role Playing</td>
</tr>
<tr>
<td>Diagrams, Charts, Tables, and Graphs</td>
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<tr>
<td>Anticipate Likely and Possible Student Responses</td>
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<td>Collect Different Student Approaches</td>
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<td>Multiple Response Strategies</td>
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<tr>
<td>Asking Assessing and Advancing Questions</td>
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<td>Revoicing</td>
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<td>Marking</td>
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<td>Recapping</td>
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<tr>
<td>Challenging</td>
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<tr>
<td>Pressing for Accuracy and Reasoning</td>
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<tr>
<td>Maintain the Cognitive Demand</td>
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</table>
### Educational Technology

#### Standards

**8.1.8.A.1, 8.1.8.A.4, 8.1.8.E.1, 8.2.8.C.8**

#### Technology Operations and Concepts

- **Demonstrate knowledge of a real world problem using digital tools**
  
  **Example:** Students will be able to practice solving for central tendency using http://www.eduplace.com/kids/hmcam/animath/swf/mean_median_and_mode.swf

- **Graph and calculate data within a spreadsheet and present a summary of the results**
  
  **Example:** Students can look up basketball player stats, put them into a table in excel and then create a box-and-whisker plot in the excel spreadsheet. This will coincide with the task Using NBA Statistics.

#### Research and Information Fluency

- **Effectively use a variety of search tools and filters in professional public databases to find information to solve a real world problem.**
  
  **Example:** Students can search through Learnzillion, Imagine Math Facts, and other interactive sites for appropriate instructional videos and/or information pertaining to strategies and modeling.

#### Design

- **Develop a proposal for a chosen solution that include models (physical, graphical or mathematical) to communicate the solution to peers.**
  
  **Example:** Students will utilize http://www.shodor.org/interactivate/activities/BoxPlot/ to help them draw a box-and-whisker plot and then use the information to explain the center and variability to the class.
**Career Ready Practices**

Career Ready Practices describe the career-ready skills that all educators in all content areas should seek to develop in their students. They are practices that have been linked to increase college, career, and life success. Career Ready Practices should be taught and reinforced in all career exploration and preparation programs with increasingly higher levels of complexity and expectation as a student advances through a program of study.

- **CRP2. Apply appropriate academic and technical skills.**
  Career-ready individuals readily access and use the knowledge and skills acquired through experience and education to be more productive. They make connections between abstract concepts with real-world applications, and they make correct insights about when it is appropriate to apply the use of an academic skill in a workplace situation.

  **Example:** Students will apply prior knowledge when solving real world problems. Students will make sound judgements about the use of specific tools, such as creating graphs (including dot plots, histograms, and box plots) and use the tools to explore and deepen the understanding of the concept of how to display numerical data.

- **CRP4. Communicate clearly and effectively and with reason.**
  Career-ready individuals communicate thoughts, ideas, and action plans with clarity, whether using written, verbal, and/or visual methods. They communicate in the workplace with clarity and purpose to make maximum use of their own and others’ time. They are excellent writers; they master conventions, word choice, and organization, and use effective tone and presentation skills to articulate ideas. They are skilled at interacting with others; they are active listeners and speak clearly and with purpose. Career-ready individuals think about the audience for their communication and prepare accordingly to ensure the desired outcome.

  **Example:** Students on a daily basis will communicate their reasoning behind their solution paths by making connections to the context and the quantities, using proper vocabulary, along with decontextualizing and/or contextualizing the problem. Students will create representations using objects, drawings, diagrams, and/or actions, such as the box plot to determine measures of center and variability. They will also explain the meaning behind the quantities and units involved. Students will also ask probing questions to clarify and improve arguments.

- **CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.**
  Career-ready individuals readily recognize problems in the workplace, understand the nature of the problem, and devise effective plans to solve the problem. They are aware of problems when they occur and take action quickly to address the problem; they thoughtfully investigate the root cause of the problem prior to introducing solutions. They carefully consider the options to solve the problem. Once a solution is agreed upon, they follow through to ensure the problem is solved, whether through their own actions or the actions of others.
Career Ready Practices

Example: Throughout their daily lessons, students will understand the meaning of a problem and look for entry points into solving their problems by analyzing the relationships of the quantities, constraints and goals of the task. Plans for solution paths will be made and have meaning. Students will self-monitor, evaluate and critique their process and progress as they are working and make changes as necessary.

- **CRP12. Work productively in teams while using cultural global competence.**
  Career-ready individuals positively contribute to every team, whether formal or informal. They apply an awareness of cultural difference to avoid barriers to productive and positive interaction. They find ways to increase the engagement and contribution of all team members. They plan and facilitate effective team meetings.

  Example: Students will work in collaborative and whole group settings to develop various solutions to math tasks that are presented to them. They will work together to understand the terms of the problem, ask clarifying and challenging questions among each other, and develop agreed upon solutions using a variety of strategies and models. Students will listen to, read and discuss arguments with each other with respect and courtesy at all times and will be willing to assist those that may need assistance. In this unit, students will demonstrate and explain to a peer or small group how to analyze the relationship between the dependent and independent variables and relate the equation to a given graph and to its table of values.
WIDA Proficiency Levels

At the given level of English language proficiency, English language learners will process, understand, produce or use:

<table>
<thead>
<tr>
<th>Level</th>
<th>Description</th>
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</thead>
</table>
| **6- Reaching** | - Specialized or technical language reflective of the content areas at grade level  
- A variety of sentence lengths of varying linguistic complexity in extended oral or written discourse as required by the specified grade level  
- Oral or written communication in English comparable to proficient English peers |
| **5- Bridging** | - Specialized or technical language of the content areas  
- A variety of sentence lengths of varying linguistic complexity in extended oral or written discourse, including stories, essays or reports  
- Oral or written language approaching comparability to that of proficient English peers when presented with grade level material. |
| **4- Expanding** | - Specific and some technical language of the content areas  
- A variety of sentence lengths of varying linguistic complexity in oral discourse or multiple, related sentences or paragraphs  
- Oral or written language with minimal phonological, syntactic or semantic errors that may impede the communication, but retain much of its meaning, when presented with oral or written connected discourse, with sensory, graphic or interactive support |
| **3- Developing** | - General and some specific language of the content areas  
- Expanded sentences in oral interaction or written paragraphs  
- Oral or written language with phonological, syntactic or semantic errors that may impede the communication, but retain much of its meaning, when presented with oral or written, narrative or expository descriptions with sensory, graphic or interactive support |
| **2- Beginning** | - General language related to the content area  
- Phrases or short sentences  
- Oral or written language with phonological, syntactic, or semantic errors that often impede of the communication when presented with one to multiple-step commands, directions, or a series of statements with sensory, graphic or interactive support |
| **1- Entering** | - Pictorial or graphic representation of the language of the content areas  
- Words, phrases or chunks of language when presented with one-step commands directions, WH-, choice or yes/no questions, or statements with sensory, graphic or interactive support |
# Language Development Supports For English Language Learners

To Increase Comprehension and Communication Skills

## Environment

- Welcoming and stress-free
- Respectful of linguistic and cultural diversity
- Honors students’ background knowledge
- Sets clear and high expectations
- Includes routines and norms
- Is thinking-focused vs. answer-seeking
- Offers multiple modalities to engage in content learning and to demonstrate understanding
- Includes explicit instruction of specific language targets
- Provides participation techniques to include all learners
- Integrates learning centers and games in a meaningful way
- Provides opportunities to practice and refine receptive and productive skills in English as a new language
- Integrates meaning and purposeful tasks/activities that:
  - Are accessible by all students through multiple entry points
  - Are relevant to students’ lives and cultural experiences
  - Build on prior mathematical learning
  - Demonstrate high cognitive demand
  - Offer multiple strategies for solutions
  - Allow for a language learning experience in addition to content

## Sensory Supports*

- Real-life objects (realia) or concrete objects
- Physical models
- Manipulatives
- Pictures & photographs
- Visual representations or models such as diagrams or drawings
- Videos & films
- Newspapers or magazines
- Gestures
- Physical movements
- Music & songs

## Graphic Supports*

- Graphs
- Charts
- Timelines
- Number lines
- Graphic organizers
- Graphing paper

## Interactive Supports*

- In a whole group
- In a small group
- With a partner such as **Turn-and-Talk**
- In pairs as a group (first, two pairs work independently, then they form a group of four)
- In triads
- Cooperative learning structures such as **Think-Pair-Share**
- Interactive websites or software
- With a mentor or coach

## Verbal and Textual Supports

- Labeling
- Students’ native language
- Modeling
- Repetitions
- Paraphrasing
- Summarizing
- Guiding questions
- Clarifying questions
- Probing questions
- Leveled questions such as **What? When? Where? How? Why?**
- Questioning prompts & cues
- Word Banks
- Sentence starters
- Sentence frames
- Discussion frames
- Talk moves, including **Wait Time**

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**BUILDING EQUITY IN YOUR TEACHING PRACTICE**

How do the essential questions highlight the connection between the big ideas of the unit and equity in your teaching practice?

<table>
<thead>
<tr>
<th>CONTENT INTEGRATION</th>
<th>KNOWLEDGE CONSTRUCTION</th>
<th>PREJUDICE REDUCTION</th>
<th>EQUITABLE PEDAGOGY</th>
<th>EMPOWERING SCHOOL CULTURE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Teachers use examples and content from a variety of cultures &amp; groups.</td>
<td>Teachers help students understand how knowledge is created and influenced by cultural assumptions, perspectives &amp; biases.</td>
<td>Teachers implement lessons and activities to assert positive images of ethnic groups &amp; improve intergroup relations.</td>
<td>Teachers modify techniques and methods to facilitate the academic achievement of students from diverse backgrounds.</td>
<td>Using the other four dimensions to create a safe and healthy educational environment for all.</td>
</tr>
<tr>
<td>This unit / lesson is connected to other topics explored with students.</td>
<td>This unit / lesson provides context to the history of privilege and oppression.</td>
<td>This unit / lesson helps students question and unpack biases &amp; stereotypes.</td>
<td>The instruction has been modified to meet the needs of each student.</td>
<td>There are opportunities for students to connect with the community.</td>
</tr>
<tr>
<td>There are multiple viewpoints reflected in the content of this unit / lesson.</td>
<td>This unit / lesson addresses power relationships.</td>
<td>This unit / lesson helps students examine, research and question information and sources.</td>
<td>Students feel respected and their cultural identities are valued.</td>
<td>My classroom is welcoming and supportive for all students?</td>
</tr>
<tr>
<td>The materials and resources are reflective of the diverse identities and experiences of students.</td>
<td>This unit / lesson helps students to develop research and critical thinking skills.</td>
<td>The curriculum encourages discussion and understanding about the groups of people being represented.</td>
<td>Additional supports have been provided for students to become successful and independent learners.</td>
<td>I am aware of and sensitive to the needs of my students and their families.</td>
</tr>
<tr>
<td>The content affirms students, as well as exposes them to experiences other than their own.</td>
<td>This curriculum creates windows and mirrors for students.</td>
<td>This unit / lesson challenges dominant perspectives.</td>
<td>Opportunities are provided for student to reflect on their learning and provide feedback.</td>
<td>There are effective parent communication systems established. Parents can talk to me about issues as they arise in my classroom.</td>
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</tbody>
</table>

### Culturally Relevant Pedagogy Examples

- **Problem-Based Learning Scenarios:** Present relatable real-world problems for your students to solve, explicitly referencing cultures and communities when applicable.
  
  **Example:** Have students choose a career and have them find the median salary in ten states. Ask them to plot the salaries on a number line. The following link from the Bureau of Labor Statistics can be used to find salaries: [https://www.bls.gov/ooh/](https://www.bls.gov/ooh/)

- **Everyone has a Voice:** Create a classroom environment where students know that their contributions are expected and valued.
  
  **Example:** Establish norms for sharing that promote discourse and a growth mindset for mathematics. All students are capable of expressing mathematical thinking and contributing to the classroom community. Students learn new ways of looking at problem solving by working with and listening to each other.

- **Use Learning Stations:** Provide a range of material by setting up learning stations.
  
  **Example:** Reinforce understandings of concepts and skills by promoting learning through student interests, modalities, experiences and/or prior knowledge. Encourage the students to make content choices based upon their strengths, needs, values and experiences. Providing students with choice boards will give them a sense of ownership to their learning and understanding.

- **Present New Concepts Using Student Vocabulary:** Use student diction to capture attention and build understanding before using academic terms.
  
  **Example:** Teach math vocabulary in various modalities to increase students’ retention. Use multi-modal activities, analogies, realia, visual cues, graphic representations, gestures, pictures, practice and cognates. Inform students that some vocabulary words have multiple meanings. Have students create the Word Wall with their definitions and examples to foster ownership. Work with students to create a sorting and matching game using vocabulary words from within the unit. Students can work in teams or individually to play these games. This will allow students to familiarize themselves with the vocabulary words within the unit.
## Differentiated Instruction

Accommodate Based on Students Individual Needs: Strategies

<table>
<thead>
<tr>
<th>Time/General</th>
<th>Processing</th>
<th>Comprehension</th>
<th>Recall</th>
</tr>
</thead>
<tbody>
<tr>
<td>Extra time for assigned tasks</td>
<td>Extra Response time</td>
<td>Precise processes for balanced math instructional model</td>
<td>Teacher-made checklist</td>
</tr>
<tr>
<td>Adjust length of assignment</td>
<td>Have students verbalize steps</td>
<td>Short manageable tasks</td>
<td>Use visual graphic organizers</td>
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<tr>
<td>Timeline with due dates for reports and projects</td>
<td>Repeat, clarify or reword</td>
<td>Brief and concrete directions</td>
<td>Reference resources to promote independence</td>
</tr>
<tr>
<td>Communication system between home and school</td>
<td>directions</td>
<td>Provide immediate feedback</td>
<td>Visual and verbal reminders</td>
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<tr>
<td>Provide lecture notes/outline</td>
<td>Mini-breaks between tasks</td>
<td>Small group instruction</td>
<td>Graphic organizers</td>
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<td></td>
<td>Provide a warning for</td>
<td>Emphasize multi-sensory learning</td>
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<td>transitions</td>
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<td></td>
<td>Partnering</td>
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<thead>
<tr>
<th>Assistive Technology</th>
<th>Tests/Quizzes/Grading</th>
<th>Behavior/Attention</th>
<th>Organization</th>
</tr>
</thead>
<tbody>
<tr>
<td>Computer/whiteboard</td>
<td>Extended time</td>
<td>Consistent daily structured routine</td>
<td>Individual daily planner</td>
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<tr>
<td>Tape recorder</td>
<td>Study guides</td>
<td>Simple and clear classroom rules</td>
<td>Display a written agenda</td>
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<td>Video Tape</td>
<td>Shortened tests</td>
<td>Frequent feedback</td>
<td>Note-taking assistance</td>
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<tr>
<td></td>
<td>Read directions aloud</td>
<td></td>
<td>Color code materials</td>
</tr>
</tbody>
</table>

- **Time/General**: Extra time for assigned tasks, Adjust length of assignment, Timeline with due dates for reports and projects, Communication system between home and school, Provide lecture notes/outline
- **Processing**: Extra Response time, Have students verbalize steps, Repeat, clarify or reword directions, Mini-breaks between tasks, Provide a warning for transitions, Partnering
- **Comprehension**: Precise processes for balanced math instructional model, Short manageable tasks, Brief and concrete directions, Provide immediate feedback, Small group instruction, Emphasize multi-sensory learning
- **Recall**: Teacher-made checklist, Use visual graphic organizers, Reference resources to promote independence, Visual and verbal reminders, Graphic organizers
- **Assistive Technology**: Computer/whiteboard, Tape recorder, Video Tape
- **Tests/Quizzes/Grading**: Extended time, Study guides, Shortened tests, Read directions aloud
- **Behavior/Attention**: Consistent daily structured routine, Simple and clear classroom rules, Frequent feedback
- **Organization**: Individual daily planner, Display a written agenda, Note-taking assistance, Color code materials
<table>
<thead>
<tr>
<th>Differentiated Instruction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Accommodate Based on Content Specific Needs: Strategies</td>
</tr>
</tbody>
</table>

- Anchor charts to model strategies and use of formulas
- Reference sheets that list formulas, step-by-step procedures and model strategies
- Conceptual word wall that contains definition, translation, pictures and/or examples
- Graphic organizers (Examples include: Venn diagram, Four Square, K-W-L)
- Translation dictionary
- Teacher modeling
- Four-function calculator to assist with computations
- Students can utilize math journals to write notes, copy solution steps, and translate terms and key vocabulary
- Highlight and label the solution steps for multi-step problems in different colors
- Utilize technological programs which provide verbal and visual instruction in native and/or second language
- Review of area models for distributive property
- Review of Commutative and Associative Properties of Multiplication and Addition
- Tables to assist with understanding independent and dependent variables, converting units of measure and finding unit rate
- Graphs to understand independent and dependent variables
- Provide pre-printed frequency tables to expose students to data and how to interpret it
- Provide dot plots, histograms and number lines to interpret data
- Provide conversion charts to assist with units of measure
- Double number line to assist with converting units of measure and finding unit rate
- Cartesian Plane and graph paper to assist with graphing points and finding the distance between points
Interdisciplinary Connections

Social Studies Connection:
*Climbing the Steps of El Castillo (6.2.8.B.2.a)*
- This task is about El Castillo which is a pyramid from Chichén Itzá. Students can learn about this Mayan City which is now Mexico. Students will need to determine their height is standing on a certain step.

*Electoral College (6.1.4.A.7)*
- Students will use a table of electoral votes for each state and Washington DC for 2012, 2016 and 2020 presidential elections to analyze and answer questions. Social Students connection can be made in explaining the electoral college and the process that takes place during elections.

Science Connection:
*Water Crisis in Haiti (MS-ESS3-2)*
- Students will be able to learn about recent natural disasters, such as earthquakes, tornados, hurricanes, etc. A discussion can be included about the health risks that can occur after a natural disaster and what is it like to lose water after a natural disaster and how to go about getting clean water.

*Range of Motion (MS-LS1-3)*
- This task incorporates knowledge of human structure and function and will give students an introduction to careers in the life science field. Students will use ratio and rate reasoning to solve this real world mathematical problem which includes making tables tracking the range of motion of a patient.

ELA Connection:
*Various Tasks: (RL.6.1 & RI.6.1)*
- Students will be able to read, analyze, and cite informational text to solve problems and explain their reasoning of how the task was solved. Students will also focus on vocabulary, mechanics and grammar in effective writing.

*Shakespeare vs. Rowling: (RL.6.1)*
- Students will read excerpts from Harry Potter and the Chamber of Secrets and Macbeth. Students may be unfamiliar with who Shakespeare and JK Rowling are and a brief biography of who these authors can be included in the lesson.
**Enrichment**

What is the purpose of Enrichment?

- The purpose of enrichment is to provide extended learning opportunities and challenges to students who have already mastered, or can quickly master, the basic curriculum. Enrichment gives the student more time to study concepts with greater depth, breadth, and complexity.
- Enrichment also provides opportunities for students to pursue learning in their own areas of interest and strengths.
- Enrichment keeps advanced students engaged and supports their accelerated academic needs.
- Enrichment provides the most appropriate answer to the question, “What do you do when the student already knows it?”

Enrichment is…

- Planned and purposeful
- *Different*, or differentiated, work – not just *more* work
- Responsive to students’ needs and situations
- A promotion of high-level thinking skills and making connections within content
- The ability to apply different or multiple strategies to the content
- The ability to synthesize concepts and make real world and cross-curricular connections
- Elevated contextual complexity
- Sometimes independent activities, sometimes direct instruction
- Inquiry based or open ended assignments and projects
- Using supplementary materials in addition to the normal range of resources
- Choices for students
- Tiered/Multi-level activities with flexible groups (may change daily or weekly)

Enrichment is not…

- Just for gifted students (some gifted students may need intervention in some areas just as some other students may need frequent enrichment)
- Worksheets that are more of the same (busywork)
- Random assignments, games, or puzzles not connected to the content areas or areas of student interest
- Extra homework
- A package that is the same for everyone
- Thinking skills taught in isolation
- Unstructured free time
Assessments

Required District/State Assessments
Unit Assessments
PARCC
SGO Assessments

Suggested Formative/Summative Classroom Assessments
Describe Learning Vertically
Identify Key Building Blocks
Make Connections (between and among key building blocks)
Short/Extended Constructed Response Items
Multiple-Choice Items (where multiple answer choices may be correct)
Drag and Drop Items
Use of Equation Editor
Quizzes
Journal Entries/Reflections/Quick-Writes
Accountable talk
Projects
Portfolio
Observation
Graphic Organizers/ Concept Mapping
Presentations
Role Playing
Teacher-Student and Student-Student Conferencing
Homework
New Jersey Student Learning Standards

6.EE.C.9. Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation \( d = 65t \) to represent the relationship between distance and time.

6.SP.A.1. Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. For example, “How old am I?” is not a statistical question, but “How old are the students in my school?” is a statistical question because one anticipates variability in students’ ages.

6.SP.A.2. Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.

6.SP.A.3. Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.

6.SP.B.4. Display numerical data in plots on a number line, including dot plots, histograms, and box plots.

6.SP.B.5. Summarize numerical data sets in relation to their context, such as by:

   6.SP.B.5a. Reporting the number of observations.

   6.SP.B.5b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement.

   6.SP.B.5c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.

   6.SP.B.5d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.
New Jersey Student Learning Standards

6.RP.A.3
Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. *(benchmarked)*

6.RP.A.3a. Make tables of equivalent ratios relating quantities with whole number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.

6.RP.A.3b. Solve unit rate problems including those involving unit pricing and constant speed. For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?

6.RP.A.3c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.

6.RP.A.3d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

6.NS.C.8. Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate. *(benchmarked)*
## Mathematical Practices

1. Make sense of problems and persevere in solving them.

2. Reason abstractly and quantitatively.

3. Construct viable arguments and critique the reasoning of others.

4. Model with mathematics.

5. Use appropriate tools strategically.

6. Attend to precision.

7. Look for and make use of structure.

8. Look for and express regularity in repeated reasoning.
Grade: Six  
Unit: 4 (Four)  
Topic: Variability, Distributions, and Relationships Between Quantities

| NJSLs: 6.EE.C.9, 6.SP.A.1, 6.SP.A.2, 6.SP.A.3, 6.SP.B.4, 6.SP.B.5a,b,c,d, 6.RP.A.3a,b,c,d, 6.NS.C.8 |

**Unit Focus:**
- Represent and analyze quantitative relationships between dependent and independent variables
- Develop understanding of statistical variability
- Summarize and describe distributions
- Understand ratio concepts and use ratio reasoning to solve problems
- Apply and extend previous understandings of numbers to the system of rational numbers

### New Jersey Student Learning Standards(s):

**6.EE.C.9:** Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation* $d = 65t$ *to represent the relationship between distance and time.*

**Student Learning Objective 1:** Write an equation using two variables (independent and dependent) to represent two quantities that change in relationship to one another in a real world problem.

**Modified Student Learning Objectives/Standards:** N/A

<table>
<thead>
<tr>
<th>MPs</th>
<th>Evidence Statement Key/Clarifications</th>
<th>Skills, Strategies &amp; Concepts</th>
<th>Essential Understandings/Questions (Accountable Talk)</th>
<th>Tasks/Activities</th>
</tr>
</thead>
<tbody>
<tr>
<td>MP 2</td>
<td>6.EE.9</td>
<td>Students should be able to write an equation from a word problem and understand how the coefficient of the dependent variable is related to the graph and/or a table of values.</td>
<td>How can proportional relationships be described using the equation $y = kx$?</td>
<td>Chocolate Bar Sales</td>
</tr>
<tr>
<td>MP 4</td>
<td>Tasks that involve writing an equation should not go beyond the equation types described in 6.EE.7 ($x+p=q$)</td>
<td></td>
<td></td>
<td>Problems in Mathematical Terms</td>
</tr>
<tr>
<td>MP 6</td>
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</tr>
</tbody>
</table>

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| and \( px = q \) where \( p, q, \) and \( x \) are all nonnegative rational numbers). |
| Two quantities that change in relationship to one another may be represented with an equation that has two variables, with a graph, and with a table of values. |
| How can proportional relationships be represented using rules, tables, and graphs? |
| How Much Should That Specialty Pizza Cost? |
| Running Around Town |

**6.C.8.2**

- Present solutions to multi-step problems in the form of valid chains of reasoning, using symbols such as equals signs appropriately (for example, rubrics award less than full credit for the presence of nonsense statements such as \( 1 + 4 = 5 + 7 = 12 \), even if the final answer is correct), or identify or describe errors in solutions to multi-step problems and present corrected solutions. Content Scope: Knowledge and skills articulated in 6.EE.9

- Tasks that involve writing an equation should not go beyond the equation types described in 6.EE.7 (\( x + p = q \) and \( px = q \) where \( p, q, \) and \( x \) are all nonnegative rational numbers).

**SPED Strategies:**

Review and demonstrate how to find the relationship between two quantities to help find the value of the quantities.

Review and create anchor charts with pictures and definitions of math terms (i.e. dependent variable, independent variable, variable).

Demonstrate and practices writing equations using independent and dependent variables.

Review, demonstrate and practice evaluating the change of one variable (independent variable) in relation to the other variable (dependent variable).

The independent variable is the variable that can be changed; the dependent variable is the variable that is affected by the change in the independent variable.
Students recognize that the independent variable is graphed on the x-axis; the dependent variable is graphed on the y-axis.

Give students practice using multiple representations. Multiple representations include describing the relationship using language, a table, an equation, or a graph. Demonstrating multiple representations helps students understand that each form represents the same relationship and provides a different perspective.

**Resources UDL - Visual and Auditory Learner(s):**
6.EE.9-1.0 Multiple Variables - Common Core Standard  
[https://youtu.be/Kpb_FrHYokE](https://youtu.be/Kpb_FrHYokE)

6.EE.9 Dependent/independent variables  
[https://youtu.be/vKT1PS6r1-E](https://youtu.be/vKT1PS6r1-E)

**ELL Strategies:**
Use an adapted text with a lower readability level, a teacher-modified text (with highlighted words, explanations, and context clues), or a simplified teacher-generated text illustrating various expressions and equations.

Provide word walls with translations side by side.
Utilize a KWL-chart graphic organizer, have the parts listed in both their L1 (students’ native language) and L2 (students’ target language) to clarify understanding.

Provide a math word bank that is translated and copied for students.

Provide math reference sheets.

Provide picture cards with formulas and real world examples.

Provide audio lessons which explain how to solve equations in both languages.

**Website:**  
*LearnZillion*  
Recognize how an equation relates independent and dependent variables  
[https://learnzillion.com/lesson_plans/681](https://learnzillion.com/lesson_plans/681)
New Jersey Student Learning Standard(s):
6.EE.C.9: Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation \( d = 65t \) to represent the relationship between distance and time.

Student Learning Objective 2: Analyze the relationship between the dependent and independent variables and relate the equation to a given graph and to its table of values.

Modified Student Learning Objectives/Standards: N/A

<table>
<thead>
<tr>
<th>MPs</th>
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<th>Tasks/Activities</th>
</tr>
</thead>
<tbody>
<tr>
<td>MP 2</td>
<td>6.EE.9</td>
<td>The purpose of this standard is for students to understand the relationship between two variables, which begins with the distinction between dependent and independent variables. The independent variable is the variable that can be changed; the dependent variable is the variable that is affected by the change in the independent variable. Students recognize that the independent variable is graphed on the x-axis; the dependent variable is graphed on the y-axis. <strong>SPED Strategies:</strong> Review and demonstrate how to find the relationship between two quantities to help find the value of the quantities.</td>
<td>How can proportional relationships be described using the equation ( y = kx )? How can proportional relationships be represented using rules, tables, and graphs? How can the graph of ( y = kx ) be interpreted for different contexts? How can algebraic expressions be used to model real-world situations? How can we solve simple algebraic equations, and</td>
<td>Geek Phone Profits</td>
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<tr>
<td>MP 4</td>
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<td>Making Sense of Graphs</td>
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<tr>
<td>MP 6</td>
<td>6.C.8.2</td>
<td></td>
<td></td>
<td>Analyzing Tables</td>
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<td></td>
<td>Present solutions to multi-step problems in the form of valid chains of reasoning, using symbols such as equals signs appropriately (for example, rubrics award less than full credit for the presence of nonsense statements such as ( 1 + 4 = 5 + 7 = 12 ), even if the final answer is correct), or identify or describe errors in equations.</td>
<td>How can proportional relationships be described using the equation ( y = kx )? How can proportional relationships be represented using rules, tables, and graphs? How can the graph of ( y = kx ) be interpreted for different contexts? How can algebraic expressions be used to model real-world situations? How can we solve simple algebraic equations, and</td>
<td>Saving Your Change</td>
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<td>Multi-step Problems in the Real World</td>
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<td>Planning a Trip</td>
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<td>Planking</td>
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<td></td>
<td></td>
<td>Families of Triangles</td>
</tr>
</tbody>
</table>
- Solutions to multi-step problems and present corrected solutions. Content Scope: Knowledge and skills articulated in 6.EE.9

| Tasks that involve writing an equation should not go beyond the equation types described in 6.EE.7 (x+p =q and px = q where p, q, and x are all nonnegative rational numbers). |
| Review and create anchor charts with pictures and definitions of math terms (i.e. dependent variable, independent variable, variable). |
| Demonstrate and practices writing equations using independent and dependent variables. Review, demonstrate and practice evaluating the change of one variable (independent variable) in relation to the other variable (dependent variable). |
| The independent variable is the variable that can be changed; the dependent variable is the variable that is affected by the change in the independent variable. Students recognize that the independent variable is graphed on the x-axis; the dependent variable is graphed on the y-axis. |
| Give students practice using multiple representations. Multiple representations include describing the relationship using language, a table, an equation, or a graph. Demonstrating multiple representations helps students understand that each form represents the same relationship and provides a different perspective. |
| how do we interpret the meaning of the solutions? |
| What is the difference between the dependent variable and the independent variable? |
| How is the dependent variable affected? |
### Resources UDL - Visual and Auditory Learner(s):
6.EE.9-1. Multiple Variables - Common Core Standard
https://youtu.be/Kpb_FrHYokE

6.EE.9 Dependent/independent variables
https://youtu.be/vKT1PS6r1-E

**ELL Strategies:**
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Provide students a math word bank with translations.

Provide math reference sheets.

Provide picture cards with formulas and real world examples.
Provide audio lessons which explain how to solve equations in both languages.

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**New Jersey Student Learning Standard(s):**
6.SP.A.1: Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. *For example, “How old am I?” is not a statistical question, but “How old are the students in my school?” is a statistical question because one anticipates variability in students’ ages.*

**Student Learning Objective 3:** Distinguish questions that are statistical (anticipate variability in data) from those that are not.

**Modified Student Learning Objectives/Standards:**
M.6.SP.A.1–2: Display data on a graph or table that shows variability in the data.

<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td><strong>MP 2</strong>&lt;br&gt; <strong>MP 6</strong></td>
<td>6.SP.1&lt;br&gt;- Tasks do not assess mode and range.</td>
<td>A statistical question anticipates an answer that varies from one individual to the next and is written to account for the variability in the data. Data are the numbers produced in response to a statistical question. Data are frequently collected from surveys or other sources (i.e. documents). Students will be able to distinguish questions that are statistical (anticipate variability in data) from those that are not.</td>
<td>How can I use data to compare different groups? How can I determine the best way to organize a set of data? How can I recognize when a question is statistical and when it is not?</td>
<td><strong>Statistical Questions</strong>&lt;br&gt;<strong>Buttons: Statistical Questions</strong>&lt;br&gt;<strong>Identifying Statistical Questions</strong></td>
</tr>
</tbody>
</table>
| **SPED Strategies:**  
Review and understanding why variability in data helps to analyze data.  

Show examples of various means of data (i.e. test scores, Star records, baseball or basketball records etc.).  

Provide examples of statistical questions that might produce variable data and why they are important when analyze data.  

Practice defining variability as it refers to the spread of data.  

Review and demonstrate how to observe and record numerical or quantitative data.  

Determine if the collection was random, representative, or biased.  

Review vocabulary words with pictures for association or connection (i.e. frequency table, numerical data, observations, quantitative data, attribute, biased, investigation, random, representation).  

| **ELL Strategies:**  
Utilize body cues and hand gestures to assist with expression identification.  

Create charts with mathematical concepts.  

Clarify, compare, and make connections to math words in discussion, particularly during and after practice.

Provide translations of all content and general vocabulary words.

Connect language with concrete and pictorial experiences (visuals of graphs and data-sets).

Know, use, and make the most of student cultural and home experiences. Build on the student’s background knowledge.

Point to visuals of data-sets, while speaking, using your hands to clearly indicate the image that corresponds to your words.

Provide a variety of ways to respond: oral, choral, student boards, concrete models, pictorial models, pair share, small group share. Embed visual, non-linguistic supports for vocabulary clarification.

**Website:** Teachers First
*Adapt a Strategy. Adjusting Lessons for ESL/ELL students*
[http://www.teachersfirst.com/content/esl/adaptstrat.cfm](http://www.teachersfirst.com/content/esl/adaptstrat.cfm)
New Jersey Student Learning Standard(s):

**6.SP.A.2:** Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.

**6.SP.A.3:** Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.

**6.SP.B.4:** Display numerical data in plots on a number line, including dot plots, histograms, and box plots.

**Student Learning Objective 4:** Display numerical data in plots on the number line (including dot plots, histograms, and box plots) and summarize in relation to their context.

**Modified Student Learning Objectives/Standards:**

**M.6.SP.A.1–2:** Display data on a graph or table that shows variability in the data.

<table>
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<tr>
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<th>Tasks/Activities</th>
</tr>
</thead>
</table>
| MP 4 | **6.SP.2**  
- Tasks might present several distributions graphically and ask which two have nearly the same center, nearly the same spread, or nearly the same overall shape.  
- Tasks do not assess mode and range.  
- 6.SP.3  
- Tasks might ask students to rate statements True/False/Not Enough Information, such as, “The average height of trees in  
- 6.SP.A.2 The distribution is the arrangement of the values of a data set.  
- Distribution can be described using center (median or mean), and spread. A data set has a distribution which can be described by its center, spread, and overall shape.  
- Data collected can be represented on graphs, which will show the shape of the distribution of the data. Students examine the distribution of a data set and discuss the center, spread and overall shape with dot plots, histograms and box plots. | **6.SP.A.2** How can I describe the spread of a set of data?  
What is meant by the center of a data set, how is it found and how is it useful when analyzing data? | **6.SP.A.3** How can I describe the center of a set of data? | Test Scores  
Shuttle Run Times  
Long Jump Distances  
Sleep Study  
How Old Were You When?  
Puppy Weights  
Baseball Players |
<table>
<thead>
<tr>
<th><strong>Watson Park</strong> is 65 feet. Are there any trees in Watson Park taller than 65 feet?</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>6.SP.A.3</strong> Data sets contain many numerical values that can be summarized by one number such as a measure of center. The measure of center summarizes, with a single number, the values of an entire data set. The measure of center gives a numerical value to represent the center of the data (i.e. midpoint of an ordered list or the balancing point). Students will be able to distinguish the center from variation. Another characteristic of a data set is the variability (or spread) of the values. Measures of variability are used to describe this characteristic. Measures of variation describe, with a single number, how the values of a data set vary.</td>
</tr>
<tr>
<td><strong>6.SP.4</strong> Tasks ask students to identify which display corresponds to a given set of data. Tasks do not assess mode and range.</td>
</tr>
</tbody>
</table>
**SPED Strategies:**
Review and understanding why variability in data helps to analyze data.

Show examples of various means of data (i.e. test scores, Star records, baseball or basketball records etc.).

Provide examples of statistical questions that might produce variable data and why they are important when analyzing data.

Review samples of data that provide understanding of variability that anticipates change in the measures of central tendency.

Review and create anchor charts with pictures and definitions of math terms (i.e. data, mean, median, mode, range, set, spread of data, and variability bell curve etc.).

Describe and provide examples of a set of data based on its mean, mode, median, range and using a statistical bell curve.

Provide chunking instruction to determine the center of a numerical data set may also be known as the mean, median of the set.

Review and demonstrate that the mean of the data set represents the average of all of the values in the data set.
Practice interpreting data on number lines, dot plots, histograms, and box plots.

Choose the appropriate data display to represent numerical data and provide teacher generated notes to students as step-by-step instructions.

Review and demonstrate how to observe and record numerical or quantitative data.

Practice how to read and interpret numerical data in tables, frequency tables, line plots, and lists.

Practice how to determine values between specified intervals using a histogram.

Describe how data can be collected for an investigation.

Describe the purpose for the investigation.

Determine if the collection was random, representative, or biased.

Explain which measure of center is best to interpret the numerical data in the context of its collection.
<table>
<thead>
<tr>
<th>Resources UDL - Visual and Auditory Learner(s):</th>
</tr>
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<tbody>
<tr>
<td>6.SP.4 - Represent Data on a Box Plot (Box-and-Whisker Plot) / 5 Number Summary</td>
</tr>
<tr>
<td><a href="https://youtu.be/0CjaU4Dm-CE">https://youtu.be/0CjaU4Dm-CE</a></td>
</tr>
</tbody>
</table>

**ELL Strategies:**

Utilize body cues and hand gestures to assist with expression identification.

Create charts with mathematical concepts.

Clarify, compare, and make connections to math words in discussion, particularly during and after practice.

Provide translations of all content and general vocabulary words.

Connect language (such as mean and median) with concrete and pictorial experiences (visuals of graphs and data-sets).

Know, use, and make the most of student cultural and home experiences.

Build on the student’s background knowledge.

Point to visuals of data-sets, while speaking, using your hands to clearly
New Jersey Student Learning Standard(s):
6.SP.B.5: Summarize numerical data sets in relation to their context, such as by:
  * 6.SP.B.5a: Reporting the number of observations.
  * 6.SP.B.5b: Describing the nature of the attribute under investigation, including how it was measured and its units of measurement.

Student Learning Objective 5: Summarize numerical data in relation to their context by identifying the number of observations and describing how the data was measured.

Modified Student Learning Objectives/Standards:
M.EE.6.SP.B.5: Summarize data distributions shown in graphs or tables.

<table>
<thead>
<tr>
<th>MPs</th>
<th>Evidence Statement Key/Clarifications</th>
<th>Skills, Strategies &amp; Concepts</th>
<th>Essential Understandings/Questions (Accountable Talk)</th>
<th>Tasks/Activities</th>
</tr>
</thead>
<tbody>
<tr>
<td>MP 2</td>
<td>6.SP.5</td>
<td>Students summarize numerical data by providing background information about</td>
<td>What conclusions can be drawn from data?</td>
<td>Candy Bars</td>
</tr>
</tbody>
</table>

Website: Teachers First Adapt a Strategy. Adjusting Lessons for ESL/ELL students http://www.teachersfirst.com/content/esl/adaptstrat.cfm
<table>
<thead>
<tr>
<th><strong>MP 4</strong></th>
<th><strong>MP 5</strong></th>
<th><strong>MP 4</strong></th>
<th><strong>MP 5</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Tasks have a text-based and a graphics-based overview of a numerical data set.</td>
<td>the attribute being measured, methods and unit of measurement, the context of data collection activities (addressing random sampling), determine the number of observations, and summary statistics.</td>
<td>How can I describe the set of data?</td>
<td>How can I describe the set of data?</td>
</tr>
<tr>
<td>Tasks require students to identify/select from unambiguously true or false statements such as, “About half of the values are greater than the average”; “If this point were deleted from the data set, the median would not change”; etc.</td>
<td>Students will also describe the data in context, including how it was measured and the units of measurement.</td>
<td>How can I use data to compare different groups?</td>
<td>How can I use data to compare different groups?</td>
</tr>
<tr>
<td>Tasks do not assess mode and range.</td>
<td><strong>SPED Strategies:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Review and understand why variability in data helps to analyze data.</td>
<td></td>
<td><strong>Also covered in:</strong></td>
</tr>
<tr>
<td></td>
<td>Show examples of various means of data (i.e. test scores, Star records, baseball or basketball records etc.).</td>
<td></td>
<td>Shakespeare vs. Rowling</td>
</tr>
<tr>
<td></td>
<td>Provide examples of statistical questions that might produce variable data and why they are important when analyze data.</td>
<td></td>
<td>How Long Did We Sleep?</td>
</tr>
<tr>
<td></td>
<td>Review and demonstrate how to observe and record numerical or quantitative data.</td>
<td></td>
<td>Displaying Jump Distances</td>
</tr>
<tr>
<td></td>
<td>Practice how to read and interpret numerical data in tables, frequency tables, line plots, and lists.</td>
<td></td>
<td>Who was the Greatest Yankee Home Run Hitter?</td>
</tr>
<tr>
<td></td>
<td>Describe how data can be collected for an investigation.</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Resources UDL - Visual and Auditory Learner(s):</strong></td>
<td></td>
<td></td>
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<tr>
<td>--------------------------------------------------</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>6.SP.5.a Report the Number of Observations in a Data Set</td>
<td><img src="https://youtu.be/r5McSJX9SEY" alt="Image" /></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6.SP.5b-1.0 Methods of Data Collection - Common Core Standard</td>
<td><img src="https://youtu.be/W9Vy6nAK9Uc" alt="Image" /></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**ELL Strategies:**

Utilize body cues and hand gestures to assist with expression identification.

Create charts with mathematical concepts.

Clarify, compare, and make connections to math words in discussion, particularly during and after practice.

Provide translations of all content and general vocabulary words.

Connect language with concrete and pictorial experiences (visuals of graphs and data-sets).

Know, use, and make the most of student cultural and home experiences.

Point to visuals of data-sets, while speaking, using your hands to clearly indicate the image that corresponds to your words.
Provide a variety of ways to respond: oral, choral, student boards, concrete models, pictorial models, pair share, and small group share.

Embed visual, non-linguistic supports for vocabulary clarification.

**Website:**
Teachers First
*Adapt a Strategy. Adjusting Lessons for ESL/ELL students*
http://www.teachersfirst.com/content/esl/adaptstrat.cfm
New Jersey Student Learning Standard(s):
6.SP.B.5: Summarize numerical data sets in relation to their context, such as by:

6.SP.B.5c: Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.

6.SP.B.5d: Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.

Student Learning Objective 6: Calculate, and interpret measures of center (mean and median) and variability (interquartile range and mean absolute deviation); report measures of center and variability appropriate to the shape of the distribution and context.

Modified Student Learning Objectives/Standards: N/A

<table>
<thead>
<tr>
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<tbody>
<tr>
<td>MP 2</td>
<td>6.SP.5 • Tasks have a text-based and a graphics-based overview of a numerical data set. • Tasks require students to identify/select from unambiguously true or false statements such as, “About half of the values are greater than the average”; “If this point were deleted from the data set, the median would not change”; etc. • Tasks do not assess mode and range.</td>
<td>Given a set of data values, students summarize the measure of center with the median or mean. Students will calculate measures of center, mean, median, measures of spread, interquartile range and mean absolute deviation. They will describe the overall shape of a distribution (skewed left, skewed right, etc) and identify striking deviations (outliers). Students will choose measures of center and variability appropriate to the shape of the distribution and context.</td>
<td>How can I describe the center of a set of data? How can I decide which measure of center (i.e., mean or median) best describes the data? How can I describe the spread of a set of data? What is the Mean Absolute Deviation? Why do we need to find the Mean Absolute Deviation?</td>
<td>Is It Center or Is It Variability? Displaying Jump Distances How Long Did We Sleep? Mean or Median Mean Absolute Deviation How MAD Are You? Shakespeare vs. Rowling</td>
</tr>
<tr>
<td>SPED Strategies:</td>
<td>How is the Mean different from the Mean Absolute Deviation?</td>
<td>Using NBA Statistics</td>
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<tr>
<td>Show examples of various means of data (i.e. test scores, Star records, baseball or basketball records etc.).</td>
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<td>Who was the Greatest Yankee Home Run Hitter?</td>
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<tr>
<td>Provide examples of statistical questions that might produce variable data and why they are important when analyze data.</td>
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<tr>
<td>Review samples of data that provide understanding of variability that anticipates change in the measures of central tendency.</td>
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<tr>
<td>Practice defining variability as it refers to the spread of data.</td>
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<tr>
<td>Provide teacher generated notes that will anticipate variability in data generated by statistical questions (Higher Order Thinking Questions).</td>
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<tr>
<td>Review and anchor vocabulary words with pictures for association or connection (i.e. spread of data, variability bell curve, data, mean, median, mode, range, set).</td>
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<tr>
<td>Review and demonstrate possible mode within the data set, more than one number is repeated with the same frequency as another.</td>
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</tr>
</tbody>
</table>
Review statistical outlier in a data set that will show results which may not be true values.

Chunk instruction to determine the center of a numerical data set. This center may also be known as the mean, median of the set.

Demonstrate how the mean of a data set represents the average of all of the values in the data set.

Demonstrate how the mode represents the value which appears most frequently in the set of data.

Review the values that have the same frequency, when there is no mode. Practice determining possible data to have more than one mode if more than one value is repeated with the same frequency within the data set.

Review and demonstrate how to observe and record numerical or quantitative data.

Practice how to read and interpret numerical data in tables, frequency tables, line plots, and lists.

Describe how data can be collected for an investigation.
Describe the purpose for the investigation.

Determine if the collection was random, representative, or biased.

Review and anchor vocabulary words with pictures for association or connection (i.e. frequency table, numerical data, observations, quantitative data, attribute, biased, investigation, random, representation, central tendency, mean, median data, data sets, distribution, measures of center, variability).

Explain which measure of center is best to interpret the numerical data in the context of its collection.

**Resources UDL - Visual and Auditory Learner(s):**
6.SP.5c-2.0 Center and Variability of Data Sets - Common Core Standard
[https://youtu.be/qQoxeF2Re5k](https://youtu.be/qQoxeF2Re5k)

6.SP.5d-1.0 Best Measure of Center and Variability - Common Core Standard
[https://youtu.be/kgEaBYKti8E](https://youtu.be/kgEaBYKti8E)

**ELL Strategies:**
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Clarify, compare, and make connections to math words in discussion, particularly during and after practice.

Provide translations of all content and general vocabulary words.

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Know, use, and make the most of student cultural and home experiences.

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Website:
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New Jersey Student Learning Standard(s):
6.RP.A.3: Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. *(benchmarked)*

6.RP.A.3a: Make tables of equivalent ratios relating quantities with whole number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.

6.RP.A.3b: Solve unit rate problems including those involving unit pricing and constant speed. For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?

6.RP.A.3c: Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.

Student Learning Objective 7: Create and complete tables of equivalent ratios to solve real world and mathematical problems using ratio and rate reasoning that include making tables of equivalent ratios, solving unit rate problems, finding percent of a quantity as a rate per 100.

Modified Student Learning Objectives/Standards: N/A

<table>
<thead>
<tr>
<th>MPs</th>
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<th>Essential Understandings/Questions (Accountable Talk)</th>
<th>Tasks/Activities</th>
</tr>
</thead>
<tbody>
<tr>
<td>MP 2</td>
<td>6.RP.3a</td>
<td>Students will use ratio and rate reasoning to create tables of equivalent ratios relating quantities with whole number measurements, find missing values in tables and plot pairs of values.</td>
<td>What information do I get when I compare two numbers using a ratio?</td>
<td>Climbing the Steps of El Castillo</td>
</tr>
<tr>
<td>MP 4</td>
<td></td>
<td></td>
<td></td>
<td>Hunger Games vs. Divergent</td>
</tr>
<tr>
<td>MP 5</td>
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<tr>
<td>MP 6</td>
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<tr>
<td>MP 7</td>
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<tr>
<td>MP 8</td>
<td>numerator and denominator should be whole numbers.</td>
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<td>-----------------------------------------------------</td>
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</tr>
<tr>
<td>6.RP.3b</td>
<td>• Expectations for unit rates in this grade are limited to non-complex fractions. The initial numerator and denominator should be whole numbers.</td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>
| 6.RP.3c-1 | • Tasks may or may not contain context.  
• Expectations for ratios in this grade are limited to ratios of non-complex fractions. The initial numerator and denominator should be whole numbers. |
| 6.RP.3c-2 | • Expectations for ratios in this grade are limited to ratios of non-complex fractions. The initial numerator and denominator should be whole numbers. |

Previously, students have used additive reasoning in tables to solve problems. To begin the shift to proportional reasoning, students need to begin using multiplicative reasoning. To aid in the development of proportional reasoning the cross-product algorithm is not expected at this level. When working with ratio tables and graphs, whole number measurements are the expectation for this standard. This will allow students to understand how to compare ratios using tables of equivalent ratios.

Solve real world and mathematical problems involving unit rate including unit price and constant speed.

Calculate a percent of a quantity and solve problems by finding the whole when given the part and the percent.

**SPED Strategies:**
Create note cards for short bursts of information.

Present information through different modalities.

Adjust color of text, graphs and visual content.

<table>
<thead>
<tr>
<th>Fuel Usage</th>
<th>Why are tables important in solving real world mathematical problems?</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>What are equivalent ratios?</td>
</tr>
<tr>
<td></td>
<td>What are percentages?</td>
</tr>
</tbody>
</table>

**Walking Around the School**

**Attributes of a Stink Bug**
<table>
<thead>
<tr>
<th>Instructional Resources</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>UDL - Visual and Auditory Learner(s):</strong></td>
</tr>
<tr>
<td>Rate and Proportion with Student Bob, CCSS 6.RP.A.3 and 7.RP.A.1. <a href="https://youtu.be/pIMNpu7mGY8">https://youtu.be/pIMNpu7mGY8</a></td>
</tr>
<tr>
<td>6.RP.2/3 Unit Rates Tape Diagram/Bar Model MaTh LAB 2.1</td>
</tr>
<tr>
<td>---------------------------------------------------------</td>
</tr>
</tbody>
</table>

**ELL Strategies:**

Provide manipulatives that show measurement rates and conversions.

Group high-level and low-level ELL students in small groups/triads.

Use of word/picture wall.

Use of L1 (students’ native language) text and/or support.

Provide illustrations/diagrams/drawings.

Provide white boards for students to practice problem solving.

Use of cloze sentences with relevant terms included in text.

Use of translation dictionary or software.

Model by playing role of student and gradually fade to level of student working independently.

**Website:**

**KHAN ACADEMY:**

*Ratios and Proportions*

[https://www.khanacademy.org/commoncore/grade-6-RP](https://www.khanacademy.org/commoncore/grade-6-RP)
New Jersey Student Learning Standard(s):

6.RP.A.3d: Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

Student Learning Objective 8: Use ratio and rate reasoning to convert measurement units and to transform units appropriately when multiplying or dividing quantities.

Modified Student Learning Objectives/Standards: N/A

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</thead>
<tbody>
<tr>
<td>MP 2</td>
<td>6.RP.3d</td>
<td>A ratio can be used to compare measures of two different types, such as inches per foot, milliliters per liter and centimeters per inch.</td>
<td>How can ratios be used to convert measurement units?</td>
<td>Baking Sale Brownies</td>
</tr>
<tr>
<td>MP 4</td>
<td></td>
<td>Students convert measurement units using ratio reasoning and transform units appropriately when multiplying and dividing quantities.</td>
<td>What information do I need to convert measurement units?</td>
<td>Paper Clip Comparisons</td>
</tr>
<tr>
<td>MP 5</td>
<td></td>
<td>Students use ratios as conversion factors and the identity property for multiplication to convert ratio units.</td>
<td></td>
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<tr>
<td>MP 6</td>
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<tr>
<td>MP 7</td>
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SPED Strategies:
Create note cards for short bursts of information.

Present information through different modalities.
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<tbody>
<tr>
<td>Adjust color of text, graphs and visual content.</td>
</tr>
<tr>
<td>Create tables of equivalent ratios relating quantities with whole-number measurements.</td>
</tr>
<tr>
<td>Create visual, verbal or tactile cues or reminders.</td>
</tr>
<tr>
<td>Link new learning to prior learning. Connect to real-life experiences.</td>
</tr>
<tr>
<td>Pre-teach prerequisite skills and concepts.</td>
</tr>
<tr>
<td>Embed links to websites for additional knowledge.</td>
</tr>
<tr>
<td>Design web quests to search for background information. Teach and model fundamental skills and procedures explicitly until they become automatic.</td>
</tr>
<tr>
<td>Vary means to assess mastery of materials taught.</td>
</tr>
<tr>
<td>Include written descriptions for graphics, images and videos.</td>
</tr>
</tbody>
</table>
| Rate and Proportion with Student Bob, CCSS 6.RP.A.3 and 7.RP.A.1.  
https://youtu.be/pIMNpu7mGY8  
| 6.RP.2/3 Unit Rates Tape Diagram/Bar Model MaTh LAB 2.1  
https://youtu.be/2y2w3WFxH3w  |
|---|---|
| **ELL Strategies:** 
Produce manipulatives that show measurement rates and conversions. 
Group high-level and low-level ELL students in small groups/triads. 
Use of word/picture wall. 
Use of L1 (students’ native language) text and/or support. 
Provide illustrations/diagrams/drawings. Provide white boards for students to practice problem solving. 
Use of cloze sentences with relevant terms included in text. 
Use of translation dictionary or software. 
Model by playing role of student and gradually fade to level of student working independently. |
**New Jersey Student Learning Standard(s):**
6.NS.C.8: Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate. *(benchmarked)*

**Student Learning Objective 9:** Solve real world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Use the absolute value of the differences of their coordinates to find distances between points with the same first coordinate or same second coordinate.

**Modified Student Learning Objectives/Standards:**
M.EE.6.NS.C.5–8: Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero).

<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>MP 1</td>
<td>6.NS.8</td>
<td>Students find the distance between points when ordered pairs have the same x-coordinate or same y-coordinate. They will use absolute value to find distances between points with the same first coordinate or the same second coordinate. <strong>Example:</strong> What is the distance between (−5, 2) and (−9, 2)?</td>
<td>When are negative numbers used and why are they important? Why is it useful for me to know the absolute value of a number? How do I plot points on the coordinate plane?</td>
<td>Doubling the Dimensions</td>
</tr>
<tr>
<td>MP 2</td>
<td>Tasks may or may not contain context. Finding distances is limited to points with integer coordinates.</td>
<td></td>
<td></td>
<td>Splitting the Land</td>
</tr>
<tr>
<td>MP 5</td>
<td></td>
<td></td>
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<td>Water Crisis in Haiti</td>
</tr>
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<td></td>
<td></td>
<td></td>
<td></td>
<td>Nome, Alaska</td>
</tr>
</tbody>
</table>
Solution: The distance would be 4 units. This would be a horizontal line since the y-coordinates are the same.

In this scenario, both coordinates are in the same quadrant. The distance can be found by using a number line to find the distance between –5 and –9. Students could also recognize that –5 is 5 units from 0 (absolute value) and that –9 is 9 units from 0 (absolute value). Since both of these are in the same quadrant, the distance can be found by finding the difference between the distances 9 and 5. (| 9 | - | 5 |).

Students will be graphing in all four quadrants of the coordinate plane in order to solve real-world and mathematical problems. The use of coordinates will allow students to draw polygons on a coordinate plane and solve real-world distance, perimeter and area problems.

**SPED Strategies:**

Provide examples and practice identifying the intervals on an axis of a coordinate plane.

Provide examples and practice identifying absolute values of integers in relation to coordinates.

| How is the absolute value of a number used to determine its distance on the coordinate plane? |
| How can I use vertical and horizontal number lines to solve problems? |
| How do I compare and order rational numbers? |
| Practice and demonstrate adding and subtracting integers in relation to coordinates. |  |
| Practice drawing the conclusion that a horizontal line is formed when the y-values and x-values are the same in a set of coordinates. |  |
| Create visual, verbal or tactile cues or reminders. |  |
| Link new learning to prior learning. |  |
| Connect to real-life experiences. |  |
| Pre-teach prerequisite skills and concepts. |  |
| Teach and model fundamental skills and procedures explicitly until they become automatic. |  |
| Vary means to assess mastery of materials taught. |  |
| Present information through different modalities (i.e. visual, auditory, tactile, and kinesthetic). |  |
| **ELL Strategies:** Provide students with graph paper and have them work in groups to plot given |  |
points on a coordinate plane, and orally discuss the process to the class.
Group high-level and low-level ELL students in small groups/ triads.

Provide a variety of ways to respond: oral, choral, student boards, concrete models (e.g., fingers), pictorial models (e.g., ten-frame), pair share, and small group share.

Create a large picture wall with different shapes, and formulas, with translated names of figures and properties.

Provide students with reference sheets with L1 (students’ native language) text materials.

Utilize white boards where students can draw mathematical representations of coordinate planes, formulas for figures and illustrate problem solving skills.

Use of math journals for students to practice writing skills with math terminology.
Use of translation dictionary or software.

Teach students how to ask questions such as: “Do you agree?” and “Why do you think so?” to extend think-pair-share conversations.
Model and post conversation starters such as: “I agree because….” “Can you explain how you solved it?” “I noticed that…” “Your solution is different from/ the same as mine because…” “My mistake was to…”.
Integrated Evidence Statements

6.C.8.1: Present solutions to multi-step problems in the form of valid chains of reasoning, using symbols such as equals signs appropriately (for example, rubrics award less than full credit for the presence of nonsense statements such as $1 + 4 = 5 + 7 = 12$, even if the final answer is correct), or identify or describe errors in solutions to multi-step problems and present corrected solutions. Content Scope: Knowledge and skills articulated in 6.RP.A

- Expectations for ratios in this grade are limited to ratios of non-complex fractions. The initial numerator and denominator should be whole numbers.

6.C.9: Distinguish correct explanation/reasoning from that which is flawed, and – if there is a flaw in the argument – present corrected reasoning. (For example, some flawed ‘student’ reasoning is presented and the task is to correct and improve it.) Content Scope: Knowledge and skills articulated in 5.NBT, 5.MD.C.

- Tasks may have scaffolding, if necessary, in order to yield a degree of difficulty appropriate to Grade 6.

6.D.1: Solve multi-step contextual word problems with degree of difficulty appropriate to Grade 6, requiring application of knowledge and skills articulated in Type I, Sub-Claim A Evidence Statements.

- Tasks may have scaffolding, if necessary, in order yield a degree of difficulty appropriate to Grade 6.

6.D.2: Solve multi-step contextual problems with degree of difficulty appropriate to Grade 6, requiring application of knowledge and skills articulated in 5.NBT.B, 5.NF, 5.MD, and 5.G.A.

- Tasks may have scaffolding, if necessary, in order yield a degree of difficulty appropriate to Grade 6.


- Tasks may have scaffolding, if necessary, in order to yield a degree of difficulty appropriate to Grade 6.
# Unit 4 Vocabulary

<table>
<thead>
<tr>
<th>Expressions and Equations</th>
<th>Expressions and Equations</th>
<th>Ratios and Proportions</th>
<th>Statistics and Probability</th>
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<tbody>
<tr>
<td>- Algebraic expressions</td>
<td>- Independent variable</td>
<td>- Bar model</td>
<td>- Box-and-whisker plot</td>
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<td>- Area models</td>
<td>- Inverse operations</td>
<td>- Conversion factor</td>
<td>- Histogram</td>
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<tr>
<td>- Associative Property of Addition</td>
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<td>- Associative Property of Multiplication</td>
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<td>- Base</td>
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<td>- Base of an exponent</td>
<td>- Product</td>
<td>- Part-to-whole</td>
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<td>- Coefficient</td>
<td>- Quantity</td>
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<td>- Common factor</td>
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<tr>
<td>- Commutative Property of Addition</td>
<td>- Sum</td>
<td>- Rate</td>
<td>- Data</td>
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<tr>
<td>- Commutative Property of Multiplication</td>
<td>- Term</td>
<td>- Tape diagram</td>
<td>- First quartile</td>
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<td>- Composite numbers</td>
<td>- Variables</td>
<td>- Unit rate</td>
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<td>- Constant</td>
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<td>- Lower extreme</td>
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<tr>
<td>- Dependent variable</td>
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<td>- Lower quartile (1st quartile or Q1)</td>
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<td>- Difference</td>
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<td>- Mean</td>
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<td>- Distributive property</td>
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<td>- Mean Absolute Deviation (M.A.D.)</td>
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<td>- Equivalent expressions</td>
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<td>- Evaluate</td>
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<td>- Variability</td>
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<td>North Carolina Department of Public Instruction – Common Core standards “unpacked” for 6th Grade Mathematics</td>
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<td><a href="https://www.georgiastandards.org/Common-Core/Pages/Math-6-8.aspx">https://www.georgiastandards.org/Common-Core/Pages/Math-6-8.aspx</a></td>
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<td>EngageNY – Common Core lessons and resources</td>
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<td><a href="https://www.engageny.org/">https://www.engageny.org/</a></td>
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<td>PARCC Math Evidence Statements – Grade 6</td>
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<td>Youtube.com</td>
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<td>Khan Academy</td>
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<td><a href="https://www.khanacademy.org/commoncore/grade-6-RP">https://www.khanacademy.org/commoncore/grade-6-RP</a></td>
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Field Trip Ideas

**THE BOUNCE FACTORY (Warren, NJ)** - STEM-Inspired FUN Field Trips The Bounce Factory, Bricks 4 Kidz of Hunterdon Somerset and Team Makers of North Jersey have combined to create a unique and exciting Field Trip for students in grades 1st – 8th. It integrates STEM learning with fun, hands on activities that will focus on Science, Engineering and Math concepts. The students will build motorized models with LEGO® bricks and discuss engineering and physics principals; enter the Bounce rooms for activities that will set in motion discussions of how physics impacts their play; learn about Math and Science concepts while playing integrative teambuilding activities that build their skills and promote working together; learn strategy and the power of collaboration while playing laser tag in a state of the art facility [http://www.bouncefactorynj.com/](http://www.bouncefactorynj.com/)

**LIBERTY SCIENCE CENTER (Jersey City, NJ)** - An interactive science museum and learning center with math connections. There is a math guidebook for teachers to make connections with math: [http://lsc.org/plan-your-visit/](http://lsc.org/plan-your-visit/)

**NATIONAL MUSEUM OF MATHEMATICS (New York, NY)** - Mathematics illuminates the patterns and structures all around us. Our dynamic exhibits, gallery, and programs will stimulate inquiry, spark curiosity, and reveal the wonders of mathematics. MoMath has innovative exhibits that will engage folks from 105 to 5 years old (and sometimes younger), but with a special emphasis on activities for 4th through 8th graders.  **Requires approval from Unit Superintendent** [http://momath.org/](http://momath.org/)

**MUSEUM OF AMERICAN FINANCE (New York, NY)** – For more than 20 years, educators from around the country have been bringing students to the Museum to help them understand how finance impacts their daily lives. The Museum offers discounted admission for pre-booked groups of eight or more, as well as a variety of classes for students in middle school through college. [http://www.moaf.org/index](http://www.moaf.org/index)

**LEGOLAND DISCOVERY CENTER (Yonkers, NY)** – Merry-Go-Round Workshop (Engineering Design, Mathematics, Listening and Speaking) This workshop provides a fun, hand-on way to get students excited about engineering, design, and mathematics. Students build a LEGO merry-go-round to explore gear ratios then experiment with gear trains to see which combination enables their ride to spin the fastest. **Requires approval from Unit Superintendent** [https://www.legolanddiscoverycenter.com/westchester/education/elementary-school.aspx](https://www.legolanddiscoverycenter.com/westchester/education/elementary-school.aspx)

**BUEHLER'S CHALLENGER & SCIENCE CENTER (Paramus, NJ)** - Fly a space mission beyond your wildest dreams in the challenger simulator! Students will work on teams to complete their mission, while conducting experiments, monitoring life support, and implementing
Field Trip Ideas

navigation orders. In this dynamic environment, students use principles of science, mathematics, and technology to complete their tasks. There are 3 missions to choose from: “Rendezvous with Comet Halley”, “Return to the Moon”, “Voyage to Mars”.

**Requires approval from Unit Superintendent**

http://www.bcsc.org/5-9th-grade/

**BRANCH BROOK PARK SKATING RINK (Newark, NJ) - A unique educational experience that gets students excited about learning!**

Students will learn how the concepts of Science, Technology, Engineering and Math can be found in everyday experiences, even FUN experiences like roller skating! Our professional STEM Educators teach visiting students about how STEM principles exist in just about every part of life. The lessons focus on hands on activities that are both educational and fun! Lessons are customized based on teachers needs to directly relate back to classroom learning making this program completely unique! Following the completion of the 1 hour STEM Lesson, the students roller skate for physical fitness. While Roller Skating the concepts students learned are continually reinforced. Our lessons are designed not only to help students overcome their fear of learning STEM concepts but to show how STEM is both FUN & EXCITING!