MATHEMATICS

Grade 5: Unit 3

Topic: Fractions and Volume
Course Philosophy/Description

In mathematics, students will learn to address a range of tasks focusing on the application of concepts, skills and understandings. Students will be asked to solve problems involving the key knowledge and skills for their grade level as identified by the NJSLS; express mathematical reasoning and construct a mathematical argument and apply concepts to solve model real world problems. The balanced math instructional model will be used as the basis for all mathematics instruction.

Fifth grade Mathematics consists of the following domains: Operations and Algebraic Thinking (OA), Number and Operations in Base Ten (NBT), Number and Operations-Fractions (NF), Measurement and Data (MD), and Geometry (G). In fifth grade, instructional time should focus on three critical areas: (1) developing fluency with addition and subtraction of fractions, and developing understanding of the multiplication of fractions and of division of fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions); (2) extending division to 2-digit divisors, integrating decimal fractions into the place value system and developing understanding of operations with decimals to hundredths, and developing fluency with whole number and decimal operations; and (3) developing understanding of volume.

1) Students apply their understanding of fractions and fraction models to represent the addition and subtraction of fractions with unlike denominators as equivalent calculations with like denominators. They develop fluency in calculating sums and differences of fractions, and make reasonable estimates of them. Students also use the meaning of fractions, of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for multiplying and dividing fractions make sense. (Note: this is limited to the case of dividing unit fractions by whole numbers and whole numbers by unit fractions.)

2) Students develop understanding of why division procedures work based on the meaning of base-ten numerals and properties of operations. They finalize fluency with multi-digit addition, subtraction, multiplication, and division. They apply their understandings of models for decimals, decimal notation, and properties of operations to add and subtract decimals to hundredths. They develop fluency in these computations, and make reasonable estimates of their results. Students use the relationship between decimals and fractions, as well as the relationship between finite decimals and whole numbers (i.e., a finite decimal multiplied by an appropriate power of 10 is a whole number), to understand and explain why the procedures for multiplying and dividing finite decimals make sense. They compute products and quotients of decimals to hundredths efficiently and accurately.

3) Students recognize volume as an attribute of three-dimensional space. They understand that volume can be measured by finding the total number of same-size units of volume required to fill the space without gaps or overlaps. They understand that a 1-unit by 1-unit by 1-unit cube is the standard unit for measuring volume. They select appropriate units, strategies, and tools for solving problems that involve estimating and measuring volume. They decompose three-dimensional shapes and find volumes of right rectangular prisms by viewing them as decomposed into layers of arrays of cubes. They measure necessary attributes of shapes in order to determine volumes to solve real world and mathematical problems.
This ESL framework was designed to be used by bilingual, dual language, ESL and general education teachers. Bilingual and dual language programs use the home language and a second language for instruction. ESL teachers and general education or bilingual teachers may use this document to collaborate on unit and lesson planning to decide who will address certain components of the SLO and language objective. ESL teachers may use the appropriate leveled language objective to build lessons for ELLs which reflects what is covered in the general education program. In this way, whether it is a pull-out or push-in model, all teachers are working on the same Student Learning Objective connected to the New Jersey Student Learning Standards. The design of language objectives are based on the alignment of the World-Class Instructional Design Assessment (WIDA) Consortium’s English Language Development (ELD) standards with the New Jersey Student Learning Standards (NJSLS). WIDA’s ELD standards advance academic language development across content areas ultimately leading to academic achievement for English learners. As English learners are progressing through the six developmental linguistic stages, this framework will assist all teachers who work with English learners to appropriately identify the language needed to meet the requirements of the content standard. At the same time, the language objectives recognize the cognitive demand required to complete educational tasks. Even though listening and reading (receptive) skills differ from speaking and writing (expressive) skills across proficiency levels the cognitive function should not be diminished. For example, an Entering Level One student only has the linguistic ability to respond in single words in English with significant support from their home language. However, they could complete a Venn diagram with single words which demonstrates that they understand how the elements compare and contrast with each other or they could respond with the support of their home language (L1) with assistance from a teacher, para-professional, peer or a technology program.

http://www.state.nj.us/education/modelcurriculum/ela/ELLOverview.pdf
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<th>SLO #</th>
<th>STUDENT LEARNING OBJECTIVES</th>
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<tr>
<td>1</td>
<td>Multiply a fraction by a whole number.</td>
<td>5.NF.B.4a 5.NF.B.6</td>
<td>8-1</td>
</tr>
<tr>
<td>2</td>
<td>Multiply a whole number by a fraction.</td>
<td>5.NF.B.4a 5.NF.B.6</td>
<td>8-2</td>
</tr>
<tr>
<td>3</td>
<td>Multiply fractions and whole numbers.</td>
<td>5.NF.B.4a</td>
<td>8-3</td>
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<tr>
<td>4</td>
<td>Use models to multiply two fractions.</td>
<td>5.NF.B.4a</td>
<td>8-4</td>
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<tr>
<td>5</td>
<td>Multiply two fractions.</td>
<td>5.NF.B.4a</td>
<td>8-5</td>
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<tr>
<td>6</td>
<td>Find the area of a rectangle using fractions and diagrams.</td>
<td>5.NF.B.4b</td>
<td>8-6</td>
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<tr>
<td>7</td>
<td>Use models, equations and previously learned strategies to multiply mixed numbers.</td>
<td>5.NF.B.6</td>
<td>8-7</td>
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<td>8</td>
<td>Compare the size of the product to the size of one factor without multiplying to consider multiplication as scaling.</td>
<td>5.NF.B.5a 5.NF.B.5b</td>
<td>8-8</td>
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<td></td>
<td><strong>5.NF.B.3</strong></td>
<td><strong>5.NF.B.6</strong></td>
<td><strong>5.NF.B.7a</strong></td>
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<tr>
<td>9</td>
<td>Use previously learned knowledge to make sense of problems and persevere in solving them.</td>
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<tr>
<td>10</td>
<td>Understand how fractions are related to division.</td>
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<td>11</td>
<td>Implement division of fractions to show quotients as fractions and mixed numbers.</td>
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<tr>
<td>12</td>
<td>Use multiplication to divide a whole number by a unit fraction.</td>
<td>5.NF.B.7b</td>
<td>5.NF.B.7c</td>
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<tr>
<td>13</td>
<td>Use models, such as pictorial models or a number line, to show dividing a whole number by a unit fraction.</td>
<td>5.NF.B.7b</td>
<td>5.NF.B.7c</td>
</tr>
<tr>
<td>14</td>
<td>Use models to divide unit fractions by non-zero whole numbers.</td>
<td>5.NF.B.7a</td>
<td>5.NF.B.7c</td>
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<tr>
<td>15</td>
<td>Use models to divide whole numbers and unit fractions. Check your answer using multiplication.</td>
<td>5.NF.B.7a</td>
<td>5.NF.B.7b</td>
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<tr>
<td>16</td>
<td>Solve multi-step problems involving division with unit fractions.</td>
<td></td>
<td>5.NF.B.7c</td>
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<tr>
<td>17</td>
<td>Draw on conceptual understanding of multiplication and division.</td>
<td>5.NF.B.3</td>
<td>5.NF.B.6</td>
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**IFL Sets of Related Lessons**
- “Dividing Fractions: Understanding Division with Numbers Less Than 1”
<table>
<thead>
<tr>
<th></th>
<th>Make use of representations and tools such as fraction models, number lines and bar diagrams.</th>
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<tr>
<td>18</td>
<td>Notice repetition in calculations and generalize about how to divide whole numbers and unit fractions.</td>
<td>5.NF.B.7a 9-8</td>
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<td>19</td>
<td>Read and analyze line plots.</td>
<td>5.MD.B.2 5.NF.A.2 10-1</td>
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<td>20</td>
<td>Organize and display data in a line plot.</td>
<td>5.MD.B.2 5.NF.A.2 10-2</td>
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<tr>
<td>21</td>
<td>Solve problems using data in a line plot.</td>
<td>5.NF.A.2 5.NF.B.6 10-3</td>
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<td>22</td>
<td>Critique the reasoning of others using understanding of line plots and fractions.</td>
<td>5.NF.A.2 10-4</td>
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<td>23</td>
<td>Find the volume of solid figures.</td>
<td>5.MD.C.3a 5.MD.C.3b 5.MD.C.4 11-1</td>
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<tr>
<td>24</td>
<td>Find the volume of rectangular prisms using a formula.</td>
<td>5.MD.C.4 5.MD.C.5a 5.MD.C.5b 11-2</td>
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<tr>
<td>25</td>
<td>Draw on conceptual understanding of multiplication and volume. Make use of representations and tools such as diagrams, cubes, and bar diagrams.</td>
<td>5.MD.C.5c 5.MD.C.3 5.MD.C.4 3-ACT Math</td>
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<tr>
<td>26</td>
<td>Find the volume of a solid figure that is the combination of two or more rectangular prisms.</td>
<td>5.MD.C.5c</td>
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<td>27</td>
<td>Use models, prior knowledge of volume and previously learned strategies to solve word problems involving volume.</td>
<td>5.MD.C.5c</td>
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<tr>
<td>28</td>
<td>Use previously learned knowledge about volume to choose the appropriate tools to solve volume problems.</td>
<td>5.MD.C.3a 5.MD.C.3b 5.MD.C.4</td>
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Key: **Major Cluster** | **Supporting** | **Additional Cluster** |
Research about Teaching and Learning Mathematics

Structure teaching of mathematical concepts and skills around problems to be solved (Checkly, 1997; Wood & Sellars, 1996; Wood & Sellars, 1997)

Encourage students to work cooperatively with others (Johnson & Johnson, 1975; Davidson, 1990)

Use group problem-solving to stimulate students to apply their mathematical thinking skills (Artzt & Armour-Thomas, 1992)

Students interact in ways that support and challenge one another’s strategic thinking (Artzt, Armour-Thomas, & Curcio, 2008)

Activities structured in ways allowing students to explore, explain, extend, and evaluate their progress (National Research Council, 1999)

There are three critical components to effective mathematics instruction (Shellard & Moyer, 2002):

- Teaching for conceptual understanding
- Developing children’s procedural literacy
- Promoting strategic competence through meaningful problem-solving investigations

Teachers should be:

- Demonstrating acceptance and recognition of students’ divergent ideas
- Challenging students to think deeply about the problems they are solving, extending thinking beyond the solutions and algorithms required to solve the problem
- Influencing learning by asking challenging and interesting questions to accelerate students’ innate inquisitiveness and foster them to examine concepts further
- Projecting a positive attitude about mathematics and about students’ ability to “do” mathematics

Students should be:

- Actively engaging in “doing” mathematics
- Solving challenging problems
- Investigating meaningful real-world problems
- Making interdisciplinary connections
- Developing an understanding of mathematical knowledge required to “do” mathematics and connect the language of mathematical ideas with numerical representations
- Sharing mathematical ideas, discussing mathematics with one another, refining and critiquing each other’s ideas and understandings
- Communicating in pairs, small group, or whole group presentations
- Using multiple representations to communicate mathematical ideas
- Using connections between pictures, oral language, written symbols, manipulative models, and real-world situations
- Using technological resources and other 21st century skills to support and enhance mathematical understanding
Mathematics is not a stagnant field of textbook problems; rather, it is a dynamic way of constructing meaning about the world around us, generating knowledge and understanding about the real world every day. Students should be metaphorically rolling up their sleeves and “doing mathematics” themselves, not watching others do mathematics for them or in front of them. (Protheroe, 2007)

Balanced Mathematics Instructional Model

Balanced math consists of three different learning opportunities: guided math, shared math, and independent math. Ensuring a balance of all three approaches will build conceptual understanding, problem solving, computational fluency, and procedural fluency. Building conceptual understanding is the focal point of developing mathematical proficiency. Students should frequently work on rigorous tasks, talk about the math, explain their thinking, justify their answer or process, build models with graphs or charts or manipulatives, and use technology.

When balanced math is used in the classroom it provides students opportunities to:

- solve problems
- make connections between math concepts and real-life situations
- communicate mathematical ideas (orally, visually and in writing)
- choose appropriate materials to solve problems
- reflect and monitor their own understanding of the math concepts
- practice strategies to build procedural and conceptual confidence

Teacher builds conceptual understanding by modeling through demonstration, explicit instruction, and think alouds, as well as guiding students as they practice math strategies and apply problem solving strategies. (whole group or small group instruction)

Teacher and students practice mathematics processes together through interactive activities, problem solving, and discussion. (whole group or small group instruction)

Students practice math strategies independently to build procedural and computational fluency. Teacher assesses learning and reteaches as necessary. (whole group instruction, small group instruction, or centers)
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<td>Identify Student’s Mathematical Misunderstandings</td>
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<td>Interviews</td>
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<td>Role Playing</td>
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<td>Diagrams, Charts, Tables, and Graphs</td>
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<td>Anticipate Likely and Possible Student Responses</td>
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<td>Collect Different Student Approaches</td>
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<td>Asking Assessing and Advancing Questions</td>
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<td>Challenging</td>
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<td>Pressing for Accuracy and Reasoning</td>
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<td>Maintain the Cognitive Demand</td>
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Computer Science and Design Thinking

**Engineering Design**: Engineering design is a systematic and creative process of communicating and collaborating to meet a design challenge.

- **8.2.5.ED.2**: Collaborate with peers to collect information, brainstorm to solve a problem, and evaluate all possible solutions to provide the best results with supporting sketches or models.
  **Example**: Students will apply a design process with math problems involving determining volume of figures and composite figures.

**Interaction of Technology and Humans**: A new tool may have favorable or unfavorable results as well as both positive and negative effects on society.

- **8.2.5.ITH.3**: Analyze the effectiveness of a new product or system and identify the positive and/or negative consequences resulting from its use.
  **Example**: Students will track their progress using Imagine Math or other programs. Students will discuss the pros and cons of using the programs with the teacher.

**Nature of Technology**: Technology innovation and improvement may be influenced by a variety of factors.

- **8.2.5.NT.1**: Troubleshoot a product that has stopped working and brainstorm ideas to correct the problem.
  **Example**: Students will provide each other ideas on how to either re-start or get a digital program to start to work again.
### Career Readiness, Life Literacies and Key Skills

Career Ready Practices describe the career-ready skills that all educators in all content areas should seek to develop in their students. They are practices that have been linked to increase college, career, and life success. Career Ready Practices should be taught and reinforced in all career exploration and preparation programs with increasingly higher levels of complexity and expectation as a student advances through a program of study.

**Creativity and Innovation:** Curiosity and a willingness to try new ideas (intellectual risk-taking) contributes to the development of creativity and innovation skills.

- **9.4.5.CI.3:** Participate in a brainstorming session with individuals with diverse perspectives to expand one’s thinking about a topic of curiosity (e.g., 8.2.5.ED.2, 1.5.5.CR1a).
  
  **Example:** Students will work collaboratively in groups to solve mathematical tasks. Students will listen to or read the arguments of others and ask probing questions to clarify or improve arguments.

**Critical Thinking and Problem Solving:** The ability to solve problems effectively begins with gathering data, seeking resources, and applying critical thinking skills.

- **9.4.5.CT.3:** Describe how digital tools and technology may be used to solve problems.
  
  **Example:** Students will use virtual manipulatives to help them solve problems involving the volume of a cube. Students can place unit cubes inside a cube at [https://www.nctm.org/Classroom-Resources/Illuminations/Interactives/Cubes/](https://www.nctm.org/Classroom-Resources/Illuminations/Interactives/Cubes/)

- **9.4.5.CT.4:** Apply critical thinking and problem-solving strategies to different types of problems such as personal, academic, community and global (e.g., 6.1.5.CivicsCM.3).
  
  **Example:** Students will understand the meaning of a problem and look for entry points to its solution. They will analyze information, make conjectures, and plan a solution pathway. Students will monitor and evaluate progress and change course as necessary.
Career Readiness, Life Literacies and Key Skills

Digital Citizenship: Sending and receiving copies of media on the internet creates the opportunity for unauthorized use of data, such as personally owned video, photos, and music.

- 9.4.5.DC.4: Model safe, legal, and ethical behavior when using online or offline technology (e.g., 8.1.5.NI.2).
  
  **Example:** Teachers and students will model appropriate use of all digital platforms that are being used in the classroom. Teachers and students will provide examples of student work that exhibits proper use of various platforms. Teachers will also monitor students’ work and behavior using GoGuardian.

Information and Media Literacy: Digital tools can be used to modify and display data in various ways that can be organized to communicate ideas.

- 9.4.5.IML.2: Create a visual representation to organize information about a problem or issue (e.g., 4.MD.B.4, 8.1.5.DA.3).
  
  **Example:** Students will use a digital whiteboard, Google Slides, or other platform, graphic organizers and virtual manipulatives to show how they were able to organize their thoughts and solve a problem.
WIDA Proficiency Levels: At the given level of English language proficiency, English language learners will process, understand, produce or use:

<table>
<thead>
<tr>
<th>Level</th>
<th>Description</th>
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| 6 - Reaching | - Specialized or technical language reflective of the content areas at grade level  
- A variety of sentence lengths of varying linguistic complexity in extended oral or written discourse as required by the specified grade level  
- Oral or written communication in English comparable to proficient English peers |
| 5 - Bridging | - Specialized or technical language of the content areas  
- A variety of sentence lengths of varying linguistic complexity in extended oral or written discourse, including stories, essays or reports  
- Oral or written language approaching comparability to that of proficient English peers when presented with grade level material. |
| 4 - Expanding | - Specific and some technical language of the content areas  
- A variety of sentence lengths of varying linguistic complexity in oral discourse or multiple, related sentences or paragraphs  
- Oral or written language with minimal phonological, syntactic or semantic errors that may impede the communication, but retain much of its meaning, when presented with oral or written connected discourse, with sensory, graphic or interactive support |
| 3 - Developing | - General and some specific language of the content areas  
- Expanded sentences in oral interaction or written paragraphs  
- Oral or written language with phonological, syntactic or semantic errors that may impede the communication, but retain much of its meaning, when presented with oral or written, narrative or expository descriptions with sensory, graphic or interactive support |
| 2 - Beginning | - General language related to the content area  
- Phrases or short sentences  
- Oral or written language with phonological, syntactic, or semantic errors that often impede the communication when presented with one to multiple-step commands, directions, or a series of statements with sensory, graphic or interactive support |
| 1 - Entering | - Pictorial or graphic representation of the language of the content areas  
- Words, phrases or chunks of language when presented with one-step commands directions, WH-, choice or yes/no questions, or statements with sensory, graphic or interactive support |
# Language Development Supports For English Language Learners

To Increase Comprehension and Communication Skills

## Environment

- Welcoming and stress-free
- Respectful of linguistic and cultural diversity
- Honors students’ background knowledge
- Sets clear and high expectations
- Includes routines and norms
- Is thinking-focused vs. answer-seeking
- Offers multiple modalities to engage in content learning and to demonstrate understanding
- Includes explicit instruction of specific language targets
- Provides participation techniques to include all learners
- Integrates learning centers and games in a meaningful way
- Provides opportunities to practice and refine receptive and productive skills in English as a new language
- Integrates meaningful and purposeful tasks/activities that:
  - Are accessible by all students through multiple entry points
  - Are relevant to students’ lives and cultural experiences
  - Build on prior mathematical learning
  - Demonstrate high cognitive demand
  - Offer multiple strategies for solutions
  - Allow for a language learning experience in addition to content

## Sensory Supports*

- Real-life objects (reallia) or concrete objects
- Physical models
- Manipulatives
- Pictures & photographs
- Visual representations or models such as diagrams or drawings
- Videos & films
- Newspapers or magazines
- Gestures
- Physical movements
- Music & songs

## Graphic Supports*

- Graphs
- Charts
- Timelines
- Number lines
- Graphic organizers
- Graphing paper

## Interactive Supports*

- In a whole group
- In a small group
- With a partner such as Turn and Talk
- In pairs as a group (first, two pairs work independently, then they form a group of four)
- In triads
- Cooperative learning structures such as Think-Pair-Share
- Interactive websites or software
- With a mentor or coach

## Verbal and Textual Supports

- Labeling
- Students’ native language
- Modeling
- Repetitions
- Paraphrasing
- Summarizing
- Guiding questions
- Clarifying questions
- Prompting questions
- Leveled questions such as What? When? Where? How? Why?
- Questioning prompts & cues
- Word Banks
- Sentence starters
- Sentence frames
- Discussion frames
- Talk moves, including Wait Time

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Geliu (Halla) Imaurika, ESOL Coach, PGIPS; 2015, Revd 2016
BUILDING EQUITY IN YOUR TEACHING PRACTICE

How do the essential questions highlight the connection between the big ideas of the unit and equity in your teaching practice?

**CONTENT INTEGRATION**
Teachers use examples and content from a variety of cultures & groups.

This unit / lesson is connected to other topics explored with students.

There are multiple viewpoints reflected in the content of this unit / lesson.

The materials and resources are reflective of the diverse identities and experiences of students.

The content affirms students, as well as exposes them to experiences other than their own.

**KNOWLEDGE CONSTRUCTION**
Teachers help students understand how knowledge is created and influenced by cultural assumptions, perspectives & biases.

This unit / lesson provides context to the history of privilege and oppression.

This unit / lesson addresses power relationships.

This unit / lesson helps students to develop research and critical thinking skills.

This curriculum creates windows and mirrors* for students.

**PREJUDICE REDUCTION**
Teachers implement lessons and activities to assert positive images of ethnic groups & improve intergroup relations.

This unit / lesson helps students question and unpack biases & stereotypes.

This unit / lesson addresses power relationships.

This unit / lesson helps students to develop research and critical thinking skills.

This curriculum creates windows and mirrors* for students.

**EQUITABLE PEDAGOGY**
Teachers modify techniques and methods to facilitate the academic achievement of students from diverse backgrounds.

The instruction has been modified to meet the needs of each student.

Students feel respected and their cultural identities are valued.

Additional supports have been provided for students to become successful and independent learners.

Opportunities are provided for students to reflect on their learning and provide feedback.

**EMPOWERING SCHOOL CULTURE**
Using the other four dimensions to create a safe and healthy educational environment for all.

There are opportunities for students to connect with the community.

My classroom is welcoming and supportive for all students?

I am aware of and sensitive to the needs of my students and their families.

There are effective parent communication systems established. Parents can talk to me about issues as they arise in my classroom.
Culturally Relevant Pedagogy Examples

- **Present new concepts using student vocabulary.** Use student diction to capture attention and build understanding before using academic terms.
  **Example:** Work with students to create a variety of vocabulary sorting and matching games that relate student diction to vocabulary words in this unit. Students can also use My Word Cards, the vocabulary review page, the animated glossary or the online vocabulary game available with SAVVAS. Students can work in teams or individually to play these games for approximately 10-15 minutes each week.

- **Use Learning Stations: Provide a range of materials by setting up learning stations.**
  **Example:** Reinforce understanding of concepts and skills by promoting the learning through student interests and modalities, experiences and/or prior knowledge. Encourage the students to make choices in content based upon their strengths, needs, values and experiences. Providing students with choice boards or allowing them to choose the Pick a Project, Problem Solving Leveled Reading Mats and/or the STEM activity available with SAVVAS. Students in each station collaborate to choose an item that is of interest to the group. After choosing the item, students will be able to understand why equal groups are important and to use arrays to multiply. Each group will present their results to the class.

- **Everyone has a Voice:** Create a classroom environment where students know that their contributions are expected and valued.
  **Example:** Norms for sharing are established that communicate a growth mindset for mathematics. All students are capable of expressing mathematical thinking and contributing to the classroom community. Students learn new ways of looking at problem solving by working with and listening to each other.

- **Integrate Relevant Word Problems:** Contextualize equations using word problems that reference student interests and cultures.
  **Example:** Create and use word problems that include student interests, current events, and/or relevance to real-world situations in order to make problems relatable to students when interpreting and answering questions about data. Using content that students can relate to adds meaning, value, and connection.
SOCIAL AND EMOTIONAL LEARNING (SEL) COMPETENCIES

SELF-AWARENESS
The ability to accurately recognize one’s own emotions, thoughts, and values and how they influence behavior. The ability to accurately assess one’s strengths and limitations, with a well-grounded sense of confidence, optimism, and a “growth mindset.”

- Identifying Emotions
- Accurate Self-Perception
- Recognizing Strengths
- Self-Confidence
- Self-Efficacy

SELF-MANAGEMENT
The ability to successfully regulate one’s emotions, thoughts, and behaviors in different situations — effectively managing stress, controlling impulses, and motivating oneself. The ability to set and work toward personal and academic goals.

- Impulse Control
- Stress Management
- Self-Discipline
- Self-Motivation
- Goal Setting
- Organizational Skills

SOCIAL AWARENESS
The ability to take the perspective of and empathize with others, including those from diverse backgrounds and cultures. The ability to understand social and ethical norms for behavior and to recognize family, school, and community resources and supports.

- Perspective-Taking
- Empathy
- Appreciating Diversity
- Respect for Others

RELATIONSHIP SKILLS
The ability to establish and maintain healthy and rewarding relationships with diverse individuals and groups. The ability to communicate clearly, listen well, cooperate with others, resist inappropriate social pressure, negotiate conflict constructively, and seek and offer help when needed.

- Communication
- Social Engagement
- Relationship Building
- Teamwork

RESPONSIBLE DECISION-MAKING
The ability to make constructive choices about personal behavior and social interactions based on ethical standards, safety concerns, and social norms. The realistic evaluation of consequences of various actions, and a consideration of the well-being of oneself and others.

- Identifying Problems
- Analyzing Situations
- Solving Problems
- Evaluating
- Reflecting
- Ethical Responsibility

Diagram: Sociocultural and Institutional Roles and Responsibilities: Classroom, School, and Community Partnerships.
<table>
<thead>
<tr>
<th>SEL Competency</th>
<th>Examples</th>
<th>Content Specific Activity &amp; Approach to SEL</th>
</tr>
</thead>
</table>
| ✓ Self-Awareness | Example practices that address Self-Awareness:  
  • Clearly state classroom rules  
  • Provide students with specific feedback regarding academics and behavior  
  • Offer different ways to demonstrate understanding  
  • Create opportunities for students to self-advocate  
  • Check for student understanding / feelings about performance  
  • Check for emotional wellbeing  
  • Facilitate understanding of student strengths and challenges | Students begin the unit by drawing themselves and identifying the attributes they have that makes them a good mathematician. Students are encouraged to add more affirmative language as they learn additional math skills. Routinely tell students authentic reasons why you feel happy/optimistic for them and their future, including your optimism about their ability to succeed in a career in math, science, or technology. Encourage conversations about how they feel learning math will contribute to their success in the future. |
| ✓ Self-Management | Example practices that address Self-Management:  
  • Encourage students to take pride/ownership in work and behavior | Students reflect on barriers they may encounter when completing an assignment by asking clarifying questions, trying out others’ strategies and describing the approaches used |
<table>
<thead>
<tr>
<th>SEL Competency</th>
<th>Examples</th>
<th>Content Specific Activity &amp; Approach to SEL</th>
</tr>
</thead>
<tbody>
<tr>
<td>SEL Competency</td>
<td>Example practices that address Social-Awareness:</td>
<td>Teach lessons that develop mathematical discourse and engage students in purposeful sharing of mathematical ideas, reasoning and approaches using varied representations.</td>
</tr>
<tr>
<td>Self-Awareness</td>
<td>• Encourage students to reflect on the perspective of others</td>
<td>After students work on a task solving real-world problems involving multiplication of fractions, ask the students to chart their solutions and hang them up around the room. Then have the students go around the room and write questions or positive feedback on sticky notes and place them on the charts.</td>
</tr>
<tr>
<td>Self-Management</td>
<td>• Assign appropriate groups</td>
<td></td>
</tr>
<tr>
<td>Social-Awareness</td>
<td>• Help students to think about social strengths</td>
<td></td>
</tr>
<tr>
<td>Relationship Skills</td>
<td>• Provide specific feedback on social skills</td>
<td></td>
</tr>
<tr>
<td>Responsible Decision-Making</td>
<td>• Model positive social awareness through metacognition activities</td>
<td></td>
</tr>
<tr>
<td>SEL Competency</td>
<td>Example practices that address Relationship Skills:</td>
<td>Model and reinforce effective communication, relationship, and conflict-resolution skills that encourages students to feel comfortable taking mathematical risks.</td>
</tr>
<tr>
<td>Self-Awareness</td>
<td>• Engage families and community members</td>
<td></td>
</tr>
<tr>
<td>Self-Management</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Social-Awareness</td>
<td></td>
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<tr>
<td>Relationship Skills</td>
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<tr>
<td>Responsible Decision-Making</td>
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<tr>
<td>SEL Competency</td>
<td>Examples</td>
<td>Content Specific Activity &amp; Approach to SEL</td>
</tr>
<tr>
<td>----------------</td>
<td>----------</td>
<td>---------------------------------------------</td>
</tr>
</tbody>
</table>
| -              | • Model effective questioning and responding to students  
|                | • Plan for project-based learning  
|                | • Assist students with discovering individual strengths  
|                | • Model and promote respecting differences  
|                | • Model and promote active listening  
|                | • Help students develop communication skills  
|                | • Demonstrate value for a diversity of opinions | Give students support and guidance as needed when they are working through a conflict. Ask nonjudgmental questions about what happened and what the students might do differently to avoid the conflict. This can be done effectively in teacher-led small group instruction during the Independent phase of the Balanced Math Instructional Model. |
| Self-Awareness | Example practices that address Responsible Decision-Making:  
| Self-Management | • Support collaborative decision making for academics and behavior  
| Social-Awareness | • Foster student-centered discipline  
| Relationship Skills | • Assist students in step-by-step conflict resolution process  
<p>| ✓ Responsible Decision-Making | • Foster student independence | In mathematics classes, students can struggle with getting started on a challenging task. Teachers can motivate students’ learning of mathematics through opportunities for exploring problem solving that builds on and extends their current mathematical understanding. Support students through the steps of making a decision anytime they face a choice or decision. (For example: Have the students refer to the anchor chart on strategies for |</p>
<table>
<thead>
<tr>
<th>SEL Competency</th>
<th>Examples</th>
<th>Content Specific Activity &amp; Approach to SEL</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>• Model fair and appropriate decision making</td>
<td>finding volume and decide which strategy they are most comfortable using.)</td>
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<tr>
<td></td>
<td>• Teach good citizenship</td>
<td></td>
</tr>
</tbody>
</table>
### Differentiated Instruction

#### Accommodate Based on Students Individual Needs: Strategies

<table>
<thead>
<tr>
<th><strong>Time/General</strong></th>
<th><strong>Processing</strong></th>
<th><strong>Comprehension</strong></th>
<th><strong>Recall</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Extra time for assigned tasks</td>
<td>Extra Response time</td>
<td>Precise step-by-step directions</td>
<td>Teacher-made checklist</td>
</tr>
<tr>
<td>Adjust length of assignment</td>
<td>Have students verbalize steps</td>
<td>Short manageable tasks</td>
<td>Use visual graphic organizers</td>
</tr>
<tr>
<td>Timeline with due dates for reports and projects</td>
<td>Repeat, clarify or reword directions</td>
<td>Brief and concrete directions</td>
<td>Reference resources to promote independence</td>
</tr>
<tr>
<td>Communication system between home and school</td>
<td>Mini-breaks between tasks</td>
<td>Provide immediate feedback</td>
<td>Visual and verbal reminders</td>
</tr>
<tr>
<td>Provide lecture notes/outline</td>
<td>Provide a warning for transitions</td>
<td>Small group instruction</td>
<td>Graphic organizers</td>
</tr>
<tr>
<td></td>
<td>Reading partners</td>
<td>Emphasize multi-sensory learning</td>
<td><strong>Assistive Technology</strong></td>
</tr>
<tr>
<td><strong>Tests/Quizzes/Grading</strong></td>
<td><strong>Behavior/Attention</strong></td>
<td><strong>Organization</strong></td>
<td></td>
</tr>
<tr>
<td>Extended time</td>
<td>Consistent daily structured routine</td>
<td>Individual daily planner</td>
<td></td>
</tr>
<tr>
<td>Study guides</td>
<td>Simple and clear classroom rules</td>
<td>Display a written agenda</td>
<td></td>
</tr>
<tr>
<td>Focused/chunked tests</td>
<td>Frequent feedback</td>
<td>Note-taking assistance</td>
<td></td>
</tr>
<tr>
<td>Read directions aloud</td>
<td><strong>Behavior/Attention</strong></td>
<td>Color code materials</td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>Organization</strong></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- Computer/whiteboard
- Tape recorder
- Spell-checker
- Audio-taped books
- Teacher-made checklist
- Use visual graphic organizers
- Reference resources to promote independence
- Visual and verbal reminders
- Graphic organizers
- Consistent daily structured routine
- Simple and clear classroom rules
- Frequent feedback
- Individual daily planner
- Display a written agenda
- Note-taking assistance
- Color code materials
## Differentiated Instruction

### Accommodate Based on Content Specific Needs:

- Teacher modeling
- Use base ten manipulatives and color tiles to represent fractions while solving word problems.
- Use centimeter cubes, inch tiles, or unifix cubes with a rectangle or rectangular prism to represent and solve real world problems involving area, perimeter and volume.
- Use nesting boxes to model and explore different rectangular prisms with the same volume.
- Use an interactive technology model to fill a figure with non-overlapping right rectangular prisms.
- Number line to create number plots
- Use rectangular arrays of objects to calculate whole number quotients.
- Use concrete models to multiply and divide fractions and mixed numbers.
- Use interactive technology to find the volume.
- Teacher will create and display anchor charts.
- Chart academic vocabulary with visual representations and definitions.
- Use centimeter grid paper to draw an area model to multiply fractions and to draw composite figures and determining the volume.
## Interdisciplinary Connections

*Model interdisciplinary thinking to expose students to other disciplines.*

### Science

**envision STEM Topic 10: Wildfires (5-ESS2-1 & 5-ESS3-1)**
- Students will learn about wildfires, how they start, how they affect animals and the environment. Students will create a line plot to show their data regarding how long each ecosystem takes to recover from a wildfire.

### English Language Arts

**Problem-Solving Leveled Reading Mat (Topic 11): Sights and Monuments (NJSLSA.R.7)**
- Students read about various famous sights and monuments in the world. Students will then determine the volume of parts of a model of a castle.

**Problem-Solving Leveled Reading Mat (Topic 9): Jewels and Jade (NJSLSA.R.7)**
- Students will read about various kinds of jewels. Students will then answer fraction question regarding cutting various kinds of gems.

### Physical Education:

**Pick A Project 8C: Calcium in the Human Body (2.1.5.PGD.1)**
- Students will learn about calcium and foods that contain calcium. Students will then need to look at their favorite restaurants and determine the amount of calcium in 1 cup of different foods for breakfast, lunch and dinner and list it as a fraction.
### Enrichment

**What is the purpose of Enrichment?**

- The purpose of enrichment is to provide extended learning opportunities and challenges to students who have already mastered, or can quickly master, the basic curriculum. Enrichment gives the student more time to study concepts with greater depth, breadth, and complexity.
- Enrichment also provides opportunities for students to pursue learning in their own areas of interest and strengths.
- Enrichment keeps advanced students engaged and supports their accelerated academic needs.
- Enrichment provides the most appropriate answer to the question, “What do you do when the student already knows it?”

**Enrichment is...**

- Planned and purposeful
- *Different*, or differentiated, work – not just more work
- Responsive to students’ needs and situations
- A promotion of high-level thinking skills and making connections within content
- The ability to apply different or multiple strategies to the content
- The ability to synthesize concepts and make real world and cross-curricular connections
- Elevated contextual complexity
- Sometimes independent activities, sometimes direct instruction
- Inquiry based or open-ended assignments and projects
- Using supplementary materials in addition to the normal range of resources
- Choices for students
- Tiered/Multi-level activities with flexible groups (may change daily or weekly)

**Enrichment is not...**

- Just for gifted students (some gifted students may need intervention in some areas just as some other students may need frequent enrichment)
- Worksheets that are more of the same (busywork)
- Random assignments, games, or puzzles not connected to the content areas or areas of student interest
- Extra homework
- A package that is the same for everyone
- Thinking skills taught in isolation
- Unstructured free time
# Assessments

## Required District/State Assessments
- District Assessments
- SGO Assessments
- State Assessments

## Suggested Formative/Summative Classroom Assessments
- Describe Learning Vertically
- Identify Key Building Blocks
- Make Connections (between and among key building blocks)
- Short/Extended Constructed Response Items
- Multiple-Choice Items (where multiple answer choices may be correct)
- Drag and Drop Items
- Use of Equation Editor
- Quizzes
- Journal Entries/Reflections/Quick-Writes
- Accountable talk
  - Projects
  - Portfolio
  - Observation
- Graphic Organizers/Concept Mapping
- Presentations
- Role Playing
- Teacher-Student and Student-Student Conferencing
- Homework
New Jersey Student Learning Standards

5.NF.A.2  Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. For example, recognize an incorrect result $2/5 + 1/2 = 3/7$, by observing that $3/7 < 1/2$.

5.NF.B.3  Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. For example, interpret $3/4$ as the result of dividing 3 by 4, noting that $3/4$ multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size $3/4$. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?

5.NF.B.4  Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.

   a) Interpret the product $(a/b) \times q$ as $a$ parts of a partition of $q$ into $b$ equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. For example, use a visual fraction model to show $(2/3) \times 4 = 8/3$, and create a story context for this equation. Do the same with $(2/3) \times (4/5) = 8/15$. (In general, $(a/b) \times (c/d) = ac/bd$.)

   b) Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.

5.NF.B.5  Interpret multiplication as scaling (resizing) by:

   a) Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.

   b) Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a/b = (n\times a)/(n\times b)$ to the effect of multiplying $a/b$ by 1.
5.NF.B.6 Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

5.NF.B.7 Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.

a) Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. For example, create a story context for \((1/3) \div 4\), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that \((1/3) \div 4 = 1/12\) because \((1/12) \times 4 = 1/3\).

b) Interpret division of a whole number by a unit fraction, and compute such quotients. For example, create a story context for \(4 \div (1/5)\), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that \(4 \div (1/5) = 20\) because \(20 \times (1/5) = 4\).

c) Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. For example, how much chocolate will each person get if 3 people share 1/2 lb. of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins?

5.MD.B.2 Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots. For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.

5.MD.C.3 Recognize volume as an attribute of solid figures and understand concepts of volume measurement.

a) A cube with side length 1 unit, called a “unit cube,” is said to have “one cubic unit” of volume, and can be used to measure volume.

b) A solid figure which can be packed without gaps or overlaps using \(n\) unit cubes is said to have a volume of \(n\) cubic units.

5.MD.C.4 Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft., and non-standard units.

5.MD.C.5 Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.

a) Find the volume of a right rectangular prism with operations of multiplication and addition and solve real world and mathematical whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the
edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.

b) Apply the formulas $V = l \times w \times h$ and $V = B \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole number edge lengths in the context of solving real world and mathematical problems.

c) Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.
Mathematical Practices

1. Make sense of problems and persevere in solving them.

2. Reason abstractly and quantitatively.

3. Construct viable arguments and critique the reasoning of others.

4. Model with mathematics.

5. Use appropriate tools strategically.

6. Attend to precision.

7. Look for and make use of structure.

8. Look for and express regularity in repeated reasoning.
Grade: 5  
Unit: 3  
Topic: Fractions and Volume


Unit Focus:
- Understand concepts of volume
- Perform operations with multi-digit whole numbers and with decimals to hundredths
- Use equivalent fractions as a strategy to add and subtract fractions

Planning Resources:
Achieve the Core: Coherence Map: This shows the connections between the standards for Math. It will provide you with related and subsequent standards to help plan for intervention, remediation and enrichment. It also provides examples for the standards.
https://achievethecore.org/coherence-map/

NJSLA Mathematics Evidence Statements: This document, created by Grant Kolmer, provides PARCC and NJSLA released tested items from 2015-2019. NJSLA MATHEMATICS EVIDENCE STATEMENTS - Google Sheets

<table>
<thead>
<tr>
<th>Standard</th>
<th>Curriculum Guide Page #</th>
<th>Main Student Learning Objectives</th>
<th>Supporting Student Learning Objectives</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.NF.A.2</td>
<td>33</td>
<td></td>
<td>19, 20, 21, 22</td>
</tr>
<tr>
<td>5.NF.B.3</td>
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<td>10, 11, 17</td>
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<td>5.NF.B.4</td>
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<td>1, 2, 3, 4, 5, 6</td>
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<tr>
<td>5.NF.B.5</td>
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<td>8</td>
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<td>5.NF.B.6</td>
<td>51</td>
<td>7, 9</td>
<td>1, 2, 17, 21</td>
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<td>5.NF.B.7</td>
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<td>12, 13, 14, 15, 16, 18</td>
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<tr>
<td>5.MD.B.2</td>
<td>63</td>
<td>19, 20, 21, 22</td>
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<tr>
<td>5.MD.C.3</td>
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<tr>
<td>Standard</td>
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<tr>
<td>5.MD.C.4</td>
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<tr>
<td>5.MD.C.5</td>
<td>70</td>
<td>25, 26, 27, 28</td>
<td>24</td>
</tr>
</tbody>
</table>
New Jersey Student Learning Standard(s):
5.NF.A.2: Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2.

Prerequisite Student Learning Standards:
4.NF.A.2: Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

5.NF.A.1: Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.)

Modified Student Learning Objectives/Standards:
M.EE.5.NF.2: Identify models of thirds (1/3, 2/3, 3/3) and tenths (1/10, …, 9/10, 10/10).

<table>
<thead>
<tr>
<th>MP 1 ☒</th>
<th>MP 2 ☒</th>
<th>MP 3 ☒</th>
<th>MP 4 ☐</th>
<th>MP 5 ☐</th>
<th>MP 6 ☒</th>
<th>MP 7 ☐</th>
<th>MP 8 ☒</th>
</tr>
</thead>
<tbody>
<tr>
<td>Evidence Statement Key/Clarifications</td>
<td>Skills, Strategies &amp; Concepts</td>
<td>Essential Understandings/Questions (Accountable Talk)</td>
<td>Tasks/Activities</td>
<td></td>
<td></td>
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<tr>
<td>5.NF.2-1 5.NF.2-2</td>
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</tr>
<tr>
<td>• The situation types are those shown in Table 2, p. 9 of the OA</td>
<td>Emphasize that we only add fractions when the fractions refer to the same whole. Justify the reasonableness of a solution using estimation and benchmark fractions.</td>
<td>A fraction is relative to the size of the whole unit. Fractions of the whole being added do not overlap.</td>
<td>Envision</td>
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<td></td>
<td></td>
<td>10-1</td>
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<td></td>
<td>10-2</td>
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</tr>
</tbody>
</table>
Progression document, sampled equally.
- Result Unknown
- Change Unknown
- Start Unknown
- Total Unknown
- Both Addends Unknown
- Addend Unknown
- Difference Unknown
- Bigger Unknown
- Smaller Unknown

- Prompts do not provide visual fraction models; students may at their discretion draw visual fraction models as a strategy.

- Tasks may involve fractions greater than one, including mixed numbers.

<table>
<thead>
<tr>
<th>Linear Model:</th>
<th>Bar Model:</th>
<th>Every fraction can be represented by an infinite set of different but equivalent fractions.</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Linear Model" /></td>
<td><img src="image2.png" alt="Bar Model" /></td>
<td>A number line can be used to determine if estimates are reasonable.</td>
</tr>
<tr>
<td>This model shows $1 \frac{3}{4}$ subtracted from $3 \frac{1}{6}$ leaving $1 \frac{1}{4} = \frac{5}{6}$ which a student can then change to $1 + \frac{3}{12} + \frac{2}{12} = \frac{5}{12}$. $3 \frac{1}{6}$ can be expressed with a denominator of 12. Once this is done a student can complete the problem, $2 \frac{14}{12} - 1 \frac{9}{12} = 1 \frac{5}{12}$.</td>
<td>Sums and differences can be estimated by rounding to the nearest whole number or by using benchmark fractions.</td>
<td></td>
</tr>
<tr>
<td>Models can be used to show different ways of adding mixed numbers.</td>
<td>What is a reasonable estimate for the answer?</td>
<td></td>
</tr>
<tr>
<td>How do operations with fractions relate to operations with whole numbers?</td>
<td>What models or pictures could aid in understanding a mathematical or real world problem and the relationships among the quantities?</td>
<td></td>
</tr>
</tbody>
</table>
Equivalent Fractions:
\[
\frac{3}{5} - \frac{1}{10} = \frac{6}{10} - \frac{1}{10} = \frac{5}{10}
\]
\[
\frac{3}{5} + \frac{5}{10} = \frac{6}{10} + \frac{5}{10} = \frac{11}{10}
\]

Students often mix models when adding, subtracting or comparing fractions. They may use a circle for thirds and a rectangle for fourths when comparing thirds and fourths. Remind students that the representations need to be from the same whole models with the same shape and size.

Pattern blocks make great fraction manipulatives, but they can be limiting since they imply part to whole more often than part to set (group). Use more than one type of manipulative (color tiles, cuisenaire rods, fraction strips and pattern blocks) so

How can I use a number line to compare relative sizes of fractions?

How can we tell if a fraction is greater than, less than, or equal to one whole?

How does the size of the whole determine the size of the fraction?

What models can we use to help us add and subtract fractions with different denominators?

What strategies can we use for adding and subtracting fractions with different denominators?
students develop flexibility in representations and computation.

Do not give students the algorithm (finding the common denominator) before they struggle with the idea of how to go about combining two fractions with unlike denominators.

**SPED Strategies:**
Let the student use a flowchart to plan strategies for problem solving.

Encourage students to explain their thinking and strategy for the solution.

Provide students with color coded notes, samples and number lines.

Review adding and subtracting fractions with like denominators.

**ELL Strategies:**
Adjusting number words and correctly pronouncing them as fractions (fifths, sixths, etc.) may be challenging.

If there are many English language learners in class, consider quickly counting together to practice enunciating word endings: halves, thirds, fourths, fifths, sixths, etc.
<table>
<thead>
<tr>
<th>After reading multi-step word problems, demonstrate understanding by adding or subtracting fractions with unlike denominators, and explain if the answer is reasonable in L1 (student’s target language) and/or using step-by-step problems and gestures, drawings and selected technical words.</th>
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</thead>
<tbody>
<tr>
<td>Elaborate on the problem-solving process:</td>
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<tr>
<td>• Read word problems aloud.</td>
</tr>
<tr>
<td>• Post a visual display of the problem-solving process.</td>
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<tr>
<td>• Have students check off or highlight each step as they work.</td>
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<tr>
<td>• Talk through the problem-solving process step-by-step to demonstrate thinking process.</td>
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<tr>
<td>• Before students solve, ask questions for comprehension, such as, “What unit are we counting? What happened to the units in the story?”</td>
</tr>
<tr>
<td>• Teach students to use self-questioning techniques, such as, “Does my answer make sense?”</td>
</tr>
</tbody>
</table>
New Jersey Student Learning Standard(s):
5.NF.B.3: Interpret a fraction as division of the numerator by the denominator (a/b = a ÷ b). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?

Prerequisite Student Learning Standards:
3.OA.A.2: Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.

3.OA.B.6: Understand division as an unknown-factor problem. For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.

4.MD.A.2: Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.

4.OA.A.3: Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

4.OA.A.1: Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.

4.OA.A.2: Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.

Modified Student Learning Objectives/Standards: N/A
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<tr>
<th>Evidence Statement Key/ Clarifications</th>
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<th>Essential Understandings/ Questions (Accountable Talk)</th>
<th>Tasks/Activities</th>
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<tbody>
<tr>
<td>5.NF.3-1</td>
<td>Equal sharing problems offer students the opportunity to use what they know about partitioning and division to learn fractions.</td>
<td>A fraction describes division ( \frac{a}{b} = \frac{a}{b} ). It can be interpreted on a number line in two ways. ( 2 \div 3 ) can be interpreted as 2 segments where each is 1/3 of a unit ((2 \times \frac{1}{3})) or 1/3 of 2 whole units ((\frac{1}{3} \times 2)). Ability to recognize that a fraction is a representation of division. A fraction or mixed number can represent the quotient of two whole numbers. What does it mean to divide? How can we model dividing a unit fraction by a whole number with manipulatives and diagrams?</td>
<td><strong>Envision</strong> 9-1 9-2 3-ACT Math (Topic 9)</td>
</tr>
<tr>
<td><strong>5.NF.3-2</strong></td>
<td>Students are able to demonstrate their understanding of fractions using concrete materials, drawings and models.</td>
<td>Example: Your teacher gives you 7 packs of paper to share with your group of 4 students. If you share the</td>
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</tr>
</tbody>
</table>

**Example:**
Your teacher gives you 7 packs of paper to share with your group of 4 students. If you share the
paper equally, how much paper does each student get?

Fractions can be explained in multiple contexts.

They read 3/8 as “three eighths” and after many experiences with sharing problems, learn that 3/8 can also be interpreted as “3 divided by 8.” Also, 3 ÷ 8 is the number you multiply by 8 to get 3 and 3/8 is the number you get by taking 3 copies of the unit fraction 1/8.

Define a fraction as division of the numerator by its denominator.

Explain between what two whole numbers the fraction solution lies.

**SPED Strategies:**
Encourage students to explain their thinking and strategy for the solution.

Provide students with color coded notes, samples and number lines.

**ELL Strategies:**
Adjusting number words and correctly pronouncing them as fractions (fifths, sixths, etc.) may be challenging.

What does dividing a unit fraction by a whole number look like?

What does dividing a whole number by a unit fraction look like?

How can we model dividing a unit fraction by a whole number with manipulatives and diagrams?
After reading multi-step word problems, demonstrate understanding by dividing fractions and explain if the answer is reasonable in L1 (student’s native language) and/or using step-by-step problems and gestures, drawings and selected technical words.

New Jersey Student Learning Standard(s):

**5.NF.B.4:** Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.

  a) Interpret the product \((a/b) \times q\) as \(a\) parts of a partition of \(q\) into \(b\) equal parts; equivalently, as the result of a sequence of operations \(a \times q \div b\). For example, use a visual fraction model to show \((2/3) \times 4 = 8/3\), and create a story context for this equation. Do the same with \((2/3) \times (4/5) = 8/15\). (In general, \((a/b) \times (c/d) = ac/bd\).)

  b) Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.

Prerequisite Student Learning Standards:

**4.NF.B.4:** Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.

**3.MD.C.7:** Relate area to the operations of multiplication and addition.

Modified Student Learning Objectives/Standards:

- MP 1 ☒
- MP 2 ☒
- MP 3 ☒
- MP 4 ☒
- MP 5 ☒
- MP 6 ☒
- MP 7 ☒
- MP 8 ☐
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<tr>
<td><strong>5.NF.4a-1</strong></td>
<td>Students may believe that multiplication always results in a larger number and that division always results in a smaller number. Using models when multiplying and dividing with fractions will enable students to see that the results can be larger or smaller. Focus on problems with 4, 8, 3, 6, 10 and 12 equal shares, but include other numbers of shares as well, such as 15, 20, and 100. Represent children’s solutions with equations, with an emphasis on linking addition and multiplication and on equations that reflect a multiplicative understanding of fractions. (1/8 + 1/8 + 1/8 = 3/8, 3 x 1/8 = 3/8).</td>
<td>Models can be used to show that the product of a fraction and a whole number can be interpreted as repeated addition. Multiplying a whole number and a fraction involves both multiplication and division. To multiply a whole number and a fraction, write a fraction ( \frac{a}{b} ) as a product of ( a \times \frac{1}{b} ), multiply the whole numbers, and write the product as a fraction or mixed number.</td>
<td><strong>Envision</strong></td>
</tr>
<tr>
<td>• Tasks require finding a fractional part of a whole number quantity. The result is equal to a whole number in 20% of tasks; these are practice forward for MP.7. Tasks have “thin context” or no context.</td>
<td></td>
<td></td>
<td>8-1</td>
</tr>
<tr>
<td>• Tasks have “thin context” or no context.</td>
<td></td>
<td></td>
<td>8-2</td>
</tr>
<tr>
<td><strong>5.NF.4a-2</strong></td>
<td>The result is equal to a whole number in 20% of tasks; these are practice forward for MP.7.</td>
<td>The meaning of multiplying a whole number by a fraction can be extended to multiplying a fraction by a fraction. Different models can be used to show this connection. To find the product of two fractions, multiply the numerators</td>
<td><strong>IFL Sets of Related Lessons</strong> – “Multiplication with Fractions: Finding Portions of Numbers”</td>
</tr>
<tr>
<td>• Tasks require finding a product of two fractions (neither of the factors equal to a whole number).</td>
<td></td>
<td></td>
<td>8-3</td>
</tr>
<tr>
<td>• The result is equal to a whole number in 20% of tasks; these are practice forward for MP.7.</td>
<td></td>
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<td>8-4</td>
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<td>8-5</td>
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<td></td>
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<td>8-6</td>
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</tbody>
</table>
5.NF.4b-1

- 50% of the tasks present students with the rectangle dimensions and ask students to find the area; 50% of the tasks give the factions and the product and ask students to show a rectangle to model the problem.

| Look for students who believe that when you multiply, the product is always larger than the factors. |
| Decomposing fractions help us to multiply fractions and model fraction multiplication. |
| Provide assistance with the distributive property with whole numbers using arrays and partial products before moving to fractions. |
| Factor sizes and products can be discovered and generalized as students study partial products. They will be better able to attend to precision because they will know if their partial products are reasonable. |
| Encourage representation of thinking. This allows students to talk through their process which in turn enables students the opportunity to attend to precision as they explain and reason mathematically. |
| Performing operations with fractions can be enhanced with the use of virtual manipulatives from web sites such as and then multiply the denominators. |
| Recognize when a product is less than or greater than 1. |
| An area model can be used to represent the product of two fractions. |
| In a rectangular array, one of the factors tells about the rows/number of units wide the figure is and the other factor tells about the number of square units in each row. |
| Either factor (the number of rows or number of square units in each row) can be worked with separately to determine the area of a portion of the figure. |
| When multiplying with at least one factor greater than one, the product will be greater than at least one of the factors because more than one group of the other factor is being utilized. |
the National Library of Virtual Manipulatives.

Grid paper, and color tiles or other manipulatives assist students with modeling.

Draw a fraction model to illustrate a product of a fraction by a whole number and a fraction by a fraction.

When multiplying two factors less than one, the product will be less than either factor because less than one whole group of less than one is being utilized (a portion of a portion).

How does multiplying fractions relate to real world problems?

What patterns do you notice in the products?

How are the products related to the factors in the problems?
SPED Strategies:
Visual fraction models (area models, tape diagrams, number lines) should be used and created by students during their work with this standard.

Present information through different modalities.
Scaffold complex concepts and provide leveled problems for multiple entry points.
In groups, students should create a flip book with rules and samples.

ELL Strategies:
Vary the grouping in the classroom, such as sometimes using small group
instruction to help ELLs learn to negotiate vocabulary with classmates and other times using native language support to allow a student to master full proficiency of the mathematics first.

Provide sufficient wait time to allow the student to process the meanings in the different languages. Present information through different modalities.

Describe or explain how to find the area of a rectangle with fractional side lengths by tiling unit squares and multiplying side lengths orally and in writing in L1 (student’s native language) and/or use gestures, pictures and selected words.

Visual fraction models (area models, tape diagrams, number lines) should be used and created by students during their work with this standard.

Mathematics Reference Sheet in native language with examples should be made available.
New Jersey Student Learning Standard(s):

5.NF.B.5: Interpret multiplication as scaling (resizing) by:

a) Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.

b) Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence \( \frac{a}{b} = \frac{(n \times a)}{(n \times b)} \) to the effect of multiplying \( \frac{a}{b} \) by 1.

Prerequisite Student Learning Standards:

3.OA.A.1: Interpret products of whole numbers, e.g., interpret \( 5 \times 7 \) as the total number of objects in 5 groups of 7 objects each. For example, describe a context in which a total number of objects can be expressed as \( 5 \times 7 \).

3.OA.A.2: Interpret whole-number quotients of whole numbers, e.g., interpret \( 56 \div 8 \) as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. For example, describe a context in which a number of shares or a number of groups can be expressed as \( 56 \div 8 \).

4.MD.A.2: Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.

4.OA.A.3: Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

4.NF.A.1: Explain why a fraction \( \frac{a}{b} \) is equivalent to a fraction \( \frac{(n \times a)}{(n \times b)} \) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

5.NF.B.4: Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.
### Modified Student Learning Objectives/Standards: N/A

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| 5.NF.5a                              | Being able to estimate and mentally multiply whole numbers will allow students to determine reasonable answers and the relationship of the product to the factors used. Students examine the magnitude of products in terms of the relationship of two factors. This understanding can be used to develop rules that assist in determining the magnitude of products given the size of each factor. Students must investigate what happens to the product when one factor remains the same and one factor changes. Students can use concrete models (such as cuisenaire rods or circle pieces) and iconic models (such as shading rectangles) to connect these models to the equations. | When multiplying with at least one factor greater than one, the product will be greater than at least one of the factors because more than one group of the other factor is being utilized. Multiplication and division can increase, decrease, or keep a number the same size. When multiplying two factors less than one, the product will be less than either factor because less than one whole group of less than one is being utilized (a portion of a portion). The relative size of factors can be used to determine the relative size of the product. | **Envision**  
8-8 | **IFL Sets of Related Lessons**  
“Multiplication with Fractions: Finding Portions of Numbers” |
| Compare the size of a product to the size of one of its factors, considering the size of the other factor (at least one factor is a fraction).  
**SPED Strategies:**  
Teach from simple to complex, moving from concrete to representation to abstract at the student’s pace.  
Cultivate peer-assisted learning interventions for instruction (e.g., dictation) and practice, particularly for computation work (e.g., peer modeling).  
Have students work together to solve and then check their solutions.  
**ELL Strategies:**  
Vary the grouping in the classroom, such as sometimes using small group instruction to help ELLs learn to negotiate vocabulary with classmates and other times using native language support to allow a student to find full proficiency of the mathematics first.  
Allow ELL students to talk to a peer in their native language when necessary to clarify understanding and clear up misunderstandings. | Why is it useful to compare numbers?  
How are fractions parts of a whole? |
New Jersey Student Learning Standard(s):

5.NF.B.6: Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

Prerequisite Student Learning Standards:

3.OA.B.6: Understand division as an unknown-factor problem. For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.

3.OA.A.2: Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.

4.MD.A.2: Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.

4.OA.A.1: Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.

4.OA.A.2: Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.

Modified Student Learning Objectives/Standards: N/A

<p>| MP 1 ☒ | MP 2 ☐ | MP 3 ☒ | MP 4 ☒ | MP 5 ☐ | MP 6 ☐ | MP 7 ☐ | MP 8 ☒ |</p>
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<tr>
<td>5.NF.6-1</td>
<td>Represent children’s solutions with equations, with an emphasis on linking addition and multiplication and on equations that reflect a multiplicative understanding of fractions. (1/8 + 1/8 + 1/8 = 3/8, 3 x 1/8 = 3/8 and 3 divided by 8 is 3/8). Provide a wide variety of multiplication problems with fractions and mixed numbers using situations, such as equal groups multiplication. Enforce the use of a variety of strategies, including make a model, draw a picture, make a table, and look for a pattern to solve problems that provide a context for multiplying fractions and mixed numbers. Allow students the opportunity to think about the reasonableness of their solutions in terms of the context and the numbers. Watch for misconceptions from previous multiplication and students who struggle understanding what information in a word problem indicates that they should multiply by</td>
<td>Multiplication does not always make the product larger than the factors and division does not always make the quotient smaller than the divisor. The order of the factors in a multiplication problem can be rearranged (flipped) because: the product is still the same. (Commutative Property). When multiplying with at least one factor greater than one, the product will be greater than at least one of the factors because more than one group of the other factor is being utilized. Multiplying mixed numbers is an extension of multiplying fractions. How can multiplying fractions be modeled using area, a number line, or measurement models?</td>
<td><strong>Envision</strong> 8-1 8-2 8-7 8-9 3-ACT Math (Topic 9) IFL Sets of Related Lessons – “Multiplication with Fractions: Finding Portions of Numbers”</td>
</tr>
<tr>
<td>5.NF.6-2</td>
<td>Tasks do not involve mixed numbers. Situations include area and comparison/times as much, with product unknown. (See Table 2, Common multiplication and division situations, p. 89 of CCSS.) Prompts do not provide visual fraction models; students may at their discretion draw visual fraction models as a strategy.</td>
<td></td>
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<tr>
<td></td>
<td><strong>Tasks/Activities</strong></td>
<td><strong>Envision</strong> 8-1 8-2 8-7 8-9 3-ACT Math (Topic 9) IFL Sets of Related Lessons – “Multiplication with Fractions: Finding Portions of Numbers”</td>
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</table>
- Situations include area and comparison/times as much, with product unknown.

- Prompts do not provide visual fraction models; students may at their discretion draw visual fraction models as a strategy.

- Providing more experiences with visual representations, breaking the problems into smaller parts, and explain their thinking as they complete each part of the problem.

Model multiplication of fractions through visual models.

Represent the product in simplest terms.

Solve word problems involving multiplication of fractions and mixed numbers.

**Example:**
There are 2 ½ bus loads of students standing in the parking lot. The students are getting ready to go on a field trip. 2/5 of the students on each bus are girls. How many busses would it take to carry only the girls?

<table>
<thead>
<tr>
<th>2/5</th>
<th>2/5</th>
<th>1/5</th>
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= 5/5 = 1 whole bus.

**SPED Strategies:**
Introduce essential terms and vocabulary prior to the mathematics instruction.
Clarify, compare, and make connections to math words in discussion, particularly during and after practice.

Highlight critical vocabulary in discussion. For example, show a picture of ‘half.’

Connect language (fractions) with concrete and pictorial experiences (creating a number line, book marks with pictures representing fractions, fraction bars, fraction tower cubes, fraction circles. Sample fractions created out of paper plates, etc.)

**ELL Strategies:**
Teacher-talk with illustrative gestures.

Vary your voice to guide comprehension. Speak dynamically with expression. Make eye-to-eye contact and speak slowly and distinctly.

Vary the grouping in the classroom, such as sometimes using small group instruction to help ELLs learn to negotiate vocabulary with classmates and other times using native language support to allow a student to find full proficiency of the mathematics first.
New Jersey Student Learning Standard(s):
**5.NF.B.7:** Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.

a) Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. *For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) × 4 = 1/3.*

b) Interpret division of a whole number by a unit fraction, and compute such quotients. *For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 × (1/5) = 4.*

c) Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, how much chocolate will each person get if 3 people share 1/2 lb. of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins?*

Prerequisite Student Learning Standards:
**3.NF.A.1:** Understand a fraction 1/b as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size 1/b.

**3.OA.B.6:** Understand division as an unknown-factor problem. For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.

**4.NF.B.4:** Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.

Modified Student Learning Objectives/Standards: N/A
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<tr>
<td>5.NF.7a</td>
<td>This is the students’ first experience with division of fractions. They use their understanding of whole number division to visualize what happens when they are dividing whole numbers by fractions and fractions by whole numbers. Review partitive model of division of whole numbers. Give students problems to solve and have them explain their thinking by relating the division of a fraction by a whole number. Look for connections between divisions with fractions using previous experiences with the relationship between multiplication and division. Different real world interpretations can be associated with division calculations involving fractions. Facilitate discussions that illuminate different</td>
<td>Models can be used to show how many dividing a whole number by a fraction related to multiplication. Visual fraction models can be used to represent and solve problems involving whole numbers divided by unit fractions. To divide a number, a, by another number, b, means to determine how many iterations of b fit inside a, including when a and b are fractions or decimals. A fraction describes the division of a whole (region, set, segment) into equal parts. What does dividing a unit fraction by a whole number look like?</td>
<td>Envision 9-3 9-4 9-5 9-6 9-7 9-8</td>
</tr>
<tr>
<td>5.NF.B.7</td>
<td>• Tasks involve equal group (partition) situations with part size unknown and number of parts unknown. • Prompts do not provide visual fraction models; students may at their discretion draw visual fraction models as a strategy.</td>
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<tr>
<td>5.NF.7c</td>
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</table>

IFL Sets of Related Lessons - “Dividing Fractions: Understanding Division with Numbers Less Than 1”
<table>
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<tr>
<th>Viewpoints students have that explain dividing fractions.</th>
<th>Define a unit fraction as fraction with a numerator of 1.</th>
</tr>
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<tr>
<td>Have students compare and discuss two problems (one that divides a fraction by a whole and one that divides a whole by a fraction). Students should express their understanding in writing, with a pair and in a whole group setting to reveal multiple viewpoints.</td>
<td>The inverse relationship between multiplication and division can be used to divide with fractions.</td>
</tr>
<tr>
<td>Provide students with representations and ask them to decide if the representations are accurate.</td>
<td>Dividing a number by a whole number results in a quotient that is a smaller number than the dividend because the dividend is being partitioned into parts.</td>
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<tr>
<td>Emphasize the use of visual models to understand the dividing of fractions that is taking place.</td>
<td>When dividing a number less than one, the quotient will be more than the dividend because either: you are making groups of an amount less than 1; or you are making less than one group.</td>
</tr>
<tr>
<td>One misconception to be addressed is that students always have to “divide the bigger number by the smaller number”. Make the connection to the structure of a fraction in which $\frac{3}{4}$ also means 3 divided by 4.</td>
<td>Dividing a unit fraction by a non-zero whole number can be modeled by showing part of a whole divided into equal parts.</td>
</tr>
<tr>
<td>Draw/show division of a unit fraction by a whole number as dividing the unit fraction into smaller parts.</td>
<td>A fraction describes the division of a whole (region, set, segment) into equal parts.</td>
</tr>
<tr>
<td></td>
<td>Area models and number lines can be used to represent and solve</td>
</tr>
</tbody>
</table>
For Example: $\frac{1}{3} \div 4$

Define the reciprocal of a unit fraction for the purpose of division.

Simplify quotients to the lowest term.

Discuss patterns students see in written equations and connect those equations with division problems involving whole numbers and unit fractions.

Some problems can be solved by first finding and solving one or more sub-problems, and then using the answer(s) to solve the original problem.

How does my knowledge of division of whole numbers apply to the division of fractions?

What do I need to know and be able to do to divide fractions by whole numbers?
to the relationship between multiplication and division.

\[ 3 \div \frac{1}{4} = \_\_\_\_ \quad \frac{1}{4} \times \_\_\_\_ = 3 \]

Model questions that students can ask themselves to help interpret the problem. For example, how many groups of \(\frac{1}{2}\) can I make out of 8? This question can be related to the visual representation students use to solve the problem.

The use of models and drawings will eliminate the confusion that eight divided by on-half (\(8 \div \frac{1}{2}\)) and eight divided by half (\(8 \div 2\)) produce the same results.
Have students compare and discuss two problems (one that divides a fraction by a whole and one that divides a whole by a fraction). Students should express their understanding in writing, with a pair and in a whole group setting to reveal multiple viewpoints.

Students will require experiences with fractional numbers that result in whole-number products and quotients.

Define the reciprocal of a unit fraction for the purpose of division.

Students can use a number line to develop an understanding of dividing a whole number by a unit fraction.

For example:

Using key words is not helpful and removes making sense from the process. Rather have students model the problem using pictures and ask supporting
questions, such as “What do you know? What do you want to find out? How can you show that in your picture?”

The real world situation determines how a remainder needs to be interpreted when solving a problem.

Explain the effects of dividing a whole number by a unit fraction (vice versa) in the context of a word problem.

Students may struggle determining which number goes where in the division problem. “Am I dividing the fraction by the whole number or the whole number by the fraction?” Drawing a picture using the information in the problem and focusing on what they want to find out will help.

Simplify quotients to the lowest forms.

Justify the reasonableness of answers in terms of the context of the problem.

**SPED Strategies:**
Create a visual model of the lesson expectations as a class.

Review prerequisite skills before introducing new concepts.
| Multiplication and division facts, identifying fractions, dividing whole numbers and multiplying whole numbers by a fraction |
| Utilize interactive journals to review examples of previous concepts. |
| Teach in small chunks so students get a lot of practice with one step at a time. |
| Color-code or highlight key words in math word problems. |
| Let the student use a flowchart to plan strategies for problem solving. |
| Provide a variety of ways to respond: oral; choral; student boards; concrete models, pictorial models (e.g., ten-frame) |

**ELL Strategies:**

Review in L1 (student’s native language) content-specific vocabulary words (i.e., quotient, equivalent, divisor). Create a visual model as a class to use as a future reference.

Utilize interactive journals to review examples of previous concepts and to demonstrate growth in math writing and reasoning.
| Teach in small chunks so students get a lot of practice with one step at a time. |
| Re-teach the same concept with a variety of fluency games. |
| Allow students to lead group and pair-share activities. |
| Have students paraphrase and write complex concepts in their own words (individually, pairs, or whole class) in their interactive math journal to demonstrate growth in math writing and reasoning. |
| Color-code or highlight key words in math word problems. |
| Let the student use a flowchart to plan strategies for problem solving. |
| Teach content-specific vocabulary words (i.e., quotient, equivalent, divisor) |

**New Jersey Student Learning Standard(s):**

**5.MD.B.2:** Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots. *For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.*
**Prerequisite Student Learning Standards:**

4.MD.B.4: Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots. *For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.*

5.NF.A.2: Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2.*

5.NF.B.6: Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

5.NF.B.7: Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.

   a) Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. *For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) × 4 = 1/3.*

   b) Interpret division of a whole number by a unit fraction, and compute such quotients. *For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 × (1/5) = 4.*

   c) Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, how much chocolate will each person get if 3 people share 1/2 lb. of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins?*

**Modified Student Learning Objectives/Standards:**

M.EE.5.MD.2 Represent and interpret data on a picture, line plot or bar graph.
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<tr>
<th>Evidence Statement Key/Clarifications</th>
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<th>Essential Understandings/Questions (Accountable Talk)</th>
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<tbody>
<tr>
<td><strong>5.MD.2-2</strong></td>
<td>Create and label a line plot to display a data set containing fractions.</td>
<td>Line plots are one way to organize and represent numerical data collected in a survey.</td>
<td><strong>Envision</strong></td>
</tr>
<tr>
<td>• Tasks requiring students to produce a line plot should only involve fractions 1/2, 1/4, or 1/8.</td>
<td>Simplify fractions to lowest terms.</td>
<td>You can use a line plot to see how data are distributed.</td>
<td>10-1</td>
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<tr>
<td></td>
<td>Using measurement information presented in line plots, add, subtract, multiply and divide fractions in order to solve problems.</td>
<td>You can use line plots to solve problems that involve data.</td>
<td>10-2</td>
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<td></td>
<td>Students might confuse a line graph with a line plot. Review the purpose of a line plot (a graphic representation that shows the frequency of data using x’s or dots along a number line) versus that of a line graph (a graphic representation that shows how data changed over time).</td>
<td>How is data collected and displayed on a line plot?</td>
<td>10-3</td>
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<td></td>
<td>Students may not understand that in order to share the items equally, you must first find the total number of items. This portion of the standard gives them a visual model and becomes the background for finding the mean in grade 6.</td>
<td>What strategies help when solving problems with line plots?</td>
<td>10-4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>How can graphic representation of data help solve problems?</td>
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<td></td>
<td></td>
<td>How do people use data?</td>
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</tbody>
</table>
Have students use the data from each person in the group to create a line plot. Students measured objects in their desk to the nearest 1/2, 1/4, or 1/8 of an inch then displayed data collected on a line plot. How many objects measured 1/4? 1/2? If you put all the objects together end to end what would be the total length of all the objects?

**SPED Strategies:**
Demonstrate understanding of making a line plot to display a data set of measurements in fractions of a unit. Explain how to solve problems involving information presented in line plots using key vocabulary in simple sentences.

Select numbers and tasks that are “just right” for learners.

Model each step for students and provide examples.

Provide a checklist of steps.
Scaffold complex concepts and provide leveled problems for multiple entry points.

Use visual fraction strips and fraction bars to represent problems and solve problems involving addition and subtraction of fractions.

**ELL Strategies:**
Demonstrate understanding of making a line plot to display a data set of measurements in fractions of a unit.
Explain how to solve problems involving information presented in line plots in L1 (student’s native language) and/or accompanied by gestures, pictures and selected words.

Use cognates or flip books written in native language to use as a reference.

Create an interactive math journal in L1 (student’s native language) to demonstrate growth in math writing and reasoning.

Provide a checklist of steps in L1 (student’s native language) on how to solve problems involving the four operations.

Give directions step-by-step (orally and in writing) before assigning students to do
independent, pair, or group work. Ask a student to repeat the directions aloud for the rest of the class to assess whether all the students understand the assignment.

**New Jersey Student Learning Standard(s):**
5.MD.C.3: Recognize volume as an attribute of solid figures and understand concepts of volume measurement.

   a) A cube with side length 1 unit, called a “unit cube,” is said to have “one cubic unit” of volume, and can be used to measure volume.

   b) A solid figure which can be packed without gaps or overlaps using $n$ unit cubes is said to have a volume of $n$ cubic units.

**Prerequisite Student Learning Standards:**
3.MD.C.5: Recognize area as an attribute of plane figures and understand concepts of area measurement.

**Modified Student Learning Objectives/Standards:**
M.EE.5.MD.3: Identify common three-dimensional shapes.

<table>
<thead>
<tr>
<th>Evidence Statement Key/Clarifications</th>
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<th>Essential Understandings/Questions (Accountable Talk)</th>
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</tr>
</thead>
<tbody>
<tr>
<td>5.MD.3</td>
<td>Volume is the amount of space inside a solid (3-dimensional) figure. Cubes with side length of 1 unit, called “a unit cube” is said to have “one cubic unit of volume, and can be used to measure volume.</td>
<td>Area measures 2 dimensions (length and width) and volume measures 3 dimensions (length, width and height).</td>
<td><strong>Envision</strong> 11-1 3-ACT Math (Topic 11)</td>
</tr>
</tbody>
</table>
Solid figures which can be packed without gaps or overlaps using $n$ unit cubes is said to have a volume of $n$ cubic units.

**SPED Strategies:**
Create flip books with pictures and examples.

Review identifying different types of shapes and their attributes.

Use real-life examples and concrete materials as needed.

**ELL Strategies:**
Describe or explain orally and in writing, how to measure volume by counting the total number of same size cubic units required filling a figure in L1 (student’s native language) and/or using gestures, pictures and selected, technical words.

Create flip books with pictures and examples in native language.

Introduce essential terms and vocabulary prior to the mathematics instruction.

Volume can be measured by counting the number of cubic units needed to fill a three-dimensional figure.

How do we represent the inside of a 3 dimensional figure?

How do you describe a three dimensional shape or solid?

What is volume and how is it used in real life?
Provide students with a list of important terms in their native language.

New Jersey Student Learning Standard(s):
5.MD.C.4: Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft., and non-standard units.

Prerequisite Student Learning Standards:
5.MD.C.3: Recognize volume as an attribute of solid figures and understand concepts of volume measurement.

- a) A cube with side length 1 unit, called a “unit cube,” is said to have “one cubic unit” of volume, and can be used to measure volume.
- b) A solid figure which can be packed without gaps or overlaps using \( n \) unit cubes is said to have a volume of \( n \) cubic units.

Modified Student Learning Objectives/Standards:
M.EE.5.MD.4: Determine the volume of a rectangular prism by counting units of measure (unit cubes).

<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>5.MD.4</td>
<td>Students need to practice these concrete activities and reinforce the concept with representations of the same activities. Volume of a solid can be determined using unit cubes of other dimensions.</td>
<td>Area measures 2 dimensions (length and width) and volume measures 3 dimensions (length, width and height). The measurement chosen depends on the size of the figure (e.g.,</td>
<td>Envision 11-1</td>
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<td></td>
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<td></td>
<td>11-2</td>
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<td>3-ACT Math (Topic 11) 11-5</td>
</tr>
</tbody>
</table>
Calculate the volume of a solid figure by counting the unit sides.

**SPED Strategies:**
Create flip books with pictures and examples.

Review identifying different types of shapes and their attributes.

Use real-life examples and concrete materials as needed.

**ELL Strategies:**
Describe or explain orally and in writing, how to measure volume by counting the total number of same size cubic units required filling a figure in L1 (student’s native language) and/or using gestures, pictures and selected, technical words.

Create flip books with pictures and examples in native language.

Introduce essential terms and vocabulary prior to the mathematics instruction.

Provide students with a list of important terms in their native language.

Formulas can be used to find the volume of rectangular prisms and cubes.

How does the area of rectangles relate to the volume of rectangular prisms?

Why is volume represented with cubic units and area represented with square units?
New Jersey Student Learning Standard(s):

5.MD.C.5: Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.

a) Find the volume of a right rectangular prism with operations of multiplication and addition and solve real world and mathematical whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.

b) Apply the formulas \( V = l \times w \times h \) and \( V = B \times h \) for rectangular prisms to find volumes of right rectangular prisms with whole number edge lengths in the context of solving real world and mathematical problems.

c) Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.

Prerequisite Student Learning Standards:

3.OA.B.5: Apply properties of operations as strategies to multiply and divide.* Examples: If \( 6 \times 4 = 24 \) is known, then \( 4 \times 6 = 24 \) is also known. (Commutative property of multiplication.) \( 3 \times 5 \times 2 \) can be found by \( 3 \times 5 = 15 \), then \( 15 \times 2 = 30 \), or by \( 5 \times 2 = 10 \), then \( 3 \times 10 = 30 \). (Associative property of multiplication.) Knowing that \( 8 \times 5 = 40 \) and \( 8 \times 2 = 16 \), one can find \( 8 \times 7 \) as \( 8 \times (5 + 2) = (8 \times 5) + (8 \times 2) = 40 + 16 = 56 \). (Distributive property.)

4.MD.A.3: Apply the area and perimeter formulas for rectangles in real world and mathematical problems. For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.

5.MD.C.3: Recognize volume as an attribute of solid figures and understand concepts of volume measurement.

5.MD.C.4: Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft., and non-standard units.

Modified Student Learning Objectives/Standards:

M.EE.5.MD.5: Determine the volume of a rectangular prism by counting units of measure (unit cubes).
<table>
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<tr>
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<th>Tasks/Activities</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.MD.5b</td>
<td>Volume is additive: volumes of composite solids can be determined by adding the volumes of each solid.</td>
<td>Area measures 2 dimensions (length and width) and volume measures 3 dimensions (length, width and height).</td>
<td>Envision 3-ACT Math (Topic 11) 11-3</td>
</tr>
<tr>
<td></td>
<td>If a cube has a length of 1 unit, a width of 1 unit and a height of 1 unit it is called a cubic unit. Use of concrete models and representations is needed to build a solid understanding of what a unit cube is. Relative volumes can be calculated geometrically, filling the larger box with smaller boxes or arithmetically using the given dimensions.</td>
<td>The measurement chosen depends on the size of the figure (e.g., miles, yards feet or inches). Multiple rectangular prisms can have the same volume. Volume is additive. The length, width and height of a rectangular prism shape can be used in any order regardless of which we name the dimensions. Volume is derived from the addition of multiple bases and/or the numerical computation of three edge lengths. The volume of a solid figure composed of rectangular prisms</td>
<td>11-4 11-5</td>
</tr>
</tbody>
</table>

- Tasks are with and without contexts.
- 50% of tasks involve use of $V = l \times w \times h$ and 50% of tasks involve use of $V = B \times h$.
- Tasks may require students to measure to find edge lengths to the nearest cm, mm or in.
Students need to practice concrete activities and reinforce the concept with representations of the same activities.

Calculate the volume of a right rectangular prism by packing it with unit cubes.

Up of repeated layers of the base. Students need to practice with concrete, representations and each volume formula.

Reinforce the use of commutative and associative properties employed to determine different shapes filling the same volume without gaps.

Use snap cubes, unfix cubes, isomeric paper or cube-shaped blocks to: compose two prisms with different measurements and the same cubic volume.

Can be found by adding the volumes of the rectangular prisms.

Some problems can be solved by first finding and solving one or more sub-problems, and then using the answer(s) to solve the original problem.

How do we represent the inside of three-dimensional figure?

How do you describe a three-dimensional shape or solid?

What is volume and how is it used in real life?

How does the area of rectangles relate to the volume of rectangular prisms?
Calculate the volumes of a right rectangular prism in the context of a word problem.

**SPED Strategies:**
With a partner, create flip books with pictures and examples of the different formulas.

Mathematics Reference Sheet with terms and examples of formulas should be provided.

Model how to use formulas. When needed, use explicit directions with steps and procedures enumerated.

Guide students through initial practice promoting gradual independence.

Students should have access to unit cubes to model volume of 3-dimensional figures.

Describe or explain orally and in writing, formulas to solve real world problems involving volumes of right rectangular prisms and composites of same using key, technical vocabulary in simple sentences.
Allow students to use mathematics reference sheet with step-by-step directions and how to apply formulas.

Teach in small chunks so students get extensive practice with one step at a time.

**ELL Strategies:**
With a partner, create flip books with pictures and examples of the different formulas in native language.

Check for understanding frequently (e.g., ‘show’) to benefit those who may shy away from asking questions.

Model how to use formulas. When needed, use explicit directions with steps and procedures enumerated.

Students should have access to unit cubes to model volume of 3-dimensional figures.

Vary the grouping in the classroom. For example: sometimes using small group instruction to help ELLs learn to negotiate vocabulary with classmates and other times using native language support.
to allow a student to find full proficiency of the mathematics first.

Provide sufficient wait time to allow the student to process the meanings in the different languages.

Listen intently to uncover the math content in the students’ speech.
Integrated Evidence Statements

5.C.1-3 Base explanations/reasoning on the properties of operations. Content Scope: Knowledge and skills articulated in 5.MD.5a
- Students need not use technical terms such as commutative, associative, distributive, or property.

5.C.2-3 Students need not use technical terms such as commutative, associative, distributive, or property. Content Scope: Knowledge and skills articulated in 5.NF.3, 5.NF.4a

5.C.2-4 Base explanations/reasoning on the relationship between multiplication and division. Content Scope: Knowledge and skills articulated in 5.NF.7

5.C.4-1 Base arithmetic explanations/reasoning on concrete referents such as diagrams (whether provided in the prompt or constructed by the student in her response), connecting the diagrams to a written (symbolic) method. Content Scope: Knowledge and skills articulated in 5.NF.2

5.C.4-2 Base arithmetic explanations/reasoning on concrete referents such as diagrams (whether provided in the prompt or constructed by the student in her response), connecting the diagrams to a written (symbolic) method. Content Scope: Knowledge and skills articulated in 5.NF.4b

5.C.5-1 Base explanations/reasoning on a number line diagram (whether provided in the prompt or constructed by the student in her response). Content Scope: Knowledge and skills articulated in 5.NF.2

5.C.5-2 Base explanations/reasoning on a number line diagram (whether provided in the prompt or constructed by the student in her response). Content Scope: Knowledge and skills articulated in 5.NF.4a

5.C.5-3 Base explanations/reasoning on a number line diagram (whether provided in the prompt or constructed by the student in her response). Content Scope: Knowledge and skills articulated in 5.NF.7a, 5.NF.7b

5.C.6 Base explanations/reasoning on concrete referents such as diagrams (whether provided in the prompt or constructed by the student in her response). Content Scope: Knowledge and skills articulated in 5.MD.C
### Integrated Evidence Statements

| 5.C.7-1 | Distinguish correct explanation/reasoning from that which is flawed, and – if there is a flaw in the argument – present corrected reasoning. (For example, some flawed ‘student’ reasoning is presented and the task is to correct and improve it.) Content Scope: Knowledge and skills articulated in 5.NF.5b |
| 5.C.7-2 | Distinguish correct explanation/reasoning from that which is flawed, and – if there is a flaw in the argument – present corrected reasoning. (For example, some flawed ‘student’ reasoning is presented and the task is to correct and improve it.) Content Scope: Knowledge and skills articulated in 5.NF.2 |
| 5.C.7-3 | Distinguish correct explanation/reasoning from that which is flawed, and – if there is a flaw in the argument – present corrected reasoning. (For example, some flawed ‘student’ reasoning is presented and the task is to correct and improve it.) Content Scope: Knowledge and skills articulated in 5.NF.1 |
| 5.C.7-4 | Distinguish correct explanation/reasoning from that which is flawed, and – if there is a flaw in the argument – present corrected reasoning. (For example, some flawed ‘student’ reasoning is presented and the task is to correct and improve it.) Content Scope: Knowledge and skills articulated in 4.NBT, 4.NF.A, 4.NF.B |
| | - Tasks may have scaffolding, if necessary, in order to yield a degree of difficulty appropriate to Grade 5. |
| 5.C.8-2 | Present solutions to multi-step problems in the form of valid chains of reasoning, using symbols such as equals signs appropriately (for example, rubrics award less than full credit for the presence of nonsense statements such as \(1 + 4 = 5 + 7 = 12\), even if the final answer is correct), or identify or describe errors in solutions to multi-step problems and present corrected solutions. Content Scope: Knowledge and skills articulated in 5.MD.5c |
| 5.D.1 | Solve multi-step contextual word problems with degree of difficulty appropriate to Grade 5, requiring application of knowledge and skills articulated in Type I, Sub-Claim A Evidence Statements. |
| | - Tasks may have scaffolding. |
| | - Multi-step problems must have at least 3 steps. |
| | - For purposes of assessment, the possibilities for multiplication are 1-digit x 2-digit, 1-digit x 3-digit, 2-digit x 3-digit, 2-digit x 4-digit, or 3-digit x 3-digit |
### Integrated Evidence Statements

5.D.2 Solve multi-step contextual problems with degree of difficulty appropriate to Grade 5, requiring application of knowledge and skills articulated in 4.OA, 4.NBT, 4.NF, 4.MD

- Tasks may have scaffolding, if necessary, in order to yield a degree of difficulty appropriate to Grade 5. ii) Multi-step problems must have at least 3 steps.
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<td>• Area Model</td>
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<td>• Area of Base</td>
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<td>• Attribute</td>
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<td>• Benchmark Fractions</td>
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<td>• Common Denominator</td>
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<td>• Compose</td>
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<td>• Composite Solid Figure</td>
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<td>• Cube</td>
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<td>• Cubic Units (cubic cm, cubic in, cubic ft, nonstandard cubic units)</td>
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<td>• Data</td>
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<td>• Rectangular Prism</td>
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<td>• Rectilinear Array</td>
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<td>• Representations</td>
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<td>• Rounding</td>
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<td>• Solid Figures</td>
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<td>• Unit</td>
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<td>• Unit Cube</td>
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<td>• Unit Fraction</td>
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<td>• Unlike/Like Fractions</td>
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<td>• Value</td>
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<td>• Visual Fraction Model</td>
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<td>• Volume</td>
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<td>• Width</td>
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## References & Suggested Instructional Websites

<table>
<thead>
<tr>
<th>Website/Resource</th>
<th>Description</th>
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<tbody>
<tr>
<td>Imagine Math</td>
<td><a href="https://math.imaginelearning.com/users/sign_in">https://math.imaginelearning.com/users/sign_in</a></td>
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<tr>
<td>Georgia Department of Education</td>
<td>contains exceptional tasks and curriculum support <a href="https://www.gdhportal.com">Georgia Standards</a></td>
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<tr>
<td>Teaching Channel</td>
<td><a href="https://www.teachingchannel.org">Inspirational Teaching Videos: Covering Common Core, Math, Science, English And More (teachingchannel.com)</a></td>
</tr>
<tr>
<td>Illustrative Mathematics</td>
<td>a library of tasks linked to Common Core State Standards [Illustrative Mathematics</td>
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<tr>
<td>NCTM Illuminations</td>
<td>contains activities and lessons, and virtual manipulatives organized by strand. <a href="https://www.nctm.org">Illuminations (nctm.org)</a></td>
</tr>
<tr>
<td>K-5 Math</td>
<td>site contains free math teaching resources, games, activities and journal tasks. <a href="https://www.teachervision.fen.com/math-teaching-resource">K-5 Math Teaching Resources</a></td>
</tr>
<tr>
<td>National Library of Virtual Manipulatives</td>
<td>has tutorials and virtual manipulatives for the classroom. <a href="https://nlvm.usu.edu">National Library of Virtual Manipulatives (usu.edu)</a></td>
</tr>
<tr>
<td>Inside Mathematics</td>
<td>site contains tools for teachers, classroom videos, common core resources, rubric scored student samples, problems of the day and performance tasks <a href="https://www.inside-mathematics.org">Inside Mathematics</a></td>
</tr>
<tr>
<td>Conceptual Vocabulary Cards</td>
<td>The vocabulary words are provided with a definition and visual representation. They are also available in Spanish, French and Chinese. [Inside Mathematics</td>
</tr>
<tr>
<td>Virtual Math Manipulatives</td>
<td><a href="https://docs.google.com/presentation/d/e/2PACX-1vSXrARvaYRKtX3ELH8VFNt19ZJ-Ku5uM-Dz_DoNBUjEwg8Q8gYaMaj6SzLAFSwREthIXIVBl4En-evH/pub?start=false&amp;loop=false&amp;delayms=3000&amp;slide=id.g27b693dca5_0_261">Virtual Math Manipulatives</a></td>
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BOARD APPROVED 08.18.21
# Field Trip Ideas

**MoMath/Museum of Mathematics:** Mathematics illuminates the patterns and structures all around us. The dynamic exhibits gallery, and programs will stimulate inquiry, spark curiosity, and reveal the wonders of mathematics. MoMath has innovative exhibits that will engage folks from 105 to 5 years old (and sometimes younger), but with a special emphasis on activities for 4th through 8th graders. [http://momath.org/](http://momath.org/)

**Liberty Science Center:** Student mastery of STEM (Science, Technology, Engineering, and Mathematics) has never been more important. Under the newest national standards, educators are required to instruct students in science and technology with active question-and-answer pedagogy and hands-on investigation. Liberty Science Center understands educators’ needs and offers a full portfolio of age-appropriate, curriculum-linked STEM programs suitable for preschoolers through technical school students, including pupils with special needs. [http://lsc.org/for-educators/](http://lsc.org/for-educators/)

**Discovery Times Square:** New York City’s first large-scale exhibition center presenting visitors with limited-run, educational and immersive exhibit experiences while exploring the world’s defining cultures, art, history, mathematics and events. [http://discoverymuseum.org/](http://discoverymuseum.org/)

**Great Falls National Park:** A Revolutionary Idea: Cotton & silk for clothing; locomotives for travel; paper for books & writing letters; airplanes, & and the mathematics needed for manufacturing. What do they have in common? They all came from the same place - Paterson, NJ. [http://www.nps.gov/pagr/index.htm](http://www.nps.gov/pagr/index.htm)

**The Paterson Museum:** Founded in 1925, it is owned and run by the city of Paterson and its mission is to preserve and display the industrial history Paterson. Manufacturing, finance and daily operation of this museum build a deeper understanding of the mathematics involved in the building and decline of this great city. [http://www.patersonnj.gov/department/index.php?structureid=16](http://www.patersonnj.gov/department/index.php?structureid=16)