MATHEMATICS

Grade 4: Unit 2
Multi-Digit Arithmetic & Fraction Equivalence
Course Philosophy/Description

In mathematics, students will learn to address a range of tasks focusing on the application of concepts, skills and understandings. Students will be asked to solve problems involving the key knowledge and skills for their grade level as identified by the NJSLS; express mathematical reasoning and construct a mathematical argument and apply concepts to solve model real world problems. The balanced math instructional model will be used as the basis for all mathematics instruction.

Fourth grade Mathematics consists of the following domains: Operations and Algebraic Thinking (OA), Number and Operations in Base Ten (NBT), Number and Operations-Fractions (NF), Measurement and Data (MD), and Geometry (G). In fourth grade, instructional time should focus on three critical areas: (1) developing understanding and fluency with multi-digit multiplication, and developing understanding of dividing to find quotients involving multi-digit dividends; (2) developing an understanding of fraction equivalence, addition and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers; (3) understanding that geometric figures can be analyzed and classified based on their properties, such as having parallel sides, perpendicular sides, particular angle measures, and symmetry.

1) Students generalize their understanding of place value to 1,000,000, understanding the relative sizes of numbers in each place. They apply their understanding of models for multiplication (equal-sized groups, arrays, and area models), place value, and properties of operations, in particular the distributive property, as they develop, discuss, and use efficient, accurate, and generalizable methods to compute products of multi-digit whole numbers. Depending on the numbers and the context, they select and accurately apply appropriate methods to estimate or mentally calculate products. They develop fluency with efficient procedures for multiplying whole numbers; understand and explain why the procedures work based on place value and properties of operations; and use them to solve problems. Students apply their understanding of models for division, place value, properties of operations, and the relationship of division to multiplication as they develop, discuss, and use efficient, accurate, and generalizable procedures to find quotients involving multi-digit dividends. They select and accurately apply appropriate methods to estimate and mentally calculate quotients, and interpret remainders based upon the context.

2) Students develop understanding of fraction equivalence and operations with fractions. They recognize that two different fractions can be equal (e.g., $15/9 = 5/3$), and they develop methods for generating and recognizing equivalent fractions. Students extend previous understandings about how fractions are built from unit fractions, composing fractions from unit fractions, decomposing fractions into unit fractions, and using the meaning of fractions and the meaning of multiplication to multiply a fraction by a whole number.

3) Students describe, analyze, compare, and classify two-dimensional shapes. Through building, drawing, and analyzing two-dimensional shapes, students deepen their understanding of properties of two-dimensional objects and the use of them to solve problems involving symmetry.
This ESL framework was designed to be used by bilingual, dual language, ESL and general education teachers. Bilingual and dual language programs use the home language and a second language for instruction. ESL teachers and general education or bilingual teachers may use this document to collaborate on unit and lesson planning to decide who will address certain components of the SLO and language objective. ESL teachers may use the appropriate leveled language objective to build lessons for ELLs which reflects what is covered in the general education program. In this way, whether it is a pull-out or push-in model, all teachers are working on the same Student Learning Objective connected to the New Jersey Student Learning Standards. The design of language objectives are based on the alignment of the World-Class Instructional Design Assessment (WIDA) Consortium’s English Language Development (ELD) standards with the New Jersey Student Learning Standards (NJSLS). WIDA’s ELD standards advance academic language development across content areas ultimately leading to academic achievement for English learners. As English learners are progressing through the six developmental linguistic stages, this framework will assist all teachers who work with English learners to appropriately identify the language needed to meet the requirements of the content standard. At the same time, the language objectives recognize the cognitive demand required to complete educational tasks. Even though listening and reading (receptive) skills differ from speaking and writing (expressive) skills across proficiency levels the cognitive function should not be diminished. For example, an Entering Level One student only has the linguistic ability to respond in single words in English with significant support from their home language. However, they could complete a Venn diagram with single words which demonstrates that they understand how the elements compare and contrast with each other or they could respond with the support of their home language (L1) with assistance from a teacher, para-professional, peer or a technology program.

http://www.state.nj.us/education/modelcurriculum/ela/ELLOverview.pdf
<table>
<thead>
<tr>
<th>#</th>
<th>Student Learning Objective</th>
<th>NJSLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Fluently add and subtract multi-digit whole numbers using the standard algorithm.</td>
<td>4.NBT.B.4</td>
</tr>
<tr>
<td>2</td>
<td>Multiply a whole number of up to four digits by a one-digit whole number and multiply two two-digit numbers; represent and explain calculations using equations, rectangular arrays, and area models.</td>
<td>4.NBT.B.5</td>
</tr>
<tr>
<td>3</td>
<td>Divide a whole number of up to four-digits by a one-digit divisor; represent and explain the calculation using equations, rectangular arrays, and area models.</td>
<td>4.NBT.B.6</td>
</tr>
<tr>
<td>4</td>
<td>Write and solve each equation (including any of the four operations) in order to solve multi-step word problems, using a letter to represent the unknown; interpret remainders in context and assess the reasonableness of answers using mental computation with estimation strategies.</td>
<td>4.OA.A.3</td>
</tr>
<tr>
<td>5</td>
<td>Solve real world problems with whole numbers by finding the area and perimeter of rectangles using formulas.</td>
<td>4.MD.A.3</td>
</tr>
<tr>
<td>6</td>
<td>Recognize and generate equivalent fractions and explain why they are equivalent using visual fraction models.</td>
<td>4.NF.A.1</td>
</tr>
<tr>
<td>7</td>
<td>Compare two fractions with different numerators or different denominators, recording comparison with &gt;, =, or &lt;, and justifying the conclusion using visual fraction models.</td>
<td>4.NF.A.2</td>
</tr>
<tr>
<td>8</td>
<td>Decompose a fraction into a sum of fractions with the same denominator in more than one way and record the decomposition as an equation; justify the decomposition with a visual fraction model.</td>
<td>4.NF.B.3a, 3b</td>
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<tr>
<td></td>
<td>Pacing Chart – Unit 2</td>
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<tr>
<td>9</td>
<td>Add and subtract mixed numbers with like denominators by replacing each mixed number with an equivalent fraction or improper fraction.</td>
<td>4.NF.B.3c</td>
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<tr>
<td>10</td>
<td>Solve word problems involving addition and subtraction of fractions having like denominators using visual fraction models and equations to represent the problem.</td>
<td>4.NF.B.3D</td>
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</tbody>
</table>
Research about Teaching and Learning Mathematics

Structure teaching of mathematical concepts and skills around problems to be solved (Checkly, 1997; Wood & Sellars, 1996; Wood & Sellars, 1997)

Encourage students to work cooperatively with others (Johnson & Johnson, 1975; Davidson, 1990)

Use group problem-solving to stimulate students to apply their mathematical thinking skills (Artzt & Armour-Thomas, 1992)

Students interact in ways that support and challenge one another’s strategic thinking (Artzt, Armour-Thomas, & Curcio, 2008)

Activities structured in ways allowing students to explore, explain, extend, and evaluate their progress (National Research Council, 1999)

There are three critical components to effective mathematics instruction (Shellard & Moyer, 2002):

- Teaching for conceptual understanding
- Developing children’s procedural literacy
- Promoting strategic competence through meaningful problem-solving investigations

Teachers should be:

- Demonstrating acceptance and recognition of students’ divergent ideas
- Challenging students to think deeply about the problems they are solving, extending thinking beyond the solutions and algorithms required to solve the problem
- Influencing learning by asking challenging and interesting questions to accelerate students’ innate inquisitiveness and foster them to examine concepts further
- Projecting a positive attitude about mathematics and about students’ ability to “do” mathematics

Students should be:

- Actively engaging in “doing” mathematics
- Solving challenging problems
- Investigating meaningful real-world problems
- Making interdisciplinary connections
- Developing an understanding of mathematical knowledge required to “do” mathematics and connect the language of mathematical ideas with numerical representations
- Sharing mathematical ideas, discussing mathematics with one another, refining and critiquing each other’s ideas and understandings
- Communicating in pairs, small group, or whole group presentations
- Using multiple representations to communicate mathematical ideas
- Using connections between pictures, oral language, written symbols, manipulative models, and real-world situations
- Using technological resources and other 21st century skills to support and enhance mathematical understanding
Mathematics is not a stagnant field of textbook problems; rather, it is a dynamic way of constructing meaning about the world around us, generating knowledge and understanding about the real world every day. Students should be metaphorically rolling up their sleeves and “doing mathematics” themselves, not watching others do mathematics for them or in front of them. (Protheroe, 2007)

**Balanced Mathematics Instructional Model**

Balanced math consists of three different learning opportunities; guided math, shared math, and independent math. Ensuring a balance of all three approaches will build conceptual understanding, problem solving, computational fluency, and procedural fluency. Building conceptual understanding is the focal point of developing mathematical proficiency. Students should frequently work on rigorous tasks, talk about the math, explain their thinking, justify their answer or process, build models with graphs or charts or manipulatives, and use technology.

When balanced math is used in the classroom it provides students opportunities to:

- solve problems
- make connections between math concepts and real-life situations
- communicate mathematical ideas (orally, visually and in writing)
- choose appropriate materials to solve problems
- reflect and monitor their own understanding of the math concepts
- practice strategies to build procedural and conceptual confidence

Teacher builds conceptual understanding by modeling through demonstration, explicit instruction, and think alouds, as well as guiding students as they practice math strategies and apply problem solving strategies. (whole group or small group instruction)

Teacher and students practice mathematics processes together through interactive activities, problem solving, and discussion. (whole group or small group instruction)

Students practice math strategies independently to build procedural and computational fluency. Teacher assesses learning and reteaches as necessary. (whole group instruction, small group instruction, or centers)
## Effective Pedagogical Routines/Instructional Strategies

<table>
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<tr>
<th>Collaborative Problem Solving</th>
<th>Analyze Student Work</th>
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<tr>
<td>Connect Previous Knowledge to New Learning</td>
<td>Identify Student’s Mathematical Understanding</td>
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<tr>
<td>Making Thinking Visible</td>
<td>Identify Student’s Mathematical Misunderstandings</td>
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<tr>
<td>Develop and Demonstrate Mathematical Practices</td>
<td>Interviews</td>
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<tr>
<td>Inquiry-Oriented and Exploratory Approach</td>
<td>Role Playing</td>
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<tr>
<td>Multiple Solution Paths and Strategies</td>
<td>Diagrams, Charts, Tables, and Graphs</td>
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<tr>
<td>Use of Multiple Representations</td>
<td>Anticipate Likely and Possible Student Responses</td>
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<td>Explain the Rationale of your Math Work</td>
<td>Collect Different Student Approaches</td>
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<td>Quick Writes</td>
<td>Multiple Response Strategies</td>
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<tr>
<td>Pair/Trio Sharing</td>
<td>Asking Assessing and Advancing Questions</td>
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<tr>
<td>Turn and Talk</td>
<td>Revoicing</td>
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<td>Charting</td>
<td>Marking</td>
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<tr>
<td>Gallery Walks</td>
<td>Recapping</td>
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<td>Small Group and Whole Class Discussions</td>
<td>Challenging</td>
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<tr>
<td>Student Modeling</td>
<td>Pressing for Accuracy and Reasoning</td>
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<td></td>
<td>Maintain the Cognitive Demand</td>
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</tbody>
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# Educational Technology

## Standards

| 8.1.5.A.1, 8.1.5.A.2, 8.1.5.D.3 |

- **Technology Operations and Concepts**
  - Select and use the appropriate digital tools and resources to accomplish a variety of tasks including problem solving.
  
  **Example:** A calculator will be used to determine if responses are correct when the students are adding, subtracting, multiplying, and dividing whole numbers.
  
  - Format a document using a word processing application to enhance text and include graphics, symbols, and/or pictures.
  
  **Example:** Microsoft Word will be utilized to create fractional visual models when decomposing fractions in more than one way.

- **Digital Citizenship**
  - Demonstrate an understanding of the need to practice cyber safety, cyber security, and cyber ethics when using technologies and social media.

  **Example:** A cyber safety chart will be posted by the computer center in the classroom so students are reminded of proper behavior when they are using the computers to complete assignments on programs such as Successmaker.

  
  or
  
### Career Ready Practices

Career Ready Practices describe the career-ready skills that all educators in all content areas should seek to develop in their students. They are practices that have been linked to increase college, career, and life success. Career Ready Practices should be taught and reinforced in all career exploration and preparation programs with increasingly higher levels of complexity and expectation as a student advances through a program of study.

<table>
<thead>
<tr>
<th>CRP2. Apply appropriate academic and technical skills.</th>
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<tbody>
<tr>
<td>Career-ready individuals readily access and use the knowledge and skills acquired through experience and education to be more productive. They make connections between abstract concepts with real-world applications, and they make correct insights about when it is appropriate to apply the use of an academic skill in a workplace situation.</td>
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</table>

**Example:** Students will apply prior knowledge when solving real world problems. Students will make sound judgements about the use of specific tools and use tools to explore and deepen understanding of concepts. Rulers can be used for measuring when finding the area and perimeter of figures and virtual manipulatives to represent fractional pieces can be used as tools to develop understanding in this unit.

<table>
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<tr>
<th>CRP4. Communicate clearly and effectively and with reason.</th>
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<tr>
<td>Career-ready individuals communicate thoughts, ideas, and action plans with clarity, whether using written, verbal, and/or visual methods. They communicate in the workplace with clarity and purpose to make maximum use of their own and others’ time. They are excellent writers; they master conventions, word choice, and organization, and use effective tone and presentation skills to articulate ideas. They are skilled at interacting with others; they are active listeners and speak clearly and with purpose. Career-ready individuals think about the audience for their communication and prepare accordingly to ensure the desired outcome.</td>
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</table>

**Example:** Students will communicate precisely using clear definitions and provide carefully formulated explanations when constructing arguments. Students will communicate and defend mathematical reasoning using objects, drawings, diagrams, and/or actions. Students will ask probing questions to clarify or improve arguments. In this unit, visual fraction models are used when comparing and decomposing fractions and rectangular arrays, area models, and equations are used when multiplying and dividing whole numbers.

<table>
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<tr>
<th>CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.</th>
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<tr>
<td>Career-ready individuals readily recognize problems in the workplace, understand the nature of the problem, and devise effective plans to solve the problem. They are aware of problems when they occur and take action quickly to address the problem; they thoughtfully investigate the root cause of the problem prior to introducing solutions. They carefully consider the options to solve the problem. Once a solution is agreed upon, they follow through to ensure the problem is solved, whether through their own actions or the actions of others.</td>
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</table>
Career Ready Practices

Example: Students will understand the meaning of a problem and look for entry points to its solution. They will analyze information, make conjectures, and plan a solution pathway. Students will monitor and evaluate progress and change course as necessary. In this unit, students will monitor their creation of area models and visual fraction models to make adjustments based on the reasonableness of the values represented.

- CRP12. Work productively in teams while using cultural global competence.
  Career-ready individuals positively contribute to every team, whether formal or informal. They apply an awareness of cultural difference to avoid barriers to productive and positive interaction. They find ways to increase the engagement and contribution of all team members. They plan and facilitate effective team meetings.

Example: Students will work collaboratively in groups to solve mathematical tasks. Students will listen to or read the arguments of others and ask probing questions to clarify and improve arguments. In this unit, students will justify decomposition, equivalence, and comparison of fractions with visual fraction models and utilize estimation strategies to determine reasonableness when computing with whole numbers.
WIDA Proficiency Levels

At the given level of English language proficiency, English language learners will process, understand, produce or use:

<table>
<thead>
<tr>
<th>Level</th>
<th>Description</th>
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<tbody>
<tr>
<td>6- Reaching</td>
<td>- Specialized or technical language reflective of the content areas at grade level&lt;br&gt;- A variety of sentence lengths of varying linguistic complexity in extended oral or written discourse as required by the specified grade level&lt;br&gt;- Oral or written communication in English comparable to proficient English peers</td>
</tr>
<tr>
<td>5- Bridging</td>
<td>- Specialized or technical language of the content areas&lt;br&gt;- A variety of sentence lengths of varying linguistic complexity in extended oral or written discourse, including stories, essays or reports&lt;br&gt;- Oral or written language approaching comparability to that of proficient English peers when presented with grade level material.</td>
</tr>
<tr>
<td>4- Expanding</td>
<td>- Specific and some technical language of the content areas&lt;br&gt;- A variety of sentence lengths of varying linguistic complexity in oral discourse or multiple, related sentences or paragraphs&lt;br&gt;- Oral or written language with minimal phonological, syntactic or semantic errors that may impede the communication, but retain much of its meaning, when presented with oral or written connected discourse, with sensory, graphic or interactive support</td>
</tr>
<tr>
<td>3- Developing</td>
<td>- General and some specific language of the content areas&lt;br&gt;- Expanded sentences in oral interaction or written paragraphs&lt;br&gt;- Oral or written language with phonological, syntactic or semantic errors that may impede the communication, but retain much of its meaning, when presented with oral or written, narrative or expository descriptions with sensory, graphic or interactive support</td>
</tr>
<tr>
<td>2- Beginning</td>
<td>- General language related to the content area&lt;br&gt;- Phrases or short sentences&lt;br&gt;- Oral or written language with phonological, syntactic, or semantic errors that often impede the communication when presented with one to multiple-step commands, directions, or a series of statements with sensory, graphic or interactive support</td>
</tr>
<tr>
<td>1- Entering</td>
<td>- Pictorial or graphic representation of the language of the content areas&lt;br&gt;- Words, phrases or chunks of language when presented with one-step commands directions, WH-, choice or yes/no questions, or statements with sensory, graphic or interactive support</td>
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</tbody>
</table>
# Language Development Supports For English Language Learners To Increase Comprehension and Communication Skills

## Environment
- Welcoming and stress-free
- Respectful of linguistic and cultural diversity
- Honors students’ background knowledge
- Sets clear and high expectations
- Includes routines and norms
- Is thinking-focused vs. answer-seeking
- Offers multiple modalities to engage in content learning and to demonstrate understanding
- Includes explicit instruction of specific language targets
- Provides participation techniques to include all learners
- Integrates learning centers and games in a meaningful way
- Provides opportunities to practice and refine receptive and productive skills in English as a new language
- Integrates meaning and purposeful tasks/activities that:
  - Are accessible by all students through multiple entry points
  - Are relevant to students’ lives and cultural experiences
  - Build on prior mathematical learning
  - Demonstrate high cognitive demand
  - Offer multiple strategies for solutions
  - Allow for a language learning experience in addition to content

## Sensory Supports*
- Real-life objects (realia) or concrete objects
- Physical models
- Manipulatives
- Pictures & photographs
- Visual representations or models such as diagrams or drawings
- Videos & films
- Newspapers or magazines
- Gestures
- Physical movements
- Music & songs

## Graphic Supports*
- Graphs
- Charts
- Timelines
- Number lines
- Graphic organizers
- Graphing paper

## Interactive Supports*
- In a whole group
- In a small group
- With a partner such as Turn-and-Talk
- In pairs as a group (first, two pairs work independently, then they form a group of four)
- In triads
- Cooperative learning structures such as Think-Pair-Share
- Interactive websites or software
- With a mentor or coach

## Verbal and Textual Supports
- Labeling
- Students’ native language
- Modeling
- Repetitions
- Paraphrasing
- Summarizing
- Guiding questions
- Clarifying questions
- Probing questions
- Leveled questions such as What? When? Where? How? Why?
- Questioning prompts & cues
- Word Banks
- Sentence starters
- Sentence frames
- Discussion frames
- Talk moves, including Wait Time

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BUILDING EQUITY IN YOUR TEACHING PRACTICE

How do the essential questions highlight the connection between the big ideas of the unit and equity in your teaching practice?

**CONTENT INTEGRATION**
Teachers use examples and content from a variety of cultures & groups.

This unit / lesson is connected to other topics explored with students.
There are multiple viewpoints reflected in the content of this unit / lesson.
The materials and resources are reflective of the diverse identities and experiences of students.
The content affirms students, as well as exposes them to experiences other than their own.

**KNOWLEDGE CONSTRUCTION**
Teachers help students understand how knowledge is created and influenced by cultural assumptions, perspectives & biases.

This unit / lesson provides context to the history of privilege and oppression.
This unit / lesson addresses power relationships.
This unit / lesson helps students to develop research and critical thinking skills.
This curriculum creates windows and mirrors* for students.

**PREJUDICE REDUCTION**
Teachers implement lessons and activities to assert positive images of ethnic groups & improve intergroup relations.

This unit / lesson helps students question and unpack biases & stereotypes.
This unit / lesson helps students examine, research and question information and sources.
The curriculum encourages discussion and understanding about the groups of people being represented.
This unit / lesson challenges dominant perspectives.

**EQUITABLE PEDAGOGY**
Teachers modify techniques and methods to facilitate the academic achievement of students from diverse backgrounds.

The instruction has been modified to meet the needs of each student.
Students feel respected and their cultural identities are valued.
Additional supports have been provided for students to become successful and independent learners.
Opportunities are provided for student to reflect on their learning and provide feedback.

**EMPOWERING SCHOOL CULTURE**
Using the other four dimensions to create a safe and healthy educational environment for all.

There are opportunities for students to connect with the community.
My classroom is welcoming and supportive for all students?
I am aware of and sensitive to the needs of my students and their families.
There are effective parent communication systems established. Parents can talk to me about issues as they arise in my classroom.

Culturally Relevant Pedagogy Examples

- **Present new concepts using student vocabulary. Use student diction to capture attention and build understanding before using academic terms.**
  
  **Example:** Work with students to create a variety of vocabulary sorting and matching games that relate student diction to vocabulary words in this unit. Students can work in teams or individually to play these games for approximately 10-15 minutes each week.

- **Bring in Guest Speakers: Invite guest speakers who can add context to your lesson and speak from a specific culture’s general perspective.**
  
  **Example:** Have an architect or a building contractor visit your class to speak about the importance of area, perimeter, fractions and measurement when designing/building a structure.

- **Everyone has a Voice: Create a classroom environment where students know that their contributions are expected and valued.**
  
  **Example:** Norms for sharing are established that communicate a growth mindset for mathematics. All students are capable of expressing mathematical thinking and contributing to the classroom community. Students learn new ways of looking at problem solving by working with and listening to each other.

- **Integrate Relevant Word Problems: Contextualize equations using word problems that reference student’s interests and cultures.**
  
  **Example:** Create and use word problems that students will be able to relate to, have prior knowledge of, includes their interests, current events and/or are relevant to real-world situations. Using content the students can relate to adds meaning, value and connection. The following link provides you with a variety of word problems that are current, relevant to real-world and student interests.

  [https://www.yummymath.com/](https://www.yummymath.com/)
Differentiated Instruction

Accommodate Based on Students Individual Needs: Strategies

<table>
<thead>
<tr>
<th>Time/General</th>
<th>Processing</th>
<th>Comprehension</th>
<th>Recall</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Extra time for assigned tasks</td>
<td>• Extra Response time</td>
<td>• Precise processes for balanced math instructional model</td>
<td>• Teacher-made checklist</td>
</tr>
<tr>
<td>• Adjust length of assignment</td>
<td>• Have students verbalize steps</td>
<td>• Short manageable tasks</td>
<td>• Use visual graphic organizers</td>
</tr>
<tr>
<td>• Timeline with due dates for reports and projects</td>
<td>• Repeat, clarify or reword directions</td>
<td>• Brief and concrete directions</td>
<td>• Reference resources to promote independence</td>
</tr>
<tr>
<td>• Communication system between home and school</td>
<td>• Mini-breaks between tasks</td>
<td>• Provide immediate feedback</td>
<td>• Visual and verbal reminders</td>
</tr>
<tr>
<td>• Provide lecture notes/outline</td>
<td>• Provide a warning for transitions</td>
<td>• Small group instruction</td>
<td>• Graphic organizers</td>
</tr>
<tr>
<td>• Partnering</td>
<td>• Partnering</td>
<td>• Emphasize multi-sensory learning</td>
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</tbody>
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<thead>
<tr>
<th>Assistive Technology</th>
<th>Tests/Quizzes/Grading</th>
<th>Behavior/Attention</th>
<th>Organization</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Computer/whiteboard</td>
<td>• Extended time</td>
<td>• Consistent daily structured routine</td>
<td>• Individual daily planner</td>
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<tr>
<td>• Tape recorder</td>
<td>• Study guides</td>
<td>• Simple and clear classroom rules</td>
<td>• Display a written agenda</td>
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<tr>
<td>• Video Tape</td>
<td>• Shortened tests</td>
<td>• Frequent feedback</td>
<td>• Note-taking assistance</td>
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<td></td>
<td>• Read directions aloud</td>
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<td>• Color code materials</td>
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Differentiated Instruction

Accommodate Based on Content Specific Needs

- Chart academic vocabulary with visual representations
- Anchor charts to model strategies
- Graphic organizers (examples include: venn diagram, 4 square graphic organizer for math word problems, K-W-L etc.)
- Translation dictionary
- Teacher modeling
- Students can utilize math journals to write notes, copy solution steps, and translate terms and key vocabulary.
- Grid paper to create accurate mathematical models and verify the area and perimeter of figures
- Utilize technological programs which provide verbal and visual instruction in native and/or second language.
- Use interactive technology to improve multiplication and division fact fluency and accuracy.
- Fraction equivalency cubes
- Fraction towers
- Use of Cuisenaire Rods and linking cubes to show the decomposing of fractions
- Fraction cubes, strips, tiles, and circles to show the size of fractional pieces
- Place value chart
- Calculators to verify computational accuracy
- Create and provide a list of properties of operations as strategies to multiply and divide whole numbers
Interdisciplinary Connections

Model interdisciplinary thinking to expose students to other disciplines.

Social Studies Connection:

Japan Trip: 6.1.4.B.1
- Students will be planning a trip through Japan and arrive at the airport in Tokyo. They should be visiting important landmarks and learn about the culture during their trip. Students can learn about Japan at http://kids.nationalgeographic.com/explore/countries/japan/ and http://www.activityvillage.co.uk/japan

General Grant’s March: 6.1.4.D.4
- Students will learn about General Ulysses S. Grant. They will investigate his role in the Civil War and as a President of the United States. Find more information at http://www.ducksters.com/biography/uspresidents/ulyssessgrant.php and http://usa4kids.com/presidents/Ulysses_S_Grant.html

Science Connection:

City Farmers: 2-LS2-1
- Students will be working on a task in which a garden is being created on the roof of a school building in a city. They will learn about how much space each vegetable needs to grow and how many can be planted within a set area. Learn more about what is needed to start a garden at http://extension.illinois.edu/firstgarden/fundamentals/index.cfm

ELA Connection:

Pizza Party: W.4.3.A,B,C,D
- Students will be writing story problems and using their created fractional representations to explain their responses. They will focus on vocabulary, mechanics, and grammar in effective writing.
### Enrichment

**What is the purpose of Enrichment?**

- The purpose of enrichment is to provide extended learning opportunities and challenges to students who have already mastered, or can quickly master, the basic curriculum. Enrichment gives the student more time to study concepts with greater depth, breadth, and complexity.
- Enrichment also provides opportunities for students to pursue learning in their own areas of interest and strengths.
- Enrichment keeps advanced students engaged and supports their accelerated academic needs.
- Enrichment provides the most appropriate answer to the question, “What do you do when the student already knows it?”

<table>
<thead>
<tr>
<th>Enrichment is…</th>
<th>Enrichment is not…</th>
</tr>
</thead>
<tbody>
<tr>
<td>Planned and purposeful</td>
<td>Just for gifted students (some gifted students may need intervention in some areas just as some other students may need frequent enrichment)</td>
</tr>
<tr>
<td>Different, or differentiated, work – not just more work</td>
<td>Worksheets that are more of the same (busywork)</td>
</tr>
<tr>
<td>Responsive to students’ needs and situations</td>
<td>Random assignments, games, or puzzles not connected to the content areas or areas of student interest</td>
</tr>
<tr>
<td>A promotion of high-level thinking skills and making connections within content</td>
<td>Extra homework</td>
</tr>
<tr>
<td>The ability to apply different or multiple strategies to the content</td>
<td>A package that is the same for everyone</td>
</tr>
<tr>
<td>The ability to synthesize concepts and make real world and cross-curricular connections</td>
<td>Thinking skills taught in isolation</td>
</tr>
<tr>
<td>Elevated contextual complexity</td>
<td>Unstructured free time</td>
</tr>
<tr>
<td>Sometimes independent activities, sometimes direct instruction</td>
<td></td>
</tr>
<tr>
<td>Inquiry based or open ended assignments and projects</td>
<td></td>
</tr>
<tr>
<td>Using supplementary materials in addition to the normal range of resources</td>
<td></td>
</tr>
<tr>
<td>Choices for students</td>
<td></td>
</tr>
<tr>
<td>Tiered/Multi-level activities with flexible groups (may change daily or weekly)</td>
<td></td>
</tr>
</tbody>
</table>
## Assessments

### Required District/State Assessments
- Unit Assessments
- PARCC
- SGO Assessments

### Suggested Formative/Summative Classroom Assessments
- Describe Learning Vertically
- Identify Key Building Blocks
- Make Connections (between and among key building blocks)
- Short/Extended Constructed Response Items
- Multiple-Choice Items (where multiple answer choices may be correct)
- Drag and Drop Items
- Use of Equation Editor
- Quizzes
- Journal Entries/Reflections/Quick-Writes
- Accountable talk
- Projects
- Portfolio
- Observation
- Graphic Organizers/Concept Mapping
- Presentations
- Role Playing
- Teacher-Student and Student-Student Conferencing
- Homework
### New Jersey Learning Standards

**4.NBT.B.4**
Fluently add and subtract multi-digit whole numbers using the standard algorithm. *(Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.)*

**4.NBT.B.5**
Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. *(Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.)*

**4.NBT.B.6**
Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. *(Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.)*

**4.OA.A.3**
Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

**4.MD.A.3**
Apply the area and perimeter formulas for rectangles in real world and mathematical problems. *For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.*

**4.NF.A.1**
Explain why a fraction $a/b$ is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions. *(Grade 4 expectations in this domain are limited to denominators of 2, 3, 4, 5, 6, 8, 10, 12 and 100.)*
New Jersey Student Learning Standards

4.NF.A.2  
Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model. [Grade 4 expectations in this domain are limited to denominators of 2, 3, 4, 5, 6, 8, 10, 12 and 100.]

4.NF.B.3  
Understand a fraction \( \frac{a}{b} \) with \( a > 1 \) as a sum of fractions \( \frac{1}{b} \). [Grade 4 expectations in this domain are limited to denominators of 2, 3, 4, 5, 6, 8, 10, 12 and 100.]

4.NF.B.3a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.

4.NF.B.3b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. Examples: \( \frac{3}{8} = \frac{1}{8} + \frac{1}{8} + \frac{1}{8} ; \frac{3}{8} = \frac{1}{8} + \frac{2}{8} ; 2 \frac{1}{8} = 1 + \frac{1}{8} = \frac{8}{8} + \frac{8}{8} + \frac{1}{8} \).

4.NF.B.3c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using the properties of operations and the relationship between addition and subtraction.

4.NF.B.3d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.
<table>
<thead>
<tr>
<th>Mathematical Practices</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Make sense of problems and persevere in solving them.</td>
</tr>
<tr>
<td>2. Reason abstractly and quantitatively.</td>
</tr>
<tr>
<td>3. Construct viable arguments and critique the reasoning of others.</td>
</tr>
<tr>
<td>4. Model with mathematics.</td>
</tr>
<tr>
<td>5. Use appropriate tools strategically.</td>
</tr>
<tr>
<td>6. Attend to precision.</td>
</tr>
<tr>
<td>7. Look for and make use of structure.</td>
</tr>
<tr>
<td>8. Look for and express regularity in repeated reasoning.</td>
</tr>
</tbody>
</table>
Grade: Four  
Unit: 2 (Two)  
Topic: Multi-digit Arithmetic & Fraction Equivalence

**NJSLS:**  
4.NBT.B.4, 4.NBT.B.5, 4.NBT.B.6, 4.OA.A.3, 4.MD.A.3, 4.NF.A.1, 4.NF.A.2, 4.NF.B.3a, 3b, 4.NF.B.3c, 4.NF.B.3d

**Unit Focus:**  
- Use place value understanding and properties of operations to perform multi-digit arithmetic  
- Use the four operations with whole numbers to solve problems  
- Solve problems involving measurement and conversion of measurements  
- Extend understanding of fraction equivalence and ordering  
- Build fractions from unit fractions

**New Jersey Student Learning Standard:**  
4.NBT.B.4: Fluently add and subtract multi-digit whole numbers using the standard algorithm. [*Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.*]

**Student Learning Objective 1:** Fluently add and subtract multi-digit whole numbers using the standard algorithm.

**Modified Student Learning Objectives/Standards:**  
M.EE.4.NBT.4: Add and subtract two-digit whole numbers.

<table>
<thead>
<tr>
<th>MPs</th>
<th>Evidence Statement Key/ Clarifications</th>
<th>Skills, Strategies &amp; Concepts</th>
<th>Essential Understandings/ Questions (Accountable Talk)</th>
<th>Tasks/Activities</th>
</tr>
</thead>
<tbody>
<tr>
<td>MP 7</td>
<td>4.NBT.4-1</td>
<td>Recognize the need for regrouping and not just subtracting the smaller digit from the larger one.</td>
<td>What strategies can we use to help us make sense of a written algorithm?</td>
<td>Make Sense of an Algorithm</td>
</tr>
<tr>
<td>MP 8</td>
<td>• The given addends are such as to require an efficient/standard algorithm (e.g., 7263 + 4875). Addends in the task do not suggest any obvious ad hoc or mental strategy (as</td>
<td>Utilize grid paper to line up similar place values when adding and subtracting.</td>
<td>How can we combine hundreds, tens, and ones in two or more numbers efficiently?</td>
<td>Reality Checking</td>
</tr>
<tr>
<td></td>
<td>Fluency refers to accuracy, efficiency (using a reasonable amount of steps and</td>
<td>What strategies can we use to help us make sense of a written algorithm?</td>
<td></td>
<td>Beehive Adventure</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Filling the Auditorium</td>
</tr>
</tbody>
</table>
would be present for example in a case such as 16,999 + 3,501).
- Tasks do not have a context and are not timed.
- Grade 4 expectations in CCSSM are limited to whole numbers less than or equal to 1,000,000; for purposes of assessment, both of the given numbers should have 4 digits.
- Tasks are not timed.

4.NBT.4-2
- The given subtrahend and minuend are such as to require an efficient/standard algorithm (e.g., 7263 – 4875 or 7406 – 4637). The subtrahend and minuend do not suggest any obvious ad hoc or mental strategy (as would be present for example in a case such as 7300 – 6301).
- Tasks do not have a context and are not timed.

<table>
<thead>
<tr>
<th>times</th>
<th>and flexibility (variety of strategies learned previously if needed).</th>
</tr>
</thead>
<tbody>
<tr>
<td>Explain why algorithms work with the use of multiples of 10, place value, and diagrams of arrays and area.</td>
<td></td>
</tr>
<tr>
<td>Computation involves taking apart and combining numbers using a variety of approaches.</td>
<td></td>
</tr>
<tr>
<td>Flexible methods of computation involve grouping numbers in strategic ways: partial sum, regrouping, and trade first.</td>
<td></td>
</tr>
<tr>
<td>One could use an alternate algorithm to check the answer to a problem.</td>
<td></td>
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</tbody>
</table>

**SPED Strategies:**
- In small groups students can practice adding and subtracting facts by using bundles, chips and other resources.
- Use of grid paper to assist students with lining up digits and emphasize place value and the meaning of each of the digits.
- Provide flashcards and or the opportunity to practice basic facts on the computer.
- Start with a students’ understanding of a certain strategy, and then make intentional connections for the student to

<table>
<thead>
<tr>
<th>value</th>
<th>determined by the place of its digits.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Addition and subtraction algorithms are abbreviations or summaries of the connection between math drawings and written numerical work.</td>
<td></td>
</tr>
<tr>
<td>Why does it help to know inverse relationships?</td>
<td></td>
</tr>
</tbody>
</table>

**Computation algorithm**
- A set of predefined steps applicable to a class of problems that gives the correct result in every case when the steps are carried out correctly.

**Computation strategy**
- Purposeful manipulations that may be chosen for specific problems, may not have a fixed order, and may be aimed at converting one problem into another.

### How Much Liquid
- If I Had a Million Dollars?
- Japan Trip
- Destination Utah
- Comparing Manti and La Sal Mountain Range Elevation
<table>
<thead>
<tr>
<th></th>
<th>Grade 4 expectations in CCSSM are limited to whole numbers less than or equal to 1,000,000; for purposes of assessment, both of the given numbers should have 4 digits.</th>
<th>the standard algorithm. This allows the student to gain understanding of the algorithm rather than just memorize certain steps to follow.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>ELL Strategies:</strong></td>
<td><strong>Sequence orally the steps needed to add and subtract two multi-digit whole numbers in L1 (student’s native language) and/or use gestures, examples, and selected, technical words.</strong></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Have students peer tutor classmates who are having difficulties applying standard algorithms in addition and subtraction.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Use technology to practice adding and subtracting single digit numbers and use grid paper to assist students with lining up digits.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Explain directions in L1 (student’s native language) to ensure understanding of the task.</td>
<td></td>
</tr>
</tbody>
</table>
New Jersey Student Learning Standard:
4.NBT.B.5: Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. [Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.]

Student Learning Objective 2: Multiply a whole number of up to four digits by a one-digit whole number and multiply two two-digit numbers; represent and explain calculations using equations, rectangular arrays, and area models.

Modified Student Learning Objectives/Standards: N/A

<table>
<thead>
<tr>
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<th>Tasks/Activities</th>
</tr>
</thead>
<tbody>
<tr>
<td>MP 7</td>
<td>4.NBT.5-1</td>
<td>Utilize the distributive property to enhance the importance of place value and to provide flexibility in decomposing numbers by their place values.</td>
<td>The distributive property allows numbers to be decomposed into base ten units and then combined.</td>
<td>School Store</td>
</tr>
<tr>
<td></td>
<td>4.NBT.5-2</td>
<td></td>
<td>Various strategies and representations (algebraic, visual, manipulatives) can be used to multiply multi-digit number and will result in the same product.</td>
<td>At the Circus</td>
</tr>
<tr>
<td></td>
<td>4.C.1-1</td>
<td>Base Ten Blocks: 25 x 24 =</td>
<td>How can we effectively explain our mathematical thinking and reasoning to others?</td>
<td>Multiplication Strategies</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(20 x 20) + (20 x 5) + (4 x 20) + (4 x 5)=</td>
<td>College Basketball Attendance</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>400 + 100 + 80 + 20 = 600</td>
<td>Additional Activities: Multiply using the Distributive Property</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Equation: 5 x 241= 5(200+40+1)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Use base ten blocks, matrix models, area models, partitioning, and compensation strategies when multiplying whole numbers.</td>
<td></td>
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</tr>
</tbody>
</table>

Utilize the distributive property to enhance the importance of place value and to provide flexibility in decomposing numbers by their place values.

Base Ten Blocks: 25 x 24 = (20 x 20) + (20 x 5) + (4 x 20) + (4 x 5) = 400 + 100 + 80 + 20 = 600

Equation: 5 x 241= 5(200+40+1)

Use base ten blocks, matrix models, area models, partitioning, and compensation strategies when multiplying whole numbers.

The distributive property allows numbers to be decomposed into base ten units and then combined.

Various strategies and representations (algebraic, visual, manipulatives) can be used to multiply multi-digit number and will result in the same product.

How can we effectively explain our mathematical thinking and reasoning to others?

What patterns do we notice when we are multiplying whole numbers that can help us multiply more effectively?
example, use $4 + 3 \times 2$ rather than $4 + (3 \times 2)$.

### Matrix Model: $25 \times 24 = $

![Matrix Model Diagram]

### Area model: $16 \times 14 = $

![Area Model Diagram]

### Rectangular Array: $6 \times 18 = $

![Rectangular Array Diagram]

Multiplication and repeated addition can be used interchangeably.

The commutative and associative properties of multiplication ensure flexibility in computations with whole numbers and provide justifications for sequences of computations with them.
<table>
<thead>
<tr>
<th>Activity</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Use words and diagrams to explain thinking while using the strategies.</td>
<td></td>
</tr>
<tr>
<td>Address the ease of accurate use (versus error proneness). The algorithm can be used reasonably easily and does not have a high frequency of error in use.</td>
<td></td>
</tr>
<tr>
<td>Address transparency (versus opacity). What do the steps of the algorithm mean mathematically and why do they advance us toward the problem solution are clearly visible?</td>
<td></td>
</tr>
<tr>
<td>Eliminate using “placeholder” because in the multiplication algorithm, the 0 does more than just hold a place; it shows the value of the ones place is 0 in multiplication by tens.</td>
<td></td>
</tr>
<tr>
<td>Practice partial products algorithms to reinforce the understanding of place value.</td>
<td></td>
</tr>
<tr>
<td>Explain the calculation by referring to the model used.</td>
<td></td>
</tr>
<tr>
<td>Correct common error of “stacking” the factors and multiplying down the columns. This algorithm mirrors algorithms used for multi-digit addition and subtraction.</td>
<td></td>
</tr>
</tbody>
</table>
Example: Common error

\[
\begin{align*}
49 & \times 40 = 160 \\
9 & \times 0 = 0 \\
4 \times 4 & = 16
\end{align*}
\]

**SPED Strategies:**
Let students use models and gestures to calculate and explain. For example, a student searching to define “multiplication” may model groups of 6 with drawings or concrete objects and write the number sentence to match.

Students may use base ten blocks, area models, partitioning, compensation strategies, etc. when multiplying.

Use songs, rhymes, or rhythms to help students remember key concepts and use direct instruction for vocabulary with visuals or concrete representations.

Adjust numbers in calculations to suit learner’s levels. For example, change (7896 multiplied by 2) to (12 multiplied by 2).

**ELL Strategies:**
Support oral or written response with sentence frames, such as “______ is hundreds, tens, and ones.”
New Jersey Student Learning Standard:
4.NBT.B.6: Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. [Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.]

Student Learning Objective 3: Divide a whole number of up to four-digits by a one-digit divisor; represent and explain the calculation using equations, rectangular arrays, and area models

Modified Student Learning Objectives/Standards: N/A

<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>MP 7</td>
<td>4.NBT.6-1</td>
<td>Find the greatest multiple less than a given number. For example, when dividing 50 by 6, the greatest multiple less than 50 would be 48 because 6 x 8=48.</td>
<td>There are two common situations where division may be used; fair sharing (given the total amount and the number of</td>
<td>Packaging Cupcakes</td>
</tr>
<tr>
<td>MP 8</td>
<td>4.NBT.6-2</td>
<td></td>
<td></td>
<td>Dividing Resources</td>
</tr>
<tr>
<td></td>
<td>Tasks do not have a context.</td>
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<td></td>
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</tr>
</tbody>
</table>

Students may use base ten blocks, area models, partitioning, compensation strategies, etc. when multiplying.

In L1 (student’s native language) use songs, rhymes, or rhythms to help students remember key concepts

Orally, and in writing, students show how to multiply whole numbers and explain the calculation by using equations, rectangular arrays, and/or area models in L1 (student’s native language) and/or use gestures, examples, and selected, technical words.
Tasks may include remainders of 0 in no more than 20% of the tasks.

4.C.1-2
- Students need not use technical terms such as commutative, associative, distributive, or property.
- Tasks do not have a context.
- Unneeded parentheses should not be used. For example, use $4 + 3 \times 2$ rather than $4 + (3 \times 2)$.

4.C.2
- Base explanations/reasoning on the relationship between multiplication and division.
- Tasks do not have a context.

Division can be explored through various strategies, such as an open array, rectangular array, or area model, and decomposing numbers using the distributive property, using base 10 blocks, place value, and multiplication.

Use words and diagrams to explain thinking while using strategies.

Identify the various problem types that require division, such as fair sharing or measurement.

Simplify division of numbers ending with zeroes by eliminating the same number of zeroes from the dividend and the divisor.

Explain the calculation by referring to the model used.

Standard algorithm for division is taught in 6th grade, but when referring to how many times a divisor goes into dividend place value has to be emphasized avoiding statements such as “How many times does 2 go into 2?” when dividing $24 \div 2$ =.

equal parts) and measurement (given the total amount and the amount in a group, determine how many groups of the same size can be created).

How are multiplication and division related to one another?

Division is the inverse of multiplication, the operation of sharing, partitioning, and repeated subtraction.

Some division situations will produce a remainder, but the remainder will always be less than the divisor.

What happens in division when there are zeros in both the divisor and the dividend?

Chaperones Needed

What is $2,500 \div 300$?

The Baker

Planning a Field Trip

Additional Activities

Sid’s Jelly Beans

Mental Division Strategy
Example A:

\[
\begin{align*}
150 \\
-60 (6 \times 10) \\
90 \\
-60 (6 \times 10) \\
30 \\
-30 (6 \times 5) \\
0 \\
\end{align*}
\]

\[150 \div 6 = 10 + 10 + 5 = 25\]

Example B:

\[1917 \div 9 = \]

\[
\begin{align*}
9 \\
1800 \\
90 \\
27 \\
\end{align*}
\]
**SPED Strategies:**

Summarize how to use strategies to divide multi-digit dividends by one-digit divisors and explain the answer using equations, rectangular arrays, and area models using key, technical vocabulary in simple sentences.

Divide up to a two or three-digit number by a one-digit number without remainders using place value materials, rectangular arrays and/or area models.

Solve division problems up to two digits by one digit, using the relationship between multiplication and division to demonstrate.

**Example:** 76 ÷ 4 = 19 because 19 x 4 = 76

Solve three-digit by one digit division problems with a quotient that is a multiple of ten, using reasoning about place value (e.g., 420 ÷ 7 = 60 because 42 ÷ 7 is 6 and 42 tens divided by 7 is 6 tens.

**ELL Strategies:**

Model the process. Talk aloud while solving problems on the smartboard or chalkboard to show the thinking process and common errors.

Summarize how to use strategies to divide multi-digit dividends by one-digit divisors.
New Jersey Student Learning Standard:
4.OA.A.3: Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

Student Learning Objective 4: Write and solve each equation (including any of the four operations) in order to solve multi-step word problems, using a letter to represent the unknown; interpret remainders in context and assess the reasonableness of answers using mental computation with estimation strategies.

Modified Student Learning Objectives/Standards:
M.EE.4.OA.A.3: Solve one-step real-world problems using addition or subtraction within 100.

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</thead>
<tbody>
<tr>
<td>MP 1 MP 2 MP 4 MP 7</td>
<td>4.OA.3-1 4.OA.3-2</td>
<td>Use graphic organizers to help identify unknowns to create equations and solve a word problem based on clues in the word problem. Some operations can be used interchangeably to create different equations that solve the same word problem. Variables can be used to represent an unknown in any part of an equation.</td>
<td>How do multiplication, division, addition, subtraction, and estimation help us solve real world problems? How can we find evidence in word problems to support our equations? Variables can be used to represent an unknown in any part of an equation.</td>
<td>IFL PBA Task: Turning Pages Task  Additional Tasks: Recycling Cans Zoo Field Trip How Many Cookies Do We Have? Birthday Treats</td>
</tr>
<tr>
<td></td>
<td>Assessing reasonableness of answer is not assessed here. Tasks involve interpreting remainders. See p. 30 of the OA Progression document.</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

and explain the answer using equations, rectangular arrays, and area models in L1 (student’s native language) and/or use gestures, examples, and selected, technical words.
- Multi-step problems must have at least 3 steps.

**4.C.5-1**
- Distinguish correct explanation/reasoning from that which is flawed, and – if there is a flaw in the argument – present corrected reasoning. (For example, some flawed ‘student’ reasoning is presented and the task is to correct and improve it.)
- Reasoning in these tasks centers on interpretation of remainders.

**4.C.6-1**
- Present solutions to multi-step problems in the form of valid chains of reasoning, using symbols such as equals signs appropriately (for example, rubrics award less than full credit for the presence of nonsense statements such as $1 + 4 = 5 + 7 = 12$, even if the final answer is correct), or identify or describe

| Emphasize the proper use of the equal (=) sign and the improper use ($3+7=10-5=5$) |
| Relate the terms in the equation to the context in the word problem. |
| Students should be able to identify the unknown(s) in the problem and use a variable to represent that term in context. |
| Solve multi-step word problems involving any of the four operations. |
| Explain why an answer is reasonable, such as using mental computation and estimation strategies. |
| Provide students with a variety of equations and have students create their own math story that is represented in the given equation. |

**SPED Strategies:**
Elaborate on the problem-solving process.
Read word problems aloud. Post a visual display of the problem-solving process.
Have students check off or highlight each step as they work. Talk through the problem-solving process step-by-step to demonstrate thinking process. Before students solve, ask questions for comprehension, such as, “What unit are we counting? What happened to the units

<p>| Ice Cream Party |</p>
<table>
<thead>
<tr>
<th>errors in solutions to multi-step problems and present corrected solutions</th>
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<tr>
<td>Tasks may involve interpreting remainders.</td>
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<td>Multi-step problems must have at least 3 steps.</td>
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<tr>
<td>in the story?” Teach students to use self-questioning techniques, such as, “Does my answer make sense?”</td>
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<tr>
<td>Interactive math journal with samples.</td>
</tr>
<tr>
<td>Replace unknown numbers with given values in two digit by two digit addition/subtraction sentences.</td>
</tr>
</tbody>
</table>

**ELL Strategies:**

In L1 (student’s native language) elaborate on the problem-solving process. Read word problems aloud. Post a visual display of the problem-solving process. Have students check off or highlight each step as they work. Talk through the problem-solving process step-by-step to demonstrate thinking process.

Provide sufficient wait time to allow the student to process the meaning in their first language.

Determine the reasonableness of an answer to an addition or subtraction problem using estimation strategies of “more” and “less” and the reasonableness of an answer to division problems using estimation strategies (e.g., successive subtraction).
New Jersey Student Learning Standard:
4.MD.A.3: Apply the area and perimeter formulas for rectangles in real world and mathematical problems. For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.

Student Learning Objective 5: Solve real world problems with whole numbers by finding the area and perimeter of rectangles using formulas.

Modified Student Learning Objectives/Standards:
M.EE.4.MD.A.3: Determine the area of a square or rectangle by counting units of measure (unit squares).

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<tr>
<td>MP 2</td>
<td>4.MD.3</td>
<td>Know and apply the formula for area (L x W) and express the answer in square units.</td>
<td>How do we find the area of a rectangle?</td>
<td>Flooring and Base Board</td>
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<tr>
<td>MP 5</td>
<td></td>
<td>Know and apply the formula for perimeter: 2L + 2W or 2(L + W) and express the answer in linear units.</td>
<td>How do we find the perimeter of a rectangle?</td>
<td>City Farmers</td>
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<td></td>
<td></td>
<td>The formulas can be used to find unknown factor. Ex. (8 x W) = 24 sq. units.</td>
<td>What is the difference between the area and the perimeter of a rectangle?</td>
<td>Rabbit Pens</td>
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<td></td>
<td></td>
<td>Communicate understanding of formula and justify why it works.</td>
<td>What is the relationship between area and perimeter when the area is fixed but the length and width vary?</td>
<td>Fair Play</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Formulas should be developed with students through experience not just memorization.</td>
<td>What is the relationship between area and perimeter when the perimeter is fixed but the length and width vary?</td>
<td>Area + Perimeter Exploration</td>
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<td></td>
<td>Solve real world problems using area and perimeter.</td>
<td>How are the units used to measure perimeter different and</td>
<td>Putting Down Carpet</td>
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<td>Fencing Yards</td>
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<td>Making a Dog Pen</td>
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<td>Karl’s Garden</td>
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<tr>
<td><strong>SPED Strategies:</strong></td>
<td><strong>ELL Strategies:</strong></td>
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<tr>
<td>Demonstrate comprehension of computing area and perimeter formulas by explaining orally and in writing how to solve real world math problems using key, technical vocabulary in simple sentences. Allow students to use rulers and yardsticks to discover relationships among these units of measurements. The use of grid or graph paper in measuring perimeter and finding area.</td>
<td>Demonstrate comprehension of computing area and perimeter formulas by explaining orally and in writing how to solve real world math problems using key, technical vocabulary in simple sentences. Allow students to use rulers and yardsticks to discover relationships among these units of measurements. Teacher creates a cloze activity where words are omitted from a math passage and students are required to fill in the blanks.</td>
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<tr>
<td>alike from the units used to measure area?</td>
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</tbody>
</table>
New Jersey Student Learning Standard:
4.NF.A.1: Explain why a fraction $a/b$ is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions. *[Grade 4 expectations in this domain are limited to denominators of 2, 3, 4, 5, 6, 8, 10, 12 and 100.]*

Student Learning Objective 6: Recognize and generate equivalent fractions and explain why they are equivalent using visual fraction models.

Modified Student Learning Objectives/Standards:
M.EE.4.NF.A.1: Identify models of one half (1/2) and one fourth (1/4).

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<td>MP 1</td>
<td>4.NF.1-2</td>
<td>Create visual fraction models using benchmark fractions to compare and determine if fractional parts are equivalent.</td>
<td>What is a fraction and how can it be represented?</td>
<td>Equivalent Pizzas</td>
</tr>
<tr>
<td>MP 4</td>
<td></td>
<td></td>
<td>How can we show that two fractions are equivalent?</td>
<td>Comparing Ropes</td>
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<tr>
<td>MP 5</td>
<td></td>
<td></td>
<td>Fractions can be represented visually and in written form.</td>
<td>Trading Blocks</td>
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<tr>
<td>MP 6</td>
<td></td>
<td></td>
<td>Fractions with different parts can be the same size when referring to the same whole.</td>
<td>Splitting to Make Equivalent Fractions</td>
</tr>
<tr>
<td>MP 7</td>
<td></td>
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<td>Only fractions of the same whole can be compared.</td>
<td>Fraction Rectangles</td>
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<td>In what ways can we model equivalent fractions?</td>
<td>Tiling the Patio</td>
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<td>Weird Pieces of Cake</td>
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<td>Equivalent Fractions</td>
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<td>Their Fair Share</td>
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<td>Fractions and Rectangles</td>
</tr>
</tbody>
</table>
reasoning. (For example, some flawed student reasoning is presented and the task is to correct and improve it.  
• Tasks have “thin context” or no context.  
• Tasks are limited to denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.  
• Tasks may include fractions that equal whole numbers. Whole numbers are limited to 0 through 5.

4.C.7-1  
• Base explanations/reasoning on a number line diagram (whether provided in the prompt or constructed by the student in the response).  
• Fractions equivalent to whole numbers are limited to 0 through 5.

Models of fractions can be used to generate a rule for writing equivalent fractions.

Equivalent fractions are the same size while the number and size of the parts differ.

Generate equivalent fractions, using fraction \( \frac{a}{b} \) as equivalent to fraction \( \frac{(n \times a)}{(n \times b)} \).

SPED Strategies:
Retell how to recognize and generate equivalent fractions and explain why they are equivalent via visual fraction models.
Demonstrate fractions equivalent to \( \frac{1}{2} \) using manipulatives (color tiles, fraction circles, fraction bars) and or technology (e.g., \( \frac{2}{4} = \frac{1}{2} \)). For example, students identify the piece for 1 whole and use it as a reference. To find fractions equivalent to \( \frac{1}{2} \), students stack fractions of like color fraction circles on the \( \frac{1}{2} \) piece to find which pieces exactly cover the \( \frac{1}{2} \).

**ELL Strategies:**
Use cognates or flip books written in native language to use as a reference.

Connect students’ prior knowledge and experiences to new learning. Find out what students already know about a topic by making a semantic web on the board. Write the topic in the center of a circle and record students’ knowledge around it.

Retell how to recognize and generate equivalent fractions and explain why they are equivalent using visual fraction models using key, technical vocabulary in simple sentences in the student's native language.
New Jersey Student Learning Standard:
4.NF.A.2: Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model. [Grade 4 expectations in this domain are limited to denominators of 2, 3, 4, 5, 6, 8, 10, 12 and 100.]

Student Learning Objective 7: Compare two fractions with different numerators or different denominators, recording comparison with >, =, or <, and justifying the conclusion using visual fraction models.

Modified Student Learning Objectives/Standards:
M.EE.4.NF.A.2: Identify models of one half $\frac{1}{2}$ and one fourth $\frac{1}{4}$.

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</table>
| MP 1 MP 4 MP 5 MP 6 MP 7 | 4.NF.2-1  
- Only the answer is required.  
- Tasks require the student to choose the comparison strategy autonomously.  
- Tasks are limited to denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.  
- Tasks may include fractions that equal whole numbers. Whole numbers are limited to 0 through 5.  
4.C.5-3  
- Distinguish correct explanation/reasoning from that which is flawed. | Visual concrete fractional models are an important initial component in developing the conceptual understanding of fractions.  
Visual fraction models or finding common denominators can be used to compare fractions. Identify and give multiple representations for the fractional parts of a whole (area model) or set.  
Utilize area models, number lines, double number lines, verbal justification, and benchmark fractions to compare fractional parts. | The size of a whole must be the same to compare fractions.  
How do we compare fractions with unlike denominators?  
Using benchmark fractions, how can we determine how much larger a fraction is from another fraction?  
Why do equivalent fractions have the same value?  
If two fractions have the same denominator, the fraction with the greater numerator is the greater fraction. If two fractions | IFL Task(s) – “Building Non-Unit Fractions”  
Part 1 Tasks:  
Pizza for the Family  
Drinking Juice  
Birthday Money  
Additional Tasks:  
Benchmark Fractions  
More or Less  
Closest to 0, $\frac{1}{2}$, or 1  
Making Fractions  
The Whole Matters |
and – if there is a flaw in the argument – present corrected reasoning. (For example, some flawed ‘student’ reasoning is presented and the task is to correct and improve it.)

- Tasks have “thin context” or no context.
- Tasks are limited to denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.
- Tasks may include fractions that equal whole numbers. Whole numbers are limited to 0 through 5.

### 4.C.7-2

- Base explanations/reasoning on a number line diagram (whether provided in the prompt or constructed by the student in her response).
- Fractions equivalent to whole numbers are limited to 0 through 5.

Fractions may only be compared when the two fractions are referring to the same whole.

**Examples:**

Fractions with common denominators may be compared using the numerators as a guide.

\[
\frac{2}{6} < \frac{3}{6} < \frac{5}{6}
\]

Fractions with common numerators may be compared and ordered using the denominators as a guide.

\[
\frac{3}{10} < \frac{3}{8} < \frac{3}{4}
\]

Fractions may be compared using \(\frac{1}{2}\) as a benchmark.

\[
0 \quad \frac{1}{2} \quad 1
\]

Fractions with the same numerator, the fraction with the lesser denominator is the greater fraction.

How can same fractional amounts be compared using symbols in different ways?

Why is it important to compare fractions as representations of equal parts of a whole or of a set?

Fractions with the same size pieces, or common denominators, within the same size whole can be compared with each other because the size of the pieces is the same. Comparison to known benchmark quantities can help determine the relative size of fractional pieces because the benchmark quantity can be seen as greater, less than, or the same as the piece.

A product can be written as two or more different but equivalent expressions as long as the total number of items in each set is the same.

\[(e.g., \frac{2}{4} \times \frac{1}{4} = \frac{4}{8} \times \frac{1}{8}).\]
Record the results of comparisons with the symbol >, =, or < and justify the conclusions.

**SPED Strategies:**
Compare two fractions; visually demonstrate which is greater and less than the benchmark of a half using manipulatives or technology. Use fraction manipulatives such as fraction circles, fraction squares, fraction bars, fraction tiles, and fraction cubes. Students identify the piece for 1 whole and use it as a reference.

Color code fraction manipulatives (e.g., all halves are red, all thirds are green, all fourths are blue).

If students find various fractional units on one number line frustrating, give them the option of plotting 1/4, 4/5, or 5/8 on two number lines placed parallel for comparison.

**ELL Strategies:**
Justify in writing how to compare two fractions with different numerators and different denominators in L1 (student’s native language) and/or use gestures, examples, and selected, technical words. If students find various fractional units on one number line frustrating, give them the option of plotting 1/4, 4/5, or
New Jersey Student Learning Standards:
4.NF.B.3: Understand a fraction \(a/b\) with \(a > 1\) as a sum of fractions \(1/b\). [Grade 4 expectations in this domain are limited to denominators of 2, 3, 4, 5, 6, 8, 10, 12 and 100.]

4.NF.B.3a: Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.
4.NF.B.3b: Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. Examples: \(3/8 = 1/8 + 1/8 + 1/8\); \(3/8 = 1/8 + 2/8\); \(2\ 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8\).

Student Learning Objective 8: Decompose a fraction into a sum of fractions with the same denominator in more than one way and record the decomposition as an equation; justify the decomposition with a visual fraction model.

Modified Student Learning Objectives/Standards:
M.EE.4.NF.B.3: Differentiate between whole and half.

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<tr>
<td>MP 1</td>
<td>4.NF.3a</td>
<td>Investigate fractions other than unit fractions by joining (composing) or separating (decomposing) fractions of the same whole.</td>
<td>Fractions can be added or subtracted by joining or separating the parts.</td>
<td>IFL Task(s) – “Building Non-Unit Fractions” Part 1 Tasks: Pizza For the Family Drinking Juice Birthday Money</td>
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<tr>
<td>MP 2</td>
<td>4.NF.3b-1</td>
<td>Decompose mixed numbers in more than one way, such as visual models.</td>
<td>Fractions can be separated into equal smaller parts.</td>
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<tr>
<td>MP 3</td>
<td>Only the answer is required.</td>
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<td>MP 4</td>
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<tr>
<td>MP 5</td>
<td>Tasks are limited to denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.</td>
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<td>MP 6</td>
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<td>MP 7</td>
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</tbody>
</table>
- Tasks may include fractions that equal whole numbers. Whole numbers are limited to 0 through 5.

4.C.4-2
- Base arithmetic explanations/reasoning on concrete referents such as diagrams (whether provided in the prompt or constructed by the student in her response), connecting the diagrams to a written (symbolic) method.
- Tasks have “thin context” or no context.
- Tasks are limited to denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.
- Tasks may include fractions that equal whole numbers. Whole numbers are limited to 0 through 5.

4.C.6-2
- Present solutions to multi-step problems in the form of valid chains of reasoning, using symbols such as equals

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<th>Emphasize the conversion of mixed numbers into improper fractions and vice versa.</th>
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<td>Create equations that identify the decomposing (iterating) of fractions.</td>
</tr>
<tr>
<td><strong>Example:</strong> ( \frac{3}{8} = \frac{1}{8} + \frac{1}{8} + \frac{1}{8} )</td>
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Visual models help understand and compare the relative size and equivalent parts of a whole.

- Apply understanding of repeated addition and subtraction of unit and non-unit fractions.
- Identify the different types of addition and subtraction situations (problems) as joining action, separating action, part-part-whole relations, or comparison situations.
- Practice counting with fractions, such as skip counting, counting on, and counting back. This will help students see patterns and develop proficiencies that will strengthen their understanding of addition and subtraction. Try asking students to count by halves starting with 0 or count by thirds starting at 10.

**SPED Strategies:**
Students use the yellow, green, blue, and red pattern blocks, reviewing the values

- Decomposing fractions can be represented visually and in written form.
- What information do we attain by composing and decomposing fractions?

Iterating a unit fraction is an interpretation of a fraction that illustrates the equal parts that the fraction is composed of \( \frac{3}{5} = \frac{1}{5} + \frac{1}{5} + \frac{1}{5} \) or revoicing of the iterations as \( 3 \times \frac{1}{5} \) [exposure level].

When decomposing a fraction into iterations of the unit fraction, the number of iterations is the same as the value of the numerator.

A fraction in which the total number of pieces in the whole is the same as the total number of the pieces is called one whole because all of the pieces of the whole are accounted for (e.g., \( \frac{4}{4}, \frac{5}{5}, \frac{6}{6} \)).

Numbers, including fractions, can be written in a variety of

**IFL PBA Task:**
Adding and Multiplying Mixed Numbers

**Additional Tasks:**
Give “Em” Chocolate Eggsactly
Tile Task
Sharing Cake
Candy Bucket
Dividing up the Land
Pizza Party
Making 22 Seventeenths in Different Ways
signs appropriately (for example, rubrics award less than full credit for the presence of nonsense statements such as $1 + 4 = 5 + 7 = 12$, even if the final answer is correct), or identify or describe errors in solutions to multi-step problems and present corrected solutions.

- Tasks have “thin context” or no context.
- Denominators are limited to grade 3 possibilities (2, 3, 4, 6, 8) so as to keep computational difficulty lower.
- Multi-step problems must have at least 3 steps.

4.C.7-3

- Base explanations/reasoning on a number line diagram (whether provided in the prompt or constructed by the student in the response).

of each block if the yellow hexagon represents 1 whole. For fractions with like denominators, students simply line up the indicated fractions and make exchanges to simplify the sum. For unlike denominators, students must trade pieces until all parts are the same color (i.e. When adding $\frac{2}{3} + \frac{1}{6}$ students line up 2 blue rhombuses and 1 green triangle, then exchange the blue rhombuses for 4 green triangles.).

Students can also piece together a hexagon congruent to the yellow hexagon and record the appropriate addition equation beneath each congruent shape (i.e., For 2 trapezoids, the student records $\frac{1}{2} + \frac{1}{2} = 1$; for one trapezoid, one rhombus, and one triangle, the student records $\frac{1}{2} + \frac{1}{3} + \frac{1}{6} = 1$).

Allow students to use fraction models and drawings to show their understanding.

**ELL Strategies:**
Interactive word wall in native language.

Allow students to use fraction models and drawings to show their understanding.

Justify in writing how to compare two fractions with different numerators and different denominators in L1 (student’s ways (decomposing and composing of numbers).
native language) and/or use gestures, examples, and selected, technical words.

Present word problems involving multiplication of a fraction by a whole number. Have students solve the problems using visual models and write equations to represent the problems.

Have students explain their thinking process aloud to a classmate while solving a problem.

Create a step-by-step anchor chart.

**New Jersey Student Learning Standard:**
4.NF.B.3c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using the properties of operations and the relationship between addition and subtraction.

**Student Learning Objective 9:** Add and subtract mixed numbers with like denominators by replacing each mixed number with an equivalent fraction or improper fraction.

**Modified Student Learning Objectives/Standards:**
M.EE.4.NF.B.3: Differentiate between whole and half.

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<td>4.NF.3c</td>
<td>Area models, bar models, number lines, converting mixed numbers to improper fractions, equations, and properties of operations can be used to add and subtract mixed numbers.</td>
<td>How can fractions and mixed numbers be added and subtracted on a number line?</td>
<td>Plastic building Blocks</td>
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<tr>
<td>MP 2</td>
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<td>Fraction Field Event</td>
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<td>MP 3</td>
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<td>Chocolate Chips Task</td>
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<td>MP 5</td>
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<tr>
<td>MP 6</td>
<td><strong>possibilities (2, 3, 4, 6, 8) so as to keep computational difficulty lower.</strong></td>
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</table>
| MP 7 | **4.C.6-2**  
- Present solutions to multi-step problems in the form of valid chains of reasoning, using symbols such as equals signs appropriately (for example, rubrics award less than full credit for the presence of nonsense statements such as $1 + 4 = 5 + 7 = 12$, even if the final answer is correct), or identify or describe errors in solutions to multi-step problems and present corrected solutions.  
- Tasks have “thin context” or no context.  
- Denominators are limited to grade 3 possibilities (2, 3, 4, 6, 8) so as to keep computational difficulty lower.  
  |
|       | **Emphasize the conversion of mixed numbers into improper fractions and vice versa.**  
Emphasize the misconception of combining both the numerators and the denominators for the total fractional amount. Common error: $\frac{11}{4} + \frac{7}{4} = \frac{18}{8}$.  
Help students understand the meaning of the numerator, the denominator, and the unit, as well as the relationship among them.  
Fractional amounts greater than 1 can be represented using a whole number and a fraction. Whole number amounts can also be represented as fractions.  
Provide a contextualized situation and an area model matching most closely to the context.  
**SPED Strategies:**  
Summarize orally and in writing how to add and subtract mixed numbers with like denominators by replacing each mixed number with an equivalent fraction using key, technical vocabulary in simple sentences.  
Provide a checklist of steps.  
Mixed numbers and improper fractions can be used interchangeably.  
A mixed number is a whole number and a fractional part which can be written as a fraction with a numerator greater than the denominator.  
What are some strategies that can be used to add and/or subtract fractions?  
Numbers, including fractions, can be written in a variety of ways (decomposing and composing of numbers).  
|       | **Fraction Cookies Bakery**  
**Square Tiles**  
**Going the Distance**  
**Cynthia’s Perfect Punch** |
| Multi-step problems must have at least 3 steps | Scaffold complex concepts and provide leveled problems for multiple entry points.  
Use a number line and/or graph paper to set up problems for adding and subtracting fractions.  
**ELL Strategies:**  
Provide a checklist of steps in L1 (student’s native language) on how to add and subtract mixed numbers with like denominators.  
Give directions step-by-step (orally and in writing) before assigning students to do independent, pair, or group work. Ask a student to repeat the directions aloud for the rest of the class to assess whether all the students understand the assignment.  
Allow ELL students to talk to a peer in their native language when necessary to clarify understanding and clear up misunderstandings.  
Summarize orally and in writing how to add and subtract mixed numbers with like denominators by replacing each mixed number with an equivalent fraction in L1 (student’s native language) and/or use gestures, examples, and selected, technical words. |
**New Jersey Student Learning Standard:**
4.NF.B.3d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.

**Student Learning Objective 10:** Solve word problems involving addition and subtraction of fractions having like denominators using visual fraction models and equations to represent the problem.

**Modified Student Learning Objectives/Standards:**
M.EE.4.NF.B.3: Differentiate between whole and half.

<table>
<thead>
<tr>
<th>MPs</th>
<th>Evidence Statement Key/Clarifications</th>
<th>Skills, Strategies &amp; Concepts</th>
<th>Essential Understandings/Questions (Accountable Talk)</th>
<th>Tasks/Activities</th>
</tr>
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<tbody>
<tr>
<td>MP 1</td>
<td>4.NF.3d</td>
<td>Models need to represent the same whole.</td>
<td>How are fractions used in problem-solving situations?</td>
<td>IFL Task(s) – “Building Non-Unit Fractions”</td>
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<tr>
<td>MP 2</td>
<td></td>
<td>Word problems need to be dissected to extract key information that will be used in adding and/or subtracting fractions.</td>
<td>Models and equations can be used to show different ways of adding and subtracting various types of fractions.</td>
<td>Part 1 Tasks: Pizza For the Family Drinking Juice Birthday Money</td>
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<td>MP 3</td>
<td>Tasks are limited to denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.</td>
<td>Make use of manipulatives, drawings, and equations to represent fractions of a whole.</td>
<td>Fractions with the same pieces, or common denominators, within the same size whole can be added or subtracted because the size of the pieces is the same.</td>
<td>Additional Tasks: Sweet Fraction Bars</td>
</tr>
<tr>
<td>MP 4</td>
<td>Addition and subtraction situations are limited to the dark- or medium-shaded types in Table 2, p. 9 of the OA Progression document; these situations are sampled equally.</td>
<td>Make use of both addition and subtraction; students employ reasoning about magnitude to compare to benchmarks.</td>
<td>How can we find evidence in word problems to support the equations created to represent the problem?</td>
<td>General Grant’s March</td>
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<td>MP 5</td>
<td>Prompts do not provide visual fraction models; students may at their discretion draw visual fraction models as a strategy.</td>
<td>Provide a contextualized situation that requires students to provide an argument; expressions/ equations and diagrams are encouraged to support the argument.</td>
<td></td>
<td>Boxing Up Leftover Brownies Pattern Blocks and Unit Fractions</td>
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<td>MP 6</td>
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<td>MP 7</td>
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Rely on the use of what students know about repeated addition or subtraction of unit and non-unit fractions.

Addition and subtraction can be divided into four categories that can be represented by different models: joining action, separating action, part-part-whole relations, and comparing situations.

**Example:**
Trevor has $4 \frac{1}{8}$ pizzas left over from his soccer party. After giving some pizza to his friend, he has $2 \frac{4}{8}$ of a pizza left. How much pizza did Trevor give to his friend?
Solution: Trevor had $4 \frac{1}{8}$ pizzas to start. This is $\frac{33}{8}$ of a pizza. The x’s show the pizza he has left which is $2 \frac{4}{8}$ pizzas or $\frac{20}{8}$ pizzas. The shaded rectangles without the x’s are the pizza he gave to his friend which is $\frac{13}{8}$ or $1 \frac{5}{8}$ pizzas.

**SPED Strategies:**
In small groups create note cards with academic vocabulary.

View videos to provide students with visual models.

In small groups, read word problems aloud, highlight key information in the text, and model how to solve.
ELL Strategies:
Scaffold complex concepts and provide leveled problems for multiple entry points.

In small groups create note cards with academic vocabulary in L1 (student’s native language).

Interactive math journal to demonstrate growth in math writing and reasoning.

Allow ELL students to talk to a peer in their native language when necessary to clarify understanding and clear up misunderstandings.
Integrated Evidence Statements

4. NBT.Int.1: Perform computations by applying conceptual understanding of place value, rather than by applying multi-digit algorithms.
   - Tasks do not have a context

4. NF.A.Int.1: Apply conceptual understanding of fraction equivalence and ordering to solve simple word problems requiring fraction comparison. Content Scope: 4.NF.A
   - Tasks have “thin context.”
   - Tasks do not require adding, subtracting, multiplying, or dividing fractions.
   - Prompts do not provide visual fraction models; students may at their discretion draw visual fraction models as a strategy.
   - Tasks are limited to denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.
   - Tasks may include fractions that equal whole numbers. Whole numbers are limited to 0 through 5

4.NF.Int. 1: Solve one-step word problems requiring integration of knowledge and skills articulated in 4.NF. Content Scope: 4.NF
   - Tasks are limited to denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.

4.Int.2: Solve one-step word problems involving multiplying two two-digit numbers.
   - The given numbers are such as to require a general strategy based on place value and the properties of operations (e.g., 63 x 44).
   - Word problems shall include a variety of grade-level appropriate applications and contexts.

4.Int.3: Solve one-step word problems involving multiplying a four-digit number by a one-digit number.
   - The given numbers are such as to require a general strategy based on place value and the properties of operations (e.g., 2392 x 8).
   - Word problems shall include a variety of grade-level appropriate applications and contexts.

4.Int.4: Solve one-step word problems involving dividing a four-digit number by a one-digit number.
   - The given numbers are such as to require a general strategy based on place value and the properties of operations (e.g., 2328 ÷ 8).
   - Quotients are whole numbers (i.e., there are no remainders).
   - Word problems shall include a variety of grade-level appropriate applications and contexts.
Integrated Evidence Statements

4.C.5-6 Distinguish correct explanation/reasoning from that which is flawed, and – if there is a flaw in the argument – present corrected reasoning. (For example, some flawed ‘student’ reasoning is presented and the task is to correct and improve it.) Content Scope: Knowledge and skills articulated in 3.OA.B, 3.NF, 3.MD.C
   • Tasks may have scaffolding if necessary in order to yield a degree of difficulty appropriate to Grade 4.

4.C.6-3: Present solutions to multi-step problems in the form of valid chains of reasoning, using symbols such as equals signs appropriately (for example, rubrics award less than full credit for the presence of nonsense statements such as $1 + 4 = 5 + 7 = 12$, even if the final answer is correct), or identify or describe errors in solutions to multi-step problems and present corrected solutions. Content Scope: Knowledge and skills articulated in 4.NF.3d, 4.NF.4c
   • Denominators are limited to grade 3 possibilities (2, 3, 4, 6, 8) so as to keep computational difficulty lower.
   • Multi-step problems must have at least 3 steps

4.D.1 Solve multi-step contextual word problems with degree of difficulty appropriate to Grade 4, requiring application of knowledge and skills articulated in Type I, Sub-Claim A Evidence Statements.
   • Tasks may have scaffolding.
   • Multi-step problems must have at least 3 steps

4.D.2 Solve multi-step contextual problems with degree of difficulty appropriate to Grade 4, requiring application of knowledge and skills articulated in 3.OA.A, 3.OA.8, 3.NBT, and/or 3.MD.
   • Tasks may have scaffolding, if necessary, in order to yield a degree of difficulty appropriate to Grade 4.
   • Multi-step problems must have at least 3 steps.
   • Tasks do not require a student to write a single equation with a letter standing for the unknown quantity in a two-step problem, and then solve that equation.
   • Tasks may require students to write an equation as part of their work to find a solution, but students are not required to use a letter for the unknown.
   • Addition, subtraction, multiplication and division situations in these problems may involve any of the basic situation types with unknowns in various positions (see NJSLS, Table 1, Common addition and subtraction, p. 93; NJSLS, Table 2, Common multiplication and division situations, p. 94; and the OA Progression document.)
## Unit 2 Vocabulary

- Algorithm
- Associative Property of Multiplication
- Area Model
- Array
- Benchmark Fractions
- Common Denominator
- Common Factor
- Common Multiple
- Commutative Property of Multiplication
- Compare
- Compatible Numbers
- Composite Number
- Decompose
- Denominator
- Distributive Property
- Divisibility Rules
- Divisor
- Dividend
- Equation
- Equivalent Fractions
- Estimate
- Expression
- Fact Family
- Fraction
- Fraction Models
- Fractional Parts of a Whole
- Identity Property of Multiplication
- Improper Fraction
- Inverse Operations
- Interpret
- Iteration
- Joining Action
- Matrix Model
- Mental Math/ Mental Calculation
- Mixed Number
- Model/ Visual Model
- Multiples
- Multiplicative Identity Property of 1
- Numerator
- Operation
- Part-Part- Whole Relations
- Partial Product
- Partial Quotient
- Perimeter
- Product
- Quotient
- Reasonableness
- Related Facts
- Remainder
- Separating Action
- Simplest Form
- Simplify
- Term
- Unit Fraction
- Unlike Denominators
- Unlike Numerators
- Variable
- Whole Number
- Zero Property of Multiplication
# References & Suggested Instructional Websites

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<tr>
<td><a href="http://www.internet4classrooms.com">www.internet4classrooms.com</a></td>
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## Field Trip Ideas

**NATIONAL MUSEUM OF MATHEMATICS:** Mathematics illuminates the patterns and structures all around us. Our dynamic exhibits, gallery, and programs will stimulate inquiry, spark curiosity, and reveal the wonders of mathematics. MoMath has innovative exhibits that will engage folks from 105 to 5 years old (and sometimes younger) but with a special emphasis on activities for 4th through 8th graders.
http://momath.org/

**THE BOUNCE FACTORY:** The Bounce Factory, Bricks 4 Kidz of Hunterdon Somerset and Team Makers of North Jersey have combined to create a unique and exciting Field Trip for students in grades 1st – 8th. It integrates STEM learning with fun, hands on activities that will focus on Science, Engineering and Math concepts. Students will build motorized models with LEGO® bricks and discuss engineering and physics principals. • Enter the Bounce rooms for activities that will set in motion discussions of how physics impacts their play. • Learn about Math and Science concepts while playing integrative team building activities that build their skills and promote working together. • Learn strategy and the power of collaboration while playing laser tag in a state of the art facility.
http://www.bouncefactorynj.com/

**LIBERTY SCIENCE CENTER** - An interactive Science museum and learning center with math connections. There is a math guidebook for teachers to make connections with math.
http://lsc.org/plan-your-visit/

**NEW JERSEY JACKALS** – Students will be able to watch a live minor league baseball game while figuring out the players batting averages, the earned run average, determine the win to loss ratio for the season, the pitch count, and other player statistics.
http://njjackals.pointstreaksites.com/view/njjackals/home-page-657

**STORMING ROBOTS** – Engage your group in STEM-oriented Robotics Engineering Exploration. Each experience may range from three to five hours. Storming Robots is listed in the NASA’s Educational Robotics Matrix. SR’s program aims to capitalize creativity with engineering disciplines. Projects captivate young minds; stimulate critical thinking and creativity with mathematics. Key Activities will include open-ended, but application-based mini-projects focusing on problem-solving skills with computational thinking skills.
http://www.stormingrobots.com/prod/techtrip.html
Field Trip Ideas

**BUEHLER CHALLENGER & SCIENCE CENTER** – Buehler Challenger & Science Center in New Jersey provides students, scouts, homeschoolers, and youth groups with the opportunity to “fly” simulated space missions using applied science and inquiry-based learning in our state-of-the-art simulators. Participants work as a team as they take on the role of astronauts and mission controllers to Rendezvous with Comet Halley, Return to the Moon, or Voyage to Mars. Students, scouts, and youth groups use team-building and hands-on learning with a focus on STEM to complete their mission goal. Programming for day field trips is available for students Pre-K through 12th grade. In addition, the Center also offers overnight camp-ins, professional development and outreach programs, including StarLab Planetarium & Living in Space Experience.


**PANTHER ACADEMY PLANETARIUM** – Since 2004, the Panther Academy Planetarium has brought the universe to the Paterson community by educating and entertaining generations of school children and adults, inspiring imaginations and expanding horizons. Located in Paterson’s downtown business district and near Passaic County Community College, it is one of the best-equipped school planetariums in the United States. The planetarium resides at the heart of PANTHER, the Paterson Academy for Earth and Space Science. The Paterson school district equipped the planetarium with an interactive computerized system that supports Panther’s mathematics and science curriculum while providing district-wide weekly programs for elementary and secondary students. In addition to its value as an educational tool, the planetarium is also an important community resource, offering a variety of special presentations for public, college and community groups.

[http://www.paterson.k12.nj.us/planetarium/](http://www.paterson.k12.nj.us/planetarium/)