Department of Accelerated Programs

Mathematical Studies SL Curriculum

10.0 Credits
# IB LEARNER PROFILE

IB Programs aim to develop internationally minded people who are striving to become:

<table>
<thead>
<tr>
<th>Role</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Inquirers</strong></td>
<td>Their natural curiosity is nurtured. They acquire the skills necessary to conduct constructive inquiry and research, and become independent active learners. They actively enjoy learning and this love of learning will be sustained throughout their lives.</td>
</tr>
<tr>
<td><strong>Knowledgeable</strong></td>
<td>They explore concepts, ideas and issues, which have global relevance and importance. In so doing, they acquire, and are able to make use of, a significant body of knowledge across a range of disciplines.</td>
</tr>
<tr>
<td><strong>Critical thinkers</strong></td>
<td>They exercise initiative in applying thinking skills critically and creatively to approach complex problems and make reasoned decisions.</td>
</tr>
<tr>
<td><strong>Communicators</strong></td>
<td>They understand and express ideas and information confidently and creatively in more than one language and in a variety of modes of communication.</td>
</tr>
<tr>
<td><strong>Risk-takers</strong></td>
<td>They approach unfamiliar situations with confidence and forethought, and have the independence of spirit to explore new roles, ideas and strategies. They are courageous and articulate in defending those things in which they believe.</td>
</tr>
<tr>
<td><strong>Principled</strong></td>
<td>They have a sound grasp of the principles of moral reasoning. They have integrity, honesty, a sense of fairness and justice and respect for the dignity of the individual.</td>
</tr>
<tr>
<td><strong>Caring</strong></td>
<td>They show empathy, compassion and respect towards the needs and feelings of others. They have a personal commitment to action and service to make a positive difference to the environment and to the lives of others.</td>
</tr>
<tr>
<td><strong>Open-minded</strong></td>
<td>Through an understanding and appreciation of their own culture, they are open to the perspectives, values and traditions of other individuals and cultures and are accustomed to seeking and considering a range of points of view.</td>
</tr>
<tr>
<td><strong>Well-balanced</strong></td>
<td>They understand the importance of physical and mental balance and personal well being for themselves and others. They demonstrate perseverance and self-discipline.</td>
</tr>
<tr>
<td><strong>Reflective</strong></td>
<td>They give thoughtful consideration to their own learning and personal development. They are able to analyze their strengths and weaknesses in a constructive manner.</td>
</tr>
</tbody>
</table>
Unit Four
Mathematical Studies Year 2

Course Description

Mathematical Studies is a course with an emphasis on applications of mathematics. It is for students with varied backgrounds and abilities. It offers students opportunities to learn important concepts and techniques and to gain an understanding of a wide variety of mathematical topics. It prepares students to be able to solve problems in a variety of settings, to develop more sophisticated mathematical reasoning and to enhance their critical thinking. The individual project is an extended piece of work based on personal research involving the collection, analysis and evaluation of data. Students taking this course are well prepared for a career in social sciences, humanities, languages or arts.
# Mathematical Studies Year 2

## Pacing Guide

<table>
<thead>
<tr>
<th>Unit</th>
<th>Topic</th>
<th>Suggested Timing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unit 1</td>
<td>Geometry and Trigonometry</td>
<td>8 Weeks</td>
</tr>
<tr>
<td>Unit 2</td>
<td>Mathematical Models</td>
<td>9 Weeks</td>
</tr>
<tr>
<td>Unit 3</td>
<td>Introduction to Differential Calculus 1</td>
<td>8 Weeks</td>
</tr>
<tr>
<td>Unit 4</td>
<td>Introduction to Differential Calculus 2</td>
<td>7 Weeks</td>
</tr>
</tbody>
</table>
Educational Technology Standards


- **Technology Operations and Concepts**
  - Create a personal digital portfolio which reflects personal and academic interests, achievements, and career aspirations by using a variety of digital tools and resources.
  - Produce and edit a multi-page digital document for a commercial or professional audience and present it to peers and/or professionals in that related area for review.

- **Creativity and Innovation**
  - Apply previous content knowledge by creating and piloting a digital learning game or tutorial.

- **Communication and Collaboration**
  - Develop an innovative solution to a real world problem or issue in collaboration with peers and experts, and present ideas for feedback through social media or in an online community.

- **Digital Citizenship**
  - Demonstrate appropriate application of copyright, fair use and/or Creative Commons to an original work.
  - Evaluate consequences of unauthorized electronic access and disclosure, and on dissemination of personal information.
  - Compare and contrast policies on filtering and censorship both locally and globally.

- **Research and Information Literacy**
  - Produce a position statement about a real world problem by developing a systematic plan of investigation with peers and experts synthesizing information from multiple sources.

- **Critical Thinking, Problem Solving, Decision Making**
  - Evaluate the strengths and limitations of emerging technologies and their impact on educational, career, personal and or social needs.
**Career Ready Practices**

Career Ready Practices describe the career-ready skills that all educators in all content areas should seek to develop in their students. They are practices that have been linked to increase college, career, and life success. Career Ready Practices should be taught and reinforced in all career exploration and preparation programs with increasingly higher levels of complexity and expectation as a student advances through a program of study.

<table>
<thead>
<tr>
<th>CRP1. Act as a responsible and contributing citizen and employee</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Career-ready individuals understand the obligations and responsibilities of being a member of a community, and they demonstrate this understanding every day through their interactions with others. They are conscientious of the impacts of their decisions on others and the environment around them. They think about the near-term and long-term consequences of their actions and seek to act in ways that contribute to the betterment of their teams, families, community and workplace. They are reliable and consistent in going beyond the minimum expectation and in participating in activities that serve the greater good.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>CRP2. Apply appropriate academic and technical skills.</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Career-ready individuals readily access and use the knowledge and skills acquired through experience and education to be more productive. They make connections between abstract concepts with real-world applications, and they make correct insights about when it is appropriate to apply the use of an academic skill in a workplace situation.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>CRP3. Attend to personal health and financial well-being.</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Career-ready individuals understand the relationship between personal health, workplace performance and personal well-being; they act on that understanding to regularly practice healthy diet, exercise and mental health activities. Career-ready individuals also take regular action to contribute to their personal financial wellbeing, understanding that personal financial security provides the peace of mind required to contribute more fully to their own career success.</td>
<td></td>
</tr>
</tbody>
</table>
CRP4. Communicate clearly and effectively and with reason.
Career-ready individuals communicate thoughts, ideas, and action plans with clarity, whether using written, verbal, and/or visual methods. They communicate in the workplace with clarity and purpose to make maximum use of their own and others’ time. They are excellent writers; they master conventions, word choice, and organization, and use effective tone and presentation skills to articulate ideas. They are skilled at interacting with others; they are active listeners and speak clearly and with purpose. Career-ready individuals think about the audience for their communication and prepare accordingly to ensure the desired outcome.

CRP5. Consider the environmental, social and economic impacts of decisions.
Career-ready individuals understand the interrelated nature of their actions and regularly make decisions that positively impact and/or mitigate negative impact on other people, organization, and the environment. They are aware of and utilize new technologies, understandings, procedures, materials, and regulations affecting the nature of their work as it relates to the impact on the social condition, the environment and the profitability of the organization.

CRP6. Demonstrate creativity and innovation.
Career-ready individuals regularly think of ideas that solve problems in new and different ways, and they contribute those ideas in a useful and productive manner to improve their organization. They can consider unconventional ideas and suggestions as solutions to issues, tasks or problems, and they discern which ideas and suggestions will add greatest value. They seek new methods, practices, and ideas from a variety of sources and seek to apply those ideas to their own workplace. They take action on their ideas and understand how to bring innovation to an organization.

CRP7. Employ valid and reliable research strategies.
Career-ready individuals are discerning in accepting and using new information to make decisions, change practices or inform strategies. They use reliable research process to search for new information. They evaluate the validity of sources when considering the use and adoption of external information or practices in their workplace situation.
Career Ready Practices

CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.
Career-ready individuals readily recognize problems in the workplace, understand the nature of the problem, and devise effective plans to solve the problem. They are aware of problems when they occur and take action quickly to address the problem; they thoughtfully investigate the root cause of the problem prior to introducing solutions. They carefully consider the options to solve the problem. Once a solution is agreed upon, they follow through to ensure the problem is solved, whether through their own actions or the actions of others.

CRP9. Model integrity, ethical leadership and effective management.
Career-ready individuals consistently act in ways that align personal and community-held ideals and principles while employing strategies to positively influence others in the workplace. They have a clear understanding of integrity and act on this understanding in every decision. They use a variety of means to positively impact the directions and actions of a team or organization, and they apply insights into human behavior to change others’ action, attitudes and/or beliefs. They recognize the near-term and long-term effects that management’s actions and attitudes can have on productivity, morals and organizational culture.

CRP10. Plan education and career paths aligned to personal goals.
Career-ready individuals take personal ownership of their own education and career goals, and they regularly act on a plan to attain these goals. They understand their own career interests, preferences, goals, and requirements. They have perspective regarding the pathways available to them and the time, effort, experience and other requirements to pursue each, including a path of entrepreneurship. They recognize the value of each step in the education and experiential process, and they recognize that nearly all career paths require ongoing education and experience. They seek counselors, mentors, and other experts to assist in the planning and execution of career and personal goals.

CRP11. Use technology to enhance productivity.
Career-ready individuals find and maximize the productive value of existing and new technology to accomplish workplace tasks and solve workplace problems. They are flexible and adaptive in acquiring new technology. They are proficient with ubiquitous technology applications. They understand the inherent risks-personal and organizational-of technology applications, and they take actions to prevent or mitigate these risks.
**Career Ready Practices**

<table>
<thead>
<tr>
<th>CRP12. Work productively in teams while using cultural global competence.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Career-ready individuals positively contribute to every team, whether formal or informal. They apply an awareness of cultural difference to avoid barriers to productive and positive interaction. They find ways to increase the engagement and contribution of all team members. They plan and facilitate effective team meetings.</td>
</tr>
</tbody>
</table>
## Differentiated Instruction

### Strategies to Accommodate Students Based on Individual Needs

<table>
<thead>
<tr>
<th>Time/General</th>
<th>Processing</th>
<th>Comprehension</th>
<th>Recall</th>
<th>Assistive Technology</th>
<th>Tests/Quizzes/Grading</th>
<th>Behavior/Attention</th>
<th>Organization</th>
</tr>
</thead>
<tbody>
<tr>
<td>Extra time for assigned tasks</td>
<td>Extra Response time</td>
<td>Precise step-by-step directions</td>
<td>Teacher-made checklist</td>
<td>Computer/whiteboard</td>
<td>Extended time</td>
<td>Consistent daily structured routine</td>
<td>Individual daily planner</td>
</tr>
<tr>
<td>Adjust length of assignment</td>
<td>Have students verbalize steps</td>
<td>Short manageable tasks</td>
<td>Use visual graphic organizers</td>
<td>Tape recorder</td>
<td>Study guides</td>
<td>Simple and clear classroom rules</td>
<td>Display a written agenda</td>
</tr>
<tr>
<td>Timeline with due dates for reports and projects</td>
<td>Repeat, clarify or reword directions</td>
<td>Brief and concrete directions</td>
<td>Reference resources to promote independence</td>
<td>Spell-checker</td>
<td>Shortened tests</td>
<td>Frequent feedback</td>
<td>Note-taking assistance</td>
</tr>
<tr>
<td>Communication system between home and school</td>
<td>Mini-breaks between tasks</td>
<td>Provide immediate feedback</td>
<td>Visual and verbal reminders</td>
<td>Audio-taped books</td>
<td>Read directions aloud</td>
<td></td>
<td>Color code materials</td>
</tr>
<tr>
<td>Provide lecture notes/outline</td>
<td>Provide a warning for transitions</td>
<td>Small group instruction</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Reading partners</td>
<td>Emphasize multi-sensory learning</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Tests/Quizzes/Grading
- Extended time
- Study guides
- Shortened tests
- Read directions aloud

### Behavior/Attention
- Consistent daily structured routine
- Simple and clear classroom rules
- Frequent feedback

### Organization
- Individual daily planner
- Display a written agenda
- Note-taking assistance
- Color code materials
Enrichment

Strategies Used to Accommodate Based on Students Individual Needs:

- Adaption of Material and Requirements
- Evaluate Vocabulary
- Elevated Text Complexity
- Additional Projects
- Independent Student Options
- Projects completed individual or with Partners
- Self Selection of Research
- Tiered/Multilevel Activities
- Learning Centers
- Individual Response Board
- Independent Book Studies
- Open-ended activities
- Community/Subject expert mentorships
## Assessments

**Suggested Formative/Summative Classroom Assessments**

- Timelines, Maps, Charts, Graphic Organizers
- Teacher-created Unit Assessments, Chapter Assessments, Quizzes
- Teacher-created DBQs, Essays, Short Answer
- Accountable Talk, Debate, Oral Report, Role Playing, Think Pair, and Share
- Projects, Portfolio, Presentations, Prezi, Gallery Walks
- Homework
- Concept Mapping
- Primary and Secondary Source analysis
- Photo, Video, Political Cartoon, Radio, Song Analysis
- Create an Original Song, Film, or Poem
- Glogster to make Electronic Posters
- Internal and External IB Assessments
## Interdisciplinary Connections

<table>
<thead>
<tr>
<th><strong>English Language Arts</strong></th>
<th><strong>Math</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>• Journal writing</td>
<td>• Research industry salaries for a geographic area and juxtapose against local cost of living</td>
</tr>
<tr>
<td>• Close reading of industry-related content</td>
<td>• Go on a geometry scavenger hunt</td>
</tr>
<tr>
<td>• Create a brochure for a specific industry</td>
<td>• Track and track various data, such as industry’s impact on the GDP, career opportunities or among of individuals currently occupying careers</td>
</tr>
<tr>
<td>• Keep a running word wall of industry vocabulary</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Social Studies</strong></th>
<th><strong>Fine &amp; Performing Arts</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>• Research the history of a given industry/profession</td>
<td>• Create a poster recruiting young people to focus their studies on a specific career or industry</td>
</tr>
<tr>
<td>• Research prominent historical individuals in a given industry/profession</td>
<td>• Design a flag or logo to represent a given career field</td>
</tr>
<tr>
<td>• Use historical references to solve problems</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>World Language</strong></th>
<th><strong>Science</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>• Translate industry-content</td>
<td>• Research the environmental impact of a given career or industry</td>
</tr>
<tr>
<td>• Create a translated index of industry vocabulary</td>
<td>• Research latest developments in industry technology</td>
</tr>
<tr>
<td>• Generate a translated list of words and phrases related to workplace safety</td>
<td>• Investigate applicable-careers in STEM fields</td>
</tr>
</tbody>
</table>
Common Core State Standards (CCSS)

F.IF.A.1: Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If \( f \) is a function and \( x \) is an element of its domain, then \( f(x) \) denotes the output of \( f \) corresponding to the input \( x \).

F.IF.A.2: Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

F.BF.B.3: Identify the effect on the graph of replacing \( f(x) \) by \( f(x) + k \), \( k f(x) \), \( f(kx) \), and \( f(x + k) \) for specific values of \( k \) (both positive and negative); find the value of \( k \) given the graphs.

F.IF.B.4: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

F.IF.B.6: Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval.

F.IF.C.7: Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

F.LE.B.5: Interpret the parameters in a linear or exponential function in terms of a context.

G.MG.A.3: Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).
**Mathematical Practices:**

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Communicate the precise answer to a real-world problem.
8. Look for and make use of structure.
9. Look for and express regularity in repeated reasoning. All of the content presented in this course has connections to the standards for mathematical practices.
### Course: Mathematical Studies
### Unit: 4
### Grade Level: 11

#### Unit Overview: Introduction to Differential Calculus
Differential Calculus is a subfield of calculus concerned with the study of the rates at which quantities change. The aim of this unit is to introduce the concept of the derivative of a function and to apply it to optimization and other problems.

#### New Jersey Core Curriculum Content Standards (NJCCCS):
- F.IF.A.1
- F.IF.A.2
- F.BF.B.3
- F.BF.B.4
- F.BF.B.6
- F.IF.C.7
- F.LE.B5
- G.MG.A.3

---

### Student Learning Objectives (SLOs)

| Analyze increasing functions and the graphical interpretation of \( f'(x) > 0 \), \( f'(x) = 0 \) and \( f'(x) < 0 \). | **CSSS:**
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>F.IF.A.1</td>
<td>F.IF.A.2</td>
</tr>
<tr>
<td>F.IF.C.7</td>
<td></td>
</tr>
</tbody>
</table>

#### Essential Questions
- What can you learn about the shape of a graph by finding it’s derivative at a point?
- When is a function differentiable?

#### Content
- Increasing and decreasing functions
- Graphical interpretations
- Finding equations of derivatives

#### Activities & Assessments
- **Activity:** Tangent line Roller Coaster Task
- Summative and Formative Assessments (Quizzes & Tests) for each topic.
- Homework and Classwork assignments based on daily lessons.

#### Resources
- Textbook: Mathematical Studies by Patrick Tobin
- Textbook: Mathematical Studies SL for the IB Diploma by Caroline Meyrick and Kwame Dwamena
- Geometer’s Sketchpad
- Desmos Online Graphing Calculator
- Texas Instruments TI-84 Plus Graphing Display Calculator
- IB Questionbank [www.occ.ibo.org](http://www.occ.ibo.org)

---

| Analyze decreasing functions and the graphical interpretation of \( f''(x) > 0 \), \( f'(x) = 0 \) and \( f'(x) < 0 \). | **CSSS:**
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>F.IF.A.1</td>
<td>F.IF.A.2</td>
</tr>
<tr>
<td>F.IF.C.7</td>
<td></td>
</tr>
</tbody>
</table>

#### Essential Questions
- Why does a derivative tell us how quickly or slowly the slope of a lines is changing?
- Finding velocity and acceleration given a displacement equation.

#### Content
- Finding velocity and acceleration given a displacement equation.

#### Activities & Assessments
- **Activity:** Roller Coaster Instantaneous Rate of Change and Acceleration.
| CSSS: F.IF.A.1 | Stationary/turning points of a function | Summative and Formative Assessments (Quizzes & Tests) for each topic. |
| CSSS: F.IF.A.2 |  | Homework and Classwork assignments based on daily lessons. |
| CSSS: F.IF.B.4 |  |  |

Formulate values of x where the gradient of a curve is zero; solution of $f'(x) = 0$

CSSS: F.IF.B.6
F.BF.B.3
F.IF.C.7

Can the average and instantaneous rates of change be compared? How can the slope of a line be interpreted?

Strategies for finding values of x when gradient of a curve is 0.
Why the solution of the equations $f'(x) = 0$ is important.

Lab: The Profit Function
Summative and Formative Assessments (Quizzes & Tests) for each topic.
Homework and Classwork assignments based on daily lessons.

What is an extrema and how can calculus be used to find them?

What stationary points are.
How to find stationary points are.
Isolate turning points
Finding local maximum & minimum
Points of inflexion

Activity: Spring Break Trip
Summative and Formative Assessments (Quizzes & Tests) for each topic.
Homework and Classwork assignments based on daily lessons.

Formulate values of x where the gradient of a curve is zero; stationary points and at local maximum and minimum points.

CSSS: F.IF.B.6
F.BF.B.3
F.IF.B.4
| F.IF.C.7 | Use second derivative to classify stationary points. | Do the domain values determine the limits? How can the first and second derivatives be used as aides in graphing functions? | Calculate second derivatives Classify stationary points from second derivative | Group Activity: Second Derivative to Classify Stationary Points. Summative and Formative Assessments (Quizzes & Tests) for each topic. Homework and Classwork assignments based on daily lessons. |
| Create and analyze optimization problems. | How can the derivative be applied to optimization problems? What are related rates problems? How can derivatives be used to solve multivariable problems where several variables change with respect to time? | Use calculus to solve optimization problems that involve maximizing or minimizing a certain quantity. | Activity: M&M’s in the box (maximum volume problem) Summative and Formative Assessments (Quizzes & Tests) for each topic. Homework and Classwork assignments based on daily lessons. |
TOK Connections

Is intuition a valid way of knowing in math?
How is it possible to reach the same conclusion from different research paths?
Can the development of Mathematics be thought of as a straight line or is it more like a tree diagram?
Does a graph without labels or scales still have meaning?

Unit 4 Vocabulary

| Normal   | Chain rule                  |
| Stationary point | Critical number          |
| Turning point     | Definite integral         |
| Minimum point/Minima | Even or odd function   |
| Maximum point/Maxima | First fundamental theorem of calculus |
| Point of inflexion | Second fundamental theorem of calculus |
| Optimization     | Remainder theorem         |
| Related rate     | Second derivative test    |
| Discontinuity    | Regression transformations of functions |
| Reiemann sum     | Rolle’s theorem           |
| Rolle’s theorem  |                           |
### Contribution to the Development of Students’ Approached to Learning Skills

Students exercise initiative in applying thinking skills critically and creatively to recognize and approach complex problems, and make reasoned ethical decisions. Inquiry approaches: investigate unfamiliar situations, both abstract and real-world, involving organizing and analyzing information or measurements, drawing conclusions, testing validity, and considering their scope and limitations.

### Contributions to the Development of the Attribute(s) of the Learner Profile

Students exercise initiative in applying thinking skills critically and creatively to recognize and approach complex problems, and make reasoned ethical decisions. Inquiry approaches: investigate unfamiliar situations, both abstract and real-world, involving organizing and analyzing information or measurements, drawing conclusions, testing validity, and considering their scope and limitations.

### Contribution to the Development of International Mindedness

Students must require the skills to better prepare for the 21st century global challenges. Sustained inquiry leads to exploration, reflection and responsible action. Connections are applied between the classroom processes with significant local and global changes.