Department of Accelerated Programs

Mathematical Studies SL Curriculum

10.0 Credits
# IB LEARNER PROFILE

IB Programs aim to develop internationally minded people who are striving to become:

<table>
<thead>
<tr>
<th>Category</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inquirers</td>
<td>Their natural curiosity is nurtured. They acquire the skills necessary to conduct constructive inquiry and research, and become independent active learners. They actively enjoy learning and this love of learning will be sustained throughout their lives.</td>
</tr>
<tr>
<td>Knowledgeable</td>
<td>They explore concepts, ideas and issues, which have global relevance and importance. In so doing, they acquire, and are able to make use of, a significant body of knowledge across a range of disciplines.</td>
</tr>
<tr>
<td>Critical thinkers</td>
<td>They exercise initiative in applying thinking skills critically and creatively to approach complex problems and make reasoned decisions.</td>
</tr>
<tr>
<td>Communicators</td>
<td>They understand and express ideas and information confidently and creatively in more than one language and in a variety of modes of communication.</td>
</tr>
<tr>
<td>Risk-takers</td>
<td>They approach unfamiliar situations with confidence and forethought, and have the independence of spirit to explore new roles, ideas and strategies. They are courageous and articulate in defending those things in which they believe.</td>
</tr>
<tr>
<td>Principled</td>
<td>They have a sound grasp of the principles of moral reasoning. They have integrity, honesty, a sense of fairness and justice and respect for the dignity of the individual.</td>
</tr>
<tr>
<td>Caring</td>
<td>They show empathy, compassion and respect towards the needs and feelings of others. They have a personal commitment to action and service to make a positive difference to the environment and to the lives of others.</td>
</tr>
<tr>
<td>Open-minded</td>
<td>Through an understanding and appreciation of their own culture, they are open to the perspectives, values and traditions of other individuals and cultures and are accustomed to seeking and considering a range of points of view.</td>
</tr>
<tr>
<td>Well-balanced</td>
<td>They understand the importance of physical and mental balance and personal well being for themselves and others. They demonstrate perseverance and self-discipline.</td>
</tr>
<tr>
<td>Reflective</td>
<td>They give thoughtful consideration to their own learning and personal development. They are able to analyze their strengths and weaknesses in a constructive manner.</td>
</tr>
</tbody>
</table>
Unit Three
Mathematical Studies Year 2

Course Description

Mathematical Studies is a course with an emphasis on applications of mathematics. It is for students with varied backgrounds and abilities. It offers students opportunities to learn important concepts and techniques and to gain an understanding of a wide variety of mathematical topics. It prepares students to be able to solve problems in a variety of settings, to develop more sophisticated mathematical reasoning and to enhance their critical thinking. The individual project is an extended piece of work based on personal research involving the collection, analysis and evaluation of data. Students taking this course are well prepared for a career in social sciences, humanities, languages or arts.
## Mathematical Studies Year 2

### Pacing Guide

<table>
<thead>
<tr>
<th>Unit</th>
<th>Topic</th>
<th>Suggested Timing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unit 1</td>
<td>Geometry and Trigonometry</td>
<td>8 Weeks</td>
</tr>
<tr>
<td>Unit 2</td>
<td>Mathematical Models</td>
<td>9 Weeks</td>
</tr>
<tr>
<td>Unit 3</td>
<td>Introduction to Differential Calculus 1</td>
<td>8 Weeks</td>
</tr>
<tr>
<td>Unit 4</td>
<td>Introduction to Differential Calculus 2</td>
<td>7 Weeks</td>
</tr>
</tbody>
</table>
Educational Technology Standards

- **Technology Operations and Concepts**
  - Create a personal digital portfolio which reflects personal and academic interests, achievements, and career aspirations by using a variety of digital tools and resources.
  - Produce and edit a multi-page digital document for a commercial or professional audience and present it to peers and/or professionals in that related area for review.

- **Creativity and Innovation**
  - Apply previous content knowledge by creating and piloting a digital learning game or tutorial.

- **Communication and Collaboration**
  - Develop an innovative solution to a real world problem or issue in collaboration with peers and experts, and present ideas for feedback through social media or in an online community.

- **Digital Citizenship**
  - Demonstrate appropriate application of copyright, fair use and/or Creative Commons to an original work.
  - Evaluate consequences of unauthorized electronic access and disclosure, and on dissemination of personal information.
  - Compare and contrast policies on filtering and censorship both locally and globally.

- **Research and Information Literacy**
  - Produce a position statement about a real world problem by developing a systematic plan of investigation with peers and experts synthesizing information from multiple sources.

- **Critical Thinking, Problem Solving, Decision Making**
  - Evaluate the strengths and limitations of emerging technologies and their impact on educational, career, personal and or social needs.
# Career Ready Practices

Career Ready Practices describe the career-ready skills that all educators in all content areas should seek to develop in their students. They are practices that have been linked to increase college, career, and life success. Career Ready Practices should be taught and reinforced in all career exploration and preparation programs with increasingly higher levels of complexity and expectation as a student advances through a program of study.

<table>
<thead>
<tr>
<th>CRP1. Act as a responsible and contributing citizen and employee</th>
</tr>
</thead>
<tbody>
<tr>
<td>Career-ready individuals understand the obligations and responsibilities of being a member of a community, and they demonstrate this understanding every day through their interactions with others. They are conscientious of the impacts of their decisions on others and the environment around them. They think about the near-term and long-term consequences of their actions and seek to act in ways that contribute to the betterment of their teams, families, community and workplace. They are reliable and consistent in going beyond the minimum expectation and in participating in activities that serve the greater good.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>CRP2. Apply appropriate academic and technical skills.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Career-ready individuals readily access and use the knowledge and skills acquired through experience and education to be more productive. They make connections between abstract concepts with real-world applications, and they make correct insights about when it is appropriate to apply the use of an academic skill in a workplace situation.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>CRP3. Attend to personal health and financial well-being.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Career-ready individuals understand the relationship between personal health, workplace performance and personal well-being; they act on that understanding to regularly practice healthy diet, exercise and mental health activities. Career-ready individuals also take regular action to contribute to their personal financial wellbeing, understanding that personal financial security provides the peace of mind required to contribute more fully to their own career success.</td>
</tr>
</tbody>
</table>
**Career Ready Practices**

**CRP4. Communicate clearly and effectively and with reason.**
Career-ready individuals communicate thoughts, ideas, and action plans with clarity, whether using written, verbal, and/or visual methods. They communicate in the workplace with clarity and purpose to make maximum use of their own and others’ time. They are excellent writers; they master conventions, word choice, and organization, and use effective tone and presentation skills to articulate ideas. They are skilled at interacting with others; they are active listeners and speak clearly and with purpose. Career-ready individuals think about the audience for their communication and prepare accordingly to ensure the desired outcome.

**CRP5. Consider the environmental, social and economic impacts of decisions.**
Career-ready individuals understand the interrelated nature of their actions and regularly make decisions that positively impact and/or mitigate negative impact on other people, organization, and the environment. They are aware of and utilize new technologies, understandings, procedures, materials, and regulations affecting the nature of their work as it relates to the impact on the social condition, the environment and the profitability of the organization.

**CRP6. Demonstrate creativity and innovation.**
Career-ready individuals regularly think of ideas that solve problems in new and different ways, and they contribute those ideas in a useful and productive manner to improve their organization. They can consider unconventional ideas and suggestions as solutions to issues, tasks or problems, and they discern which ideas and suggestions will add greatest value. They seek new methods, practices, and ideas from a variety of sources and seek to apply those ideas to their own workplace. They take action on their ideas and understand how to bring innovation to an organization.

**CRP7. Employ valid and reliable research strategies.**
Career-ready individuals are discerning in accepting and using new information to make decisions, change practices or inform strategies. They use reliable research process to search for new information. They evaluate the validity of sources when considering the use and adoption of external information or practices in their workplace situation.
Career Ready Practices

**CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.**
Career-ready individuals readily recognize problems in the workplace, understand the nature of the problem, and devise effective plans to solve the problem. They are aware of problems when they occur and take action quickly to address the problem; they thoughtfully investigate the root cause of the problem prior to introducing solutions. They carefully consider the options to solve the problem. Once a solution is agreed upon, they follow through to ensure the problem is solved, whether through their own actions or the actions of others.

**CRP9. Model integrity, ethical leadership and effective management.**
Career-ready individuals consistently act in ways that align personal and community-held ideals and principles while employing strategies to positively influence others in the workplace. They have a clear understanding of integrity and act on this understanding in every decision. They use a variety of means to positively impact the directions and actions of a team or organization, and they apply insights into human behavior to change others’ action, attitudes and/or beliefs. They recognize the near-term and long-term effects that management’s actions and attitudes can have on productivity, morals and organizational culture.

**CRP10. Plan education and career paths aligned to personal goals.**
Career-ready individuals take personal ownership of their own education and career goals, and they regularly act on a plan to attain these goals. They understand their own career interests, preferences, goals, and requirements. They have perspective regarding the pathways available to them and the time, effort, experience and other requirements to pursue each, including a path of entrepreneurship. They recognize the value of each step in the education and experiential process, and they recognize that nearly all career paths require ongoing education and experience. They seek counselors, mentors, and other experts to assist in the planning and execution of career and personal goals.

**CRP11. Use technology to enhance productivity.**
Career-ready individuals find and maximize the productive value of existing and new technology to accomplish workplace tasks and solve workplace problems. They are flexible and adaptive in acquiring new technology. They are proficient with ubiquitous technology applications. They understand the inherent risks-personal and organizational-of technology applications, and they take actions to prevent or mitigate these risks.
## Career Ready Practices

**CRP12. Work productively in teams while using cultural global competence.**
Career-ready individuals positively contribute to every team, whether formal or informal. They apply an awareness of cultural difference to avoid barriers to productive and positive interaction. They find ways to increase the engagement and contribution of all team members. They plan and facilitate effective team meetings.
## Differentiated Instruction

**Strategies to Accommodate Students Based on Individual Needs**

<table>
<thead>
<tr>
<th>Time/General</th>
<th>Processing</th>
<th>Comprehension</th>
<th>Recall</th>
</tr>
</thead>
<tbody>
<tr>
<td>Extra time for assigned tasks</td>
<td>Extra Response time</td>
<td>Precise step-by-step directions</td>
<td>Teacher-made checklist</td>
</tr>
<tr>
<td>Adjust length of assignment</td>
<td>Have students verbalize steps</td>
<td>Short manageable tasks</td>
<td>Use visual graphic organizers</td>
</tr>
<tr>
<td>Timeline with due dates for reports and projects</td>
<td>Repeat, clarify or reword directions</td>
<td>Brief and concrete directions</td>
<td>Reference resources to promote independence</td>
</tr>
<tr>
<td>Communication system between home and school</td>
<td>Mini-breaks between tasks</td>
<td>Provide immediate feedback</td>
<td>Visual and verbal reminders</td>
</tr>
<tr>
<td>Provide lecture notes/outline</td>
<td>Provide a warning for transitions</td>
<td>Small group instruction</td>
<td>Graphic organizers</td>
</tr>
<tr>
<td></td>
<td>Reading partners</td>
<td>Emphasize multi-sensory learning</td>
<td></td>
</tr>
</tbody>
</table>

### Assistive Technology
- Computer/whiteboard
- Tape recorder
- Spell-checker
- Audio-taped books

### Tests/Quizzes/Grading
- Extended time
- Study guides
- Shortened tests
- Read directions aloud

### Behavior/Attention
- Consistent daily structured routine
- Simple and clear classroom rules
- Frequent feedback

### Organization
- Individual daily planner
- Display a written agenda
- Note-taking assistance
- Color code materials
## Enrichment

Strategies Used to Accommodate Based on Students Individual Needs:

- Adaptation of Material and Requirements
- Evaluate Vocabulary
- Elevated Text Complexity
- Additional Projects
- Independent Student Options
- Projects completed individual or with Partners
- Self Selection of Research
- Tiered/Multilevel Activities
- Learning Centers
- Individual Response Board
- Independent Book Studies
- Open-ended activities
- Community/Subject expert mentorships
### Assessments

**Suggested Formative/Summative Classroom Assessments**

- Timelines, Maps, Charts, Graphic Organizers
- Teacher-created Unit Assessments, Chapter Assessments, Quizzes
- Teacher-created DBQs, Essays, Short Answer
- Accountable Talk, Debate, Oral Report, Role Playing, Think Pair, and Share
- Projects, Portfolio, Presentations, Prezi, Gallery Walks
- Homework
- Concept Mapping
- Primary and Secondary Source analysis
- Photo, Video, Political Cartoon, Radio, Song Analysis
- Create an Original Song, Film, or Poem
- Glogster to make Electronic Posters
- Internal and External IB Assessments
# Interdisciplinary Connections

## English Language Arts
- Journal writing
- Close reading of industry-related content
- Create a brochure for a specific industry
- Keep a running word wall of industry vocabulary

## Social Studies
- Research the history of a given industry/profession
- Research prominent historical individuals in a given industry/profession
- Use historical references to solve problems

## World Language
- Translate industry-content
- Create a translated index of industry vocabulary
- Generate a translated list of words and phrases related to workplace safety

## Math
- Research industry salaries for a geographic area and juxtapose against local cost of living
- Go on a geometry scavenger hunt
- Track and track various data, such as industry’s impact on the GDP, career opportunities or among of individuals currently occupying careers

## Fine & Performing Arts
- Create a poster recruiting young people to focus their studies on a specific career or industry
- Design a flag or logo to represent a given career field

## Science
- Research the environmental impact of a given career or industry
- Research latest developments in industry technology
- Investigate applicable-careers in STEM fields
Common Core State Standards (CCSS)

F.IF.A.1: Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x.

F.IF.A.2: Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

F.BF.B.3: Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs.

F.IF.B.4: For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.

F.IF.B.6: Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval.

F.IF.C.7: Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

F.LE.A.1: Distinguish between situations that can be modeled with linear functions and with exponential functions.

F.LE.B.5: Interpret the parameters in a linear or exponential function in terms of a context.

G.GMD.A.1: Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalier’s principle, and informal limit arguments.
**Mathematical Practices:**

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Communicate the precise answer to a real-world problem.
8. Look for and make use of structure.
9. Look for and express regularity in repeated reasoning. All of the content presented in this course has connections to the standards for mathematical practices.
**Course:** Mathematical Studies  
**Unit:** 3  
**Grade Level:** 11

**Unit Overview:** Introduction to Differential Calculus 1  
Differential Calculus is a subfield of calculus concerned with the study of the rates at which quantities change. The aim of this unit is to introduce the concept of the derivative of a function and to apply it to optimization and other problems.

**New Jersey Core Curriculum Content Standards (NJCCCS):**  
F.IF.A.1, F.IF.A.2, F.BF.B.3, F.BF.B.4, F.BF.B.6, F.IF.C.7  
F.LE.A.1, F.LE.B5 and G.GMD.A.1

<table>
<thead>
<tr>
<th>Student Learning Objectives (SLOs)</th>
<th>Essential Questions</th>
<th>Content</th>
<th>Activities &amp; Assessments</th>
<th>Resources</th>
</tr>
</thead>
</table>
| Demonstrate an understanding of the formal definition of the derivative of a function at a point and the notion of differentiability.  
**CSSS:** F.IF.A.1 F.IF.A.2 | Can derivatives describe the rate of change?  
What strategies can be explored to identify the derivative of a function as the limit of a difference quotient?  
➢ The two concepts of derivatives: 1) as a rate of change of a function and 2) as a gradient of a graph  
➢ The concept of the gradient of a curve as the gradient of a tangent line |  
**Activity:** Why a Derivative?  
Summative and Formative Assessments (Quizzes & Tests) for each topic.  
Homework and Classwork assignments based on daily lessons. |  
• Textbook: Mathematical Studies by Patrick Tobin  
• Textbook: Mathematical Studies SL for the IB Diploma by Caroline Meyrick and Kwame Dwamena  
• Geometer’s Sketchpad  
• Desmos Online Graphing Calculator  
• Texas Instruments TI- |

<p>| Examine the principle that ( f(x) = ax^n \Rightarrow f'(x) = ) | How can a function’s derivative, which is itself | <strong>Group Activity:</strong> Power to the Power rule |  |</p>
<table>
<thead>
<tr>
<th>Student Learning Objectives (SLOs)</th>
<th>Essential Questions</th>
<th>Content</th>
<th>Activities &amp; Assessments</th>
<th>Resources</th>
</tr>
</thead>
<tbody>
<tr>
<td>CSSS: F.IF.A.1 F.IF.A.2 F.LE.A.1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>The derivative of functions of the form ( f(x) = ax^n + bx^{n-1} + \cdots ), where all exponents are integers.</td>
<td>Determine the key features of functions and their derivatives can be identified and related to their graphical, numerical and analytical representations. How is a derivative related to velocity and acceleration?</td>
<td>➢ How to find the derivative of functions of the form ( f(x) = ax^n + bx^m + \cdots ) where all exponents are integers.</td>
<td>Lab: Finding Derivatives with Technology. Summative and Formative Assessments (Quizzes &amp; Tests) for each topic. Homework and Classwork assignments based on daily lessons.</td>
<td></td>
</tr>
<tr>
<td>CSSS: F.BF.B.3 F.IF.B.4 F.LE.A.1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Use dissection arguments, Cavalier’s principle, and informal limit arguments.</td>
<td>How can the concept of a limit be used to understand the behavior of functions? How can numerical and</td>
<td>➢ Cavalier’s principle ➢ Informal limit arguments ➢ Understand the concepts of limits ➢ Marginal cost</td>
<td>Activity: Why so Limiting? Summative and Formative Assessments (Quizzes &amp; Tests) for</td>
<td></td>
</tr>
<tr>
<td>Student Learning Objectives (SLOs)</td>
<td>Essential Questions</td>
<td>Content</td>
<td>Activities &amp; Assessments</td>
<td>Resources</td>
</tr>
<tr>
<td>-----------------------------------</td>
<td>---------------------</td>
<td>---------</td>
<td>--------------------------</td>
<td>-----------</td>
</tr>
<tr>
<td>G.GMD.A.1 F.IF.B.6 F.IF.C.7</td>
<td>Graphical information be used to estimate limits?</td>
<td>➢ The concept of a limit can be extended to include one-sided limits, limits at infinity, and infinite limits</td>
<td>each topic.</td>
<td><a href="https://www.youtube.com/watch?v=CYp824FeJ9s">https://www.youtube.com/watch?v=CYp824FeJ9s</a></td>
</tr>
<tr>
<td>Explor [ ] the gradients of curves for given values of x where ( f'(x) ) is given.</td>
<td>What role do derivatives and limits play as a foundation for calculus? What role do derivatives and limits play in practical applications?</td>
<td>➢ How to find the gradient of a curve at given values of x. ➢ How to find the values of x at which a curve has a given gradient.</td>
<td>Lab: It’s not just lines anymore. Summative and Formative Assessments (Quizzes &amp; Tests) for each topic. Homework and Classwork assignments based on daily lessons.</td>
<td><a href="https://ocw.mit.edu/resources/res-18-001-calculus-online.../textbook">https://ocw.mit.edu/resources/res-18-001-calculus-online.../textbook</a></td>
</tr>
<tr>
<td>CSSS: F.IF.B.6 F.IF.C.7</td>
<td>Calculate the gradient of curves and compute the equation of the tangent at a given point including the equation of the line perpendicular to the tangent.</td>
<td>How can limits be used to find the instantaneous rate of change of a function at a point</td>
<td>➢ How to find the equation of a tangent at a given point on a curve ➢ How to find the equation of the line perpendicular to tangent at a given point (the normal).</td>
<td><a href="https://www.wyzant.com/resources/lessons/math/calculus/differentiation/list_of_derivatives">https://www.wyzant.com/resources/lessons/math/calculus/differentiation/list_of_derivatives</a></td>
</tr>
<tr>
<td>CSSS: F.IF.B.6 F.IF.C.7</td>
<td>How can limits be used to find the instantaneous rate of change of a function at a point</td>
<td>➢ How to find the equation of a tangent at a given point on a curve ➢ How to find the equation of the line perpendicular to tangent at a given point (the normal).</td>
<td>Activity: Is Math for Real? (TOK connections) Summative and Formative Assessments (Quizzes &amp; Tests) for each topic. Homework and Classwork assignments based on daily lessons.</td>
<td></td>
</tr>
</tbody>
</table>
### Unit 3 Vocabulary

<table>
<thead>
<tr>
<th>Calculus</th>
<th>Marginal cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Derivative</td>
<td>Second derivative</td>
</tr>
<tr>
<td>Gradient of a curve</td>
<td>Second derivative test</td>
</tr>
<tr>
<td>Gradient function</td>
<td>Velocity</td>
</tr>
<tr>
<td>Differentiation</td>
<td>Displacement</td>
</tr>
<tr>
<td>Tangent line</td>
<td>Acceleration</td>
</tr>
<tr>
<td>Secant line</td>
<td>Power rule</td>
</tr>
<tr>
<td>Antiderivative</td>
<td>Product rule</td>
</tr>
<tr>
<td>Composite function</td>
<td>Quotient rule</td>
</tr>
<tr>
<td>Concavity</td>
<td></td>
</tr>
</tbody>
</table>

### TOK Connections

- Is intuition a valid way of knowing in math?
- How is it possible to reach the same conclusion from different research paths?
- Can the development of Mathematics be thought of as a straight line or is it more like a tree diagram?
- Does a graph without labels or scales still have meaning?
## Contribution to the Development of Students’ Approached to Learning Skills

Students exercise initiative in applying thinking skills critically and creatively to recognize and approach complex problems, and make reasoned ethical decisions. Inquiry approaches: investigate unfamiliar situations, both abstract and real-world, involving organizing and analyzing information or measurements, drawing conclusions, testing validity, and considering their scope and limitations.

## Contributions to the Development of the Attribute(s) of the Learner Profile

Differential calculus is a topic with a broad range of real-world applications. The learner exercises initiative in applying thinking skills critically and creatively to recognize and approach complex problems, and make reasoned ethical decisions. Inquiry approaches: investigate unfamiliar situations, both abstract and real-world, involving organizing and analyzing information or measurements, drawing conclusions, testing validity, and considering their scope and limitations.

## Contribution to the Development of International Mindedness

Students must require the skills to better prepare for the 21st century global challenges. Sustained inquiry leads to exploration, reflection, and responsible action. Connections are applied between the classroom processes with significant local and global changes.